Formal Languages and Automata Theory
Exercises Finite Automata
Unit 3

Authors:
Araceli Sanchis de Miguel
Agapito Ledezma Espino
Jose A. Iglesias Martínez
Beatriz García Jiménez
Juan Manuel Alonso Weber

* Several exercises are based on the ones proposed in the following books:

- Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón. *Teoría de autómatas y lenguajes formales.* McGraw-Hill (2007).
- Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga. *Teoría de lenguajes, gramáticas y autómatas.* Publicaciones R.A.E.C. (1997).
- Pedro Isasi, Paloma Martínez y Daniel Borrajo. *Lenguajes, Gramáticas y Autómatas. Un enfoque práctico.* Addison-Wesley (1997).

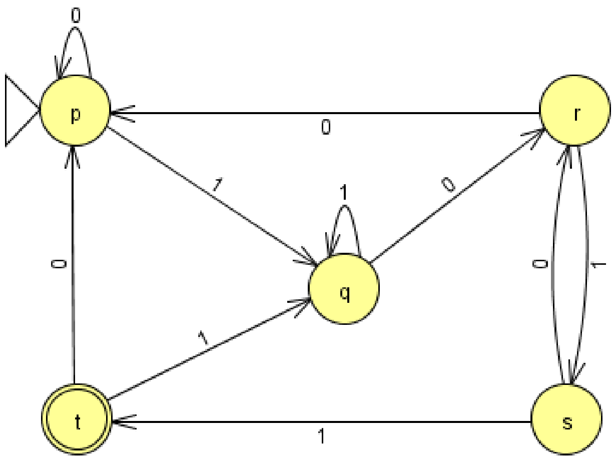
Universidad Carlos III de Madrid
www.uc3m.es

Formal Languages and Automata Theory

1. **We want to design a device that, given a string which consists of binary numbers, will be able to find if the keyword “1011” is included in the input string and it also would be used as a basis to count the number of times this keyword is included. For instance, for the input string 0101011011011, the device would detect two occurrences of the keyword (the “1” in the seventh position is not considered as the beginning of a new apparition). It is required to design the corresponding DFA.**

<u>Solution</u>:

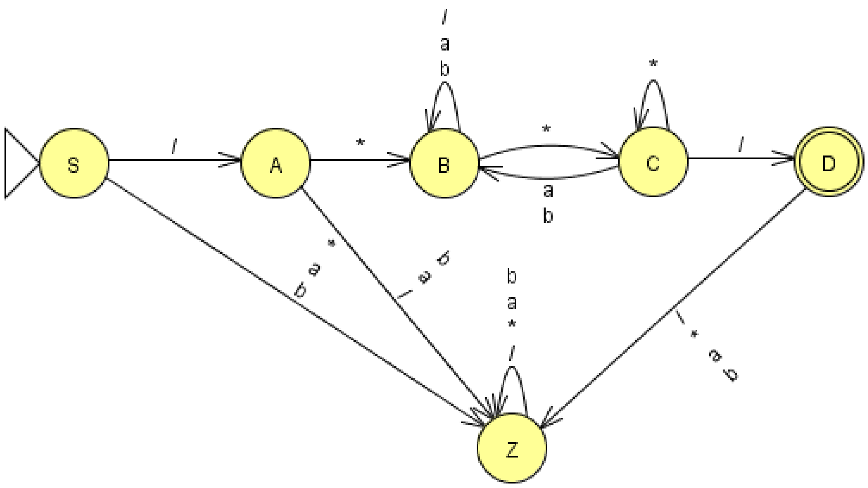
DFA=({0,1},{p,q,r,s,t},f,p,{t}), where f:



2. **In several programming languages, comments are included between the marks “/*” and “*/”. Let L be the language of every string of comments limited by these marks. Then, every element in L begins /* and ends with */, but it does not include any intermediate */. To simplify the problem, consider that the input alphabet is {a, b, /,*}. Indicate the DFA which recognizes L.**

