MACHINE THEORY
Bachelor in Mechanical Engineering

INTRODUCTION TO KINEMATICS AND MECHANISMS

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DEFINITIONS

- **Kinematic chain**: It is a linkage of elements and joints that transmit a controlled output motion related to a given input motion.
- **Mechanism**: It is a kinematic chain where one element (or more) are fixed to the reference framework (which can be in motion).
- **Machine**: Group of resistant elements (which usually contain mechanisms) thought to transmit considerable movement, forces or/and power. **Boundary is not completely clear!!**
• **Kinematic pair** – Existing connection between two elements of a mechanism that have a relative motion between them.

Kinematic pairs was classified by Reuleaux as follow:

- Lower pair – two links having a surface contact between them.
- Higher pair – two links having line or point contact between them.

• **Joint** - guarantees the contact between two members and constrains their relative motion
Classification of Kinematic Pairs by Degrees of Freedom

- 0 DOF
- 1 DOF
- 2 DOF
- 3 DOF
- 4 DOF
- 5 DOF

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<th>Variables y parámetros</th>
<th>( x \quad a \quad \alpha )</th>
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<td>Transmisión por ruedas dentadas (engranajes)</td>
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Degrees of freedom:

1. Number of independent coordinates needed to define the position of the element/mechanism...

2. ..... or number of parameters needed to determine unambiguously the geometry configuration of a system in space....

3. ..... or the number of inputs needed to obtain a predictable output of a mechanism.

GRÜBLER’S EQUATION

\[ G = 3 \cdot (N-1) - 2 \cdot f_1 - f_2 \]

Kutzbach Criterion for mobility of a planar mechanism

\[ f_1 = \text{nº pairs 1 DOF} \]

\[ f_2 = \text{nº pairs 2 DOF} \]

\[ N = \text{nº of elements} \]
Example. Obtain the number of DOF of the digger arm.

- $G=0$ Structure. No motion
- $G>0$ Mechanism. Motion
- $G < 0$ Hyperstatic Structure. No motion
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- $G=0$ Structure. No motion
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$$G = 3(12-1) - 2 \times 12 \text{(pin joints)} - 2 \times 3 \text{(slide joints)} = 3$$
Be careful!!!! Grübler equation not always works. As far as this equation does not consider shape or size of links, there are some exceptions:
FOUR-BAR LINCKAGE

- Very simple but very versatile. First option for design.

- Classification depending on the task:
  - **Function Generator.** Output rules
  - **Path Generator.** Path rules
  - **Motion Generator.** All important
4 BAR KINEMATIC INVERSIONS

- It is the method of obtaining different mechanisms by fixing different links of the same kinematic chain. **POWERFUL TOOL.** See that with the slider-crank example:

Marine engine

Whitworth mechanism. Gnome engine

Hand pump
In some mechanisms, for a given driver position, there are more than one possible configurations.

DEAD CENTER POSITION ALWAYS APPEARS in a four bar mechanism.
GRASHOF CRITERIA

- Simple relation that describes the behavior of the kinematic inversions of a four-bar mechanism.

S = length of the shortest link
L = length of the longest link.
P and Q are the other links.

\[ S + L \leq P + Q \]

If my condition is satisfied, at least one link would be able to do a full revolution with respect to another link.

CONTINOUS MOTION IS ALLOWED
If \( s + l < p + q \): Four possibilities of Grashof mechanism:

- **Crank-rocker**: Shortest link is the crank. Frame is adjacent

- **Rocker-Crank**: The shortest link is the follower.

- **Double Crank** or drag-link: Shortest link is the frame.
Double rocker: The link opposite the shortest is the frame.

Where is the full rotation of a link?
Double rocker: link opposite the shortest is the frame

Where is the full rotation of a link? The Coupler
If \( s + l > p + q \): All the kinematic inversions will be double rocker. No continuous relative motion is possible.
If \( s + l = p + q \). Grashof Special Mechanisms.

- All inversions are double-crank or crank-rocker.
- These mechanisms suffer from the change-point condition.

▶ All links become collinear creating momentarily a second DOF. OUTPUT RESPONSE IS UNDETERMINED.
Use it when a four-bar linkage does not provide the performance required.

Watt Kinematic Chain

Stephenson Kinematic Chain
Six bar kinematic chains also present kinematic inversions.

Inversions of Watt and Stephenson kinematic chains
See some real examples

http://www.youtube.com/watch?v=ZiAbpscuJdo
FOUR BAR MECHANISM

SIX BAR MECHANISM: to maintain a constant distance between the axle and bottom bracket. It is a Stephenson III six-bar linkage.

UCI XGR gravity racer

Brake pedal
Sometimes intermitent motion is needed.

Examples: Geneva mechanism, camshaft, ratchet mechanism.

http://www.youtube.com/watch?v=85BsbncfRqA

http://www.youtube.com/watch?v=ejiyLC4ZzQk&feature=related
LITERATURE

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  - http://www.youtube.com/watch?v=eiylC4ZzQk&feature=related