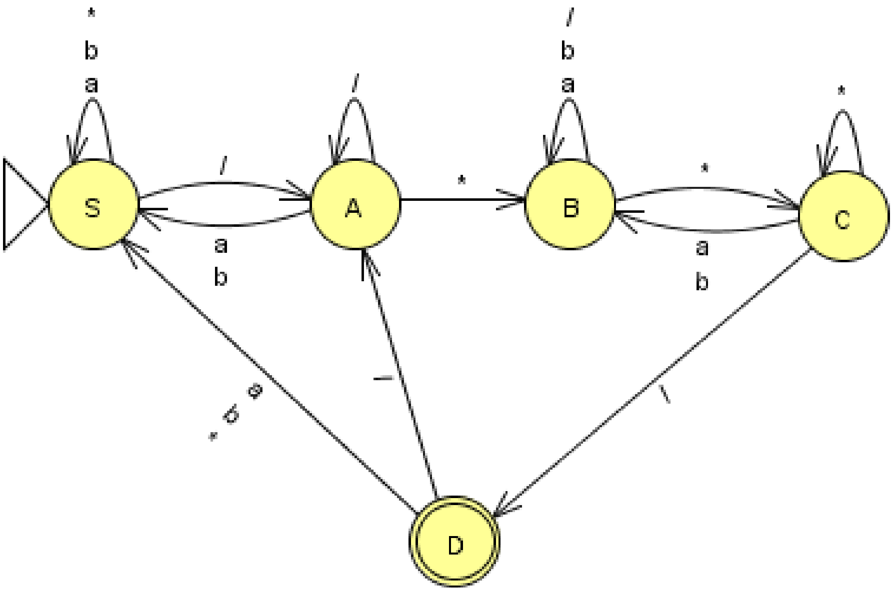
<u>Solution</u>:

DFA$_1$=({/,*,a,b},{S,A,B,C,D,Z},f,S,{D}), where f:



<u>Alternative solution</u>:

DFA$_2$=({/,*,a,b},{S,A,B,C,D},f’,S,D), where f’:

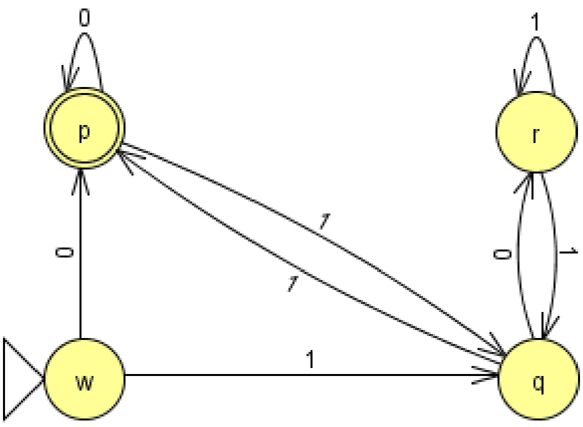


3. Design a DFA to recognize binary numbers which multiple of 3.

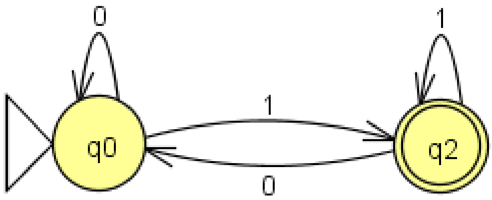
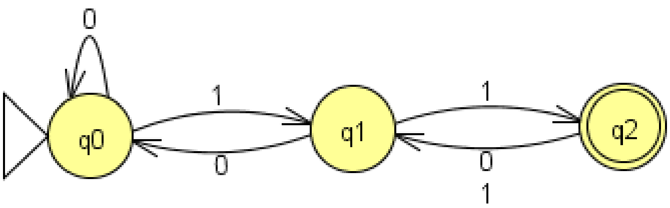
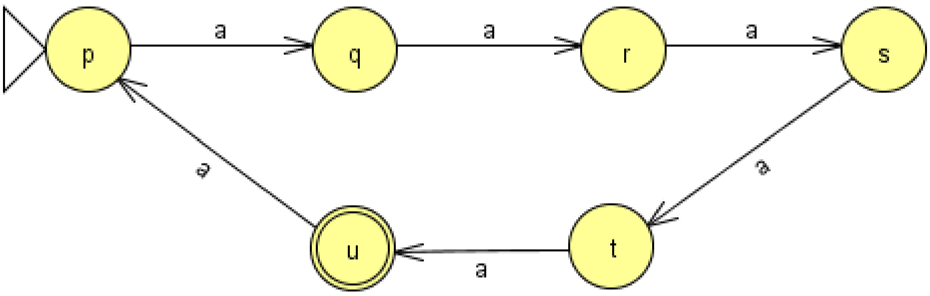
Solution:

DFA=({0,1},{p,q,r,w},f,w,p), where f:

State=t	Input=0	Input=1
→w	p	q
*p	p	q
q	q	r
r	q	r



4. Calculate the Q/E of the following automata:

a	
b	
c	

__Solution__:

a) Q/E0={{q1},{q2}}={C1,C2}=Q/E

b) Q/E0={{q0,q1},{q2}}={C1,C2}

Q/E1={{q0},{q1},{q2}}=Q/E

c) Q/E0={{p,q,r,s,t},{u}}={C1,C2}
Q/E1={{p,q,r,s},{u},{t}}={C1,C2,C3}
Q/E2={{p,q,r},{u},{t},{s}}={C1,C2,C3,C4}
Q/E3={{p,q},{u},{t},{s},{r}}={C1,C2,C3,C4,C5}
Q/E4={{p},{u},{t},{s},{r},{q}}={C1,C2,C3,C4,C5,C6}=Q/E

5. Obtain the minimal DFA for the following DFA.

DFA_1=({a,b,c},{Q0,Q1,Q2,Q3,Q4},f,Q0,Q3)

f(Q0, a) = Q1 ; f(Q0, b) = Q2 ; f(Q0, c) = Q3
f(Q1, a) = Q2 ; f(Q1, b) = Q3 ; f(Q1, c) = Q1
f(Q2, a) = Q3 ; f(Q2, b) = Q1 ; f(Q2, c) = Q3
f(Q3, a) = Q4 ; f(Q3, b) = Q4 ; f(Q3, c) = Q4
f(Q4, a) = Q4 ; f(Q4, b) = Q4 ; f(Q4, c) = Q4

DFA_2=({a,b,c}, {Q0,Q1,Q3,Q4,Q5,Q6,Q8},f,Q0,{Q3,Q4,Q6,Q8})
f(Q0, a) = Q4 ; f(Q0, b) = Q5 ; f(Q0, c) = Q1 f(Q1, a) = Q5 ; f(Q1, b) = Q5 ; f(Q1, c) = Q3 f(Q3, a) = Q5 ; f(Q3, b) = Q5 ; f(Q3, c) = Q5 f(Q4, a) = Q4 ; f(Q4, b) = Q8 ; f(Q4, c) = Q1 f(Q5, a) = Q5 ; f(Q5, b) = Q5 ; f(Q5, c) = Q5 f(Q6, a) = Q5 ; f(Q6, b) = Q8 ; f(Q6, c) = Q5 f(Q8, a) = Q5 ; f(Q8, b) = Q6 ; f(Q8, c) = Q5

DFA_3=({a,b,c},{Q0,Q1,Q2,Q3,Q4,Q6,Q7,Q8,Q9},f,Q0,{Q7,Q8})
f(Q0, a) = Q1 ; f(Q0, b) = Q6 ; f(Q0, c) = Q6 f(Q1, a) = Q7 ; f(Q1, b) = Q2 ; f(Q1, c) = Q6 f(Q7, a) = Q7 ; f(Q7, b) = Q2 ; f(Q7, c) = Q6 f(Q2, a) = Q6 ; f(Q2, b) = Q8 ; f(Q2, c) = Q6 f(Q8, a) = Q6 ; f(Q8, b) = Q8 ; f(Q8, c) = Q4 f(Q4, a) = Q6 ; f(Q4, b) = Q9 ; f(Q4, c) = Q3 f(Q9, a) = Q6 ; f(Q9, b) = Q8 ; f(Q9, c) = Q4 f(Q3, a) = Q6 ; f(Q3, b) = Q9 ; f(Q3, c) = Q4 f(Q6, a) = Q6 ; f(Q6, b) = Q6 ; f(Q6, c) = Q6

DFA_4=({c,f,d},{Q0,Q5,Q8,Q9,Q10,Q11,Q12},f,Q0,Q10)
f(Q0, c) = Q9 ; f(Q0, f) = Q10 ; f(Q0, d) = Q8 f(Q9, c) = Q9 ; f(Q9, f) = Q11 ; f(Q9, d) = Q12 f(Q10, c) = Q0 ; f(Q10, f) = Q8 ; f(Q10, d) = Q8 f(Q11, c) = Q11 ; f(Q11, f) = Q11 ; f(Q11, d) = Q8 f(Q12, c) = Q12 ; f(Q12, f) = Q5 ; f(Q12, d) = Q5 f(Q5, c) = Q5 ; f(Q5, f) = Q5 ; f(Q5, d) = Q8 f(Q8, c) = Q8 ; f(Q8, f) = Q8 ; f(Q8, d) = Q8

Solution:

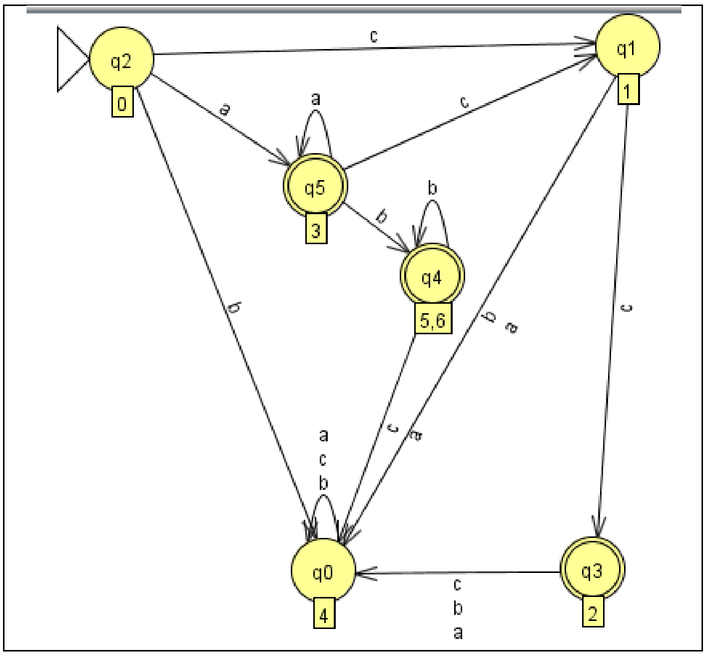
1) DFA = DFAmin

DFA	DFA minimal
`DFA=({a,b,c},{Q0,Q1,Q2,Q3,Q4},f,Q0,Q3)` f(Q0, a) = Q1 f(Q0, b) = Q2 f(Q0, c) = Q3 f(Q1, a) = Q2 f(Q1, b) = Q3 f(Q1, c) = Q1 f(Q2, a) = Q3 f(Q2, b) = Q1 f(Q2, c) = Q3 f(Q3, a) = Q4 f(Q3, b) = Q4 f(Q3, c) = Q4 f(Q4, a) = Q4 f(Q4, b) = Q4 f(Q4, c) = Q4	`Same DFA.`

2) DFA <> DFAmin

DFA	DFA minimal

`DFA=({a,b,c},{Q0,Q1,Q3,Q4,Q5,Q6,Q8},f,Q0,{Q3,Q4,Q6,Q8})`
 	
`f(Q0, a) = Q4 ; f(Q0, b) = Q5 ; f(Q0, c) = Q1`
`f(Q1, a) = Q5 ; f(Q1, b) = Q5 ; f(Q1, c) = Q3`
`f(Q3, a) = Q5 ; f(Q3, b) = Q5 ; f(Q3, c) = Q5`
`f(Q4, a) = Q4 ; f(Q4, b) = Q8 ; f(Q4, c) = Q1`
`f(Q5, a) = Q5 ; f(Q5, b) = Q5 ; f(Q5, c) = Q5`
`f(Q6, a) = Q5 ; f(Q6, b) = Q8 ; f(Q6, c) = Q5`
`f(Q8, a) = Q5 ; f(Q8, b) = Q6 ; f(Q8, c) = Q5`
	`DFAmin=({a,b,c},{Q0,Q1,Q3,Q4,Q5,Q9},f,Q0,{Q3,Q4,Q9})`
	
	f(Q0, a) = Q4 ; f(Q0, b) = Q5 ; f(Q0, c) = Q1
	f(Q1, a) = Q5 ; f(Q1, b) = Q5 ; f(Q1, c) = Q3
	f(Q3, a) = Q5 ; f(Q3, b) = Q5 ; f(Q3, c) = Q5
	f(Q4, a) = Q4 ; f(Q4, b) = Q9 ; f(Q4, c) = Q1
	f(Q5, a) = Q5 ; f(Q5, b) = Q5 ; f(Q5, c) = Q5
	f(Q9, a) = Q5 ; f(Q9, b) = Q9 ; f(Q9, c) = Q5



3) DFA <> DFAmin

DFA	DFA minimal

`DFA=({a,b,c},{Q0,Q1,Q2,Q3,Q4,Q6,Q7,Q8,Q9},f,Q0,{Q7,Q8})`
 	
`f(Q0, a) = Q1`
`f(Q0, b) = Q6`
`f(Q0, c) = Q6`
`f(Q1, a) = Q7`
`f(Q1, b) = Q2`
`f(Q1, c) = Q6`
`f(Q7, a) = Q7`
`f(Q7, b) = Q2`
`f(Q7, c) = Q6`
`f(Q2, a) = Q6`
`f(Q2, b) = Q8`
`f(Q2, c) = Q6`
`f(Q8, a) = Q6`
`f(Q8, b) = Q8`
`f(Q8, c) = Q4`
`f(Q4, a) = Q6`
`f(Q4, b) = Q9`
`f(Q4, c) = Q3`
`f(Q9, a) = Q6`
`f(Q9, b) = Q8`
`f(Q9, c) = Q4`
`f(Q3, a) = Q6`
`f(Q3, b) = Q9`
`f(Q3, c) = Q4`
`f(Q6, a) = Q6`
`f(Q6, b) = Q6`
`f(Q6, c) = Q6`
	`DFAmin=({a,b,c},{Q0,Q1,Q2,Q6,Q7,Q8,Q9,Q10},f,Q0,{Q7,Q8})`
	
	f(Q0, a) = Q1
	f(Q0, b) = Q6
	f(Q0, c) = Q6
	f(Q1, a) = Q7
	f(Q1, b) = Q2
	f(Q1, c) = Q6
	f(Q7, a) = Q7
	f(Q7, b) = Q2
	f(Q7, c) = Q6
	f(Q2, a) = Q6
	f(Q2, b) = Q8
	f(Q2, c) = Q6
	f(Q8, a) = Q6
	f(Q8, b) = Q8
	f(Q8, c) = Q10
	f(Q9, a) = Q6
	f(Q9, b) = Q8
	f(Q9, c) = Q10
	f(Q6, a) = Q6
	f(Q6, b) = Q6
	f(Q6, c) = Q6
	f(Q10, a) = Q6
	f(Q10, b) = Q9
	f(Q10, c) = Q10

4) NFA <> DFA <> DFAmin

DFA	DFA minimal
`DFA=({c,f,d},{Q0,Q5,Q8,Q9,Q10,Q11,Q12},f,Q0,Q10)` f(Q0, c) = Q9 f(Q0, f) = Q10 f(Q0, d) = Q8 f(Q9, c) = Q9 f(Q9, f) = Q11 f(Q9, d) = Q12 f(Q10, c) = Q0 f(Q10, f) = Q8 f(Q10, d) = Q8 f(Q11, c) = Q11 f(Q11, f) = Q11 f(Q11, d) = Q8 f(Q12, c) = Q12 f(Q12, f) = Q5 f(Q12, d) = Q5 f(Q5, c) = Q5 f(Q5, f) = Q5 f(Q5, d) = Q8 f(Q8, c) = Q8 f(Q8, f) = Q8 f(Q8, d) = Q8	`DFAmin=({c,f,d},{Q0,Q10,Q13},f,Q0,Q10)` f(Q0, c) = Q13 f(Q0, f) = Q10 f(Q0, d) = Q13 f(Q13, c) = Q13 f(Q13, f) = Q13 f(Q13, d) = Q13 f(Q10, c) = Q0 f(Q10, f) = Q13 f(Q10, d) = Q13

6. **Given the language $(01)^n$ with n≥0, indicate which of the following finite automata generates this language. In addition, obtain the minimal equivalent DFA for the selected automaton. FA=[{0,1}, {A,B,C,F}, f, A, {F}]**
   f(A,0)=B, f(A, λ)= λ, f(C,0)=B, f(B,1)=C, f(B,1)= λ

   a. **FA=[{0,1}, {A,B,C,F}, f, A, {F}]**
   f(A,0)=B, f(A, λ)=F, f(C,0)=B, f(B,1)=C, f(B,1)=F

   b. **FA=[{0,1}, {A,B,C,F}, f, A,{F}]**
   f(A, B)=0, f(A,F)= λ, f(C,B)=0, f(B,C)=1, f(B,F)=1

   c. **FA=[{0,1}, {A,B,C,F}, f, A, {F}]**
   f(B,0)=A, f(F, λ)=A, f(B,0)=C, f(C,1)=B, f(F,1)=B

<u>Solution</u>:

The correct option is “b”:

NFA=[{0,1}, {A,B,C,F}, f, A, {F}]
f(A,0)=B, f(A,λ)=F, f(C,0)=B, f(B,1)=C, f(B,1)=F

7. Obtain the minimal equivalent DFA for the following Non-Deterministic Finite Automata. Describe the intermediate transformations: NFA→DFA→Minimal DFA.

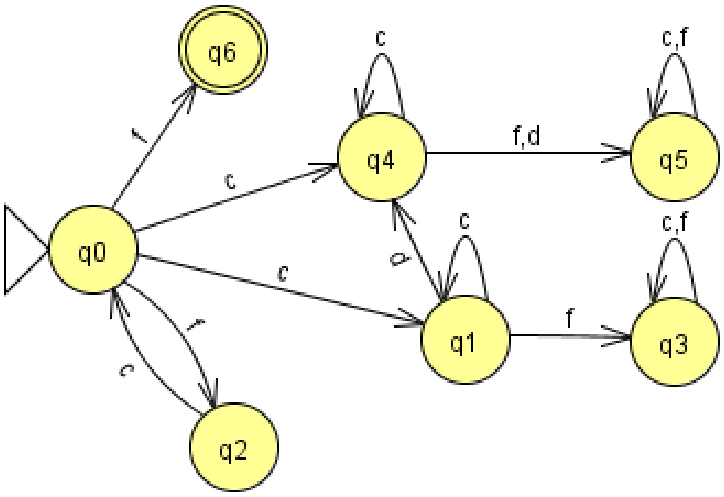
```
NFA=({c,f,d},{Q0,Q1,Q2,Q3,Q4,Q5,Q6},f,Q0,Q6)
```

f(Q0, c) = Q1,Q4; f(Q0, f) = Q2,Q6; f(Q1, c) = Q1
f(Q1, f) = Q3; f(Q1, d) = Q4; f(Q2, c) = Q0
f(Q3, c) = Q3; f(Q3, f) = Q3; f(Q4, c) = Q4
f(Q4, f) = Q5; f(Q4, d) = Q5; f(Q5, c) = Q5
f(Q5, f) = Q5

1.- NFA

NFA=({c,f,d},{Q0,Q1,Q2,Q3,Q4,Q5,Q6},f,Q0,{Q6}) where f is,

	c	f	d
→Q0	Q1,Q4	Q6,Q2	
Q1	Q1	Q3	Q4
Q2	Q0		
Q3	Q3	Q3	
Q4	Q4	Q5	Q5
Q5	Q5	Q5	
*** Q6**			



2.- NFA → DFA

		c	f	d
	→Q0	Q7	Q8	Q9
{Q1,Q4}=	**Q7**	Q7	Q10	Q11
{Q2,Q6}=	*** Q8**	Q0	Q9	Q9
{Q3,Q5}=	**Q10**	Q10	Q10	Q9
{Q4,Q5}=	**Q11**	Q11	Q5	Q5
	Q5	Q5	Q5	Q9
	Q9	Q9	Q9	Q9

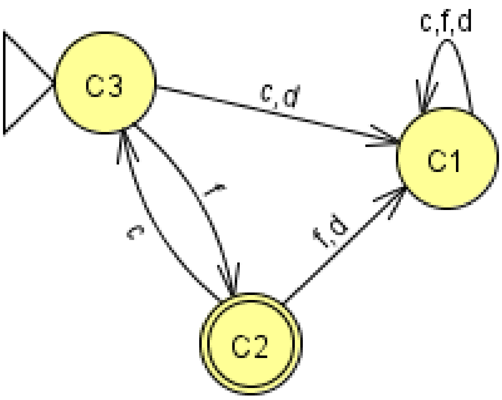
DFA=({c,f,d},{Q0,Q5,Q7,Q8,Q9,Q10,Q11},f’,Q0,{Q8})

3.- DFA → DFAmin

Q/E0 = {{Q0,Q5,Q7,Q9,Q10,Q11},{Q8}} = {C1,C2}
Q/E1 = {{Q5,Q7,Q9,Q10,Q11},{Q8},{Q0}} = {C1,C2,C3}
Q/E1 = Q/E2 = Q/E

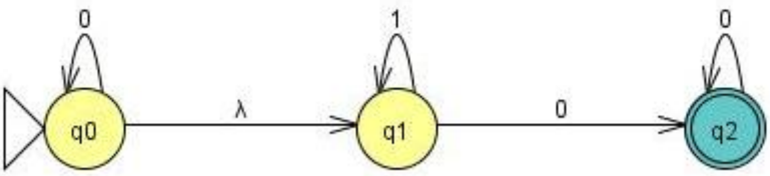
DFAmin=({c,f,d},{C1,C2,C3},f '',C3,{C2}), where f''is

	c	**f**	**d**
→C3	C1	C2	C1
C1	C1	C1	C1
*** C2**	C3	C1	C1



8. **Draw the graph of a Determinist Finite Automaton. The alphabet is {0, 1} and the language (m ≥ 0,n ≥ 0, p ≥1). The problem can be solved by directly designing the DFA, or by starting from the NFs and then obtaining the equivalent DFA.**

Solution:



	0	1	λ
→ q0	q0		q1
q1	q2	q1	
*q2	q2		

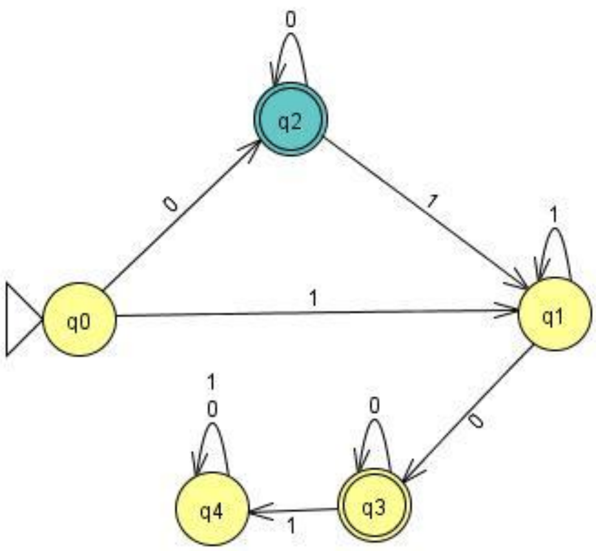
	0	1	λ	λλ	…	λ*
→ q0	q0		q1	q0,q1		q0,q1
q1	q2	q1		q1		q1
*q2	q2			q2		q2

	λ*0λ*	λ*1λ*
→ q0	q0,q1,q2	q1
q1	q2	q1
*q2	q2	

DFA:

1.

	0	1
→ {q0,q1}=q0	q3	q1
{q0,q1,q2}=*q3	q3	q1
q1	q2	q1
* q2	q2	q4
q4	q4	q4



9. Given the NFA (with lambda transitions) described by the following table, obtain the minimal equivalent DFA.

	a	**b**	**c**	**λ**
→p	p	q		q
q	q	p,r		r
r			s	p
*** s**	s			

Solution:

	a	b	c	λ	λλ	λλλ	…	λ*
→p	p	q		q	**p**,q,r	p,q,r		p,q,r
q	q	p,r		r	**q**,r,p	p,q,r		p,q,r
r			s	p	**r**,p,q	p,q,r		p,q,r
*** s**	s				**s**	s		s

	λ*aλ*	λ*bλ*	λ*cλ*
→p	p,q,r	p,q,r	s
q	p,q,r	p,q,r	s
r	p,q,r	p,q,r	s
*** s**	s		

DFA:

	a	b	c
→{p,q,r}=t	t	t	s
*** s**	s	ϕ	ϕ
ϕ	ϕ	ϕ	ϕ

DFA=({a,b,c}, {t,s,ϕ}, f, t, {s})