EXERCISES

4.1. The European contribution to the International Space Station is a science module called Columbus, whose scheme is shown in Fig. 1. Its structure is made of an aluminium alloy. To evaluate the possibility of manufacturing the module using a composite material, a symmetric carbon / epoxy laminate with a staking sequence ± 45 ° will be studied.

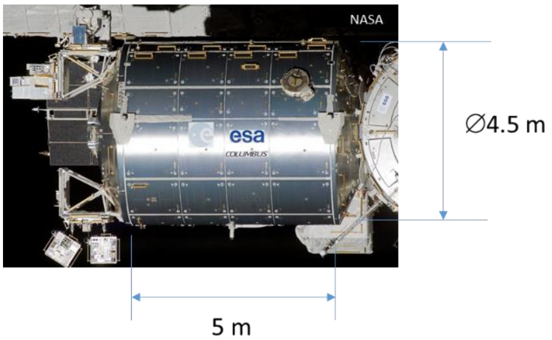

Fig.1

Modeling the module as a cylindrical structure subjected to an internal pressure of one atmosphere, calculate the thickness to withstand the pressure employed without causing failure. In addition to the internal pressure it is assumed that the module is subjected to a torque of 675 kNm, (which causes a shear stress in the laminate plane).

DATA:

- Using aluminium
  - Module thickness: 2.46 mm
  - Module weight: 486 kg (only the cylinder part)
- Using composite material

Laminate stacking sequence: ±45º
Volume of fibres: 65%
Density: 1600 kg/m^3

E_1 = 177 GPa	X = 2860 MPa
E_2 = 10.8 GPa	Y = 49 MPa
G_{12} = 7.6 GPa	S = 83 MPa
ν_{21} = 0.27	

NOTE:

- It will be assumed that a lamina fails if the Tsai-Hill criterion is verified:

$$\left(\frac{\sigma_{11}}{X}\right)^2+\left(\frac{\sigma_{22}}{Y}\right)^2+\left(\frac{\tau_{12}}{S}\right)^2-\frac{\sigma_{11}\cdot\sigma_{22}}{X^2}=1$$

- Consider a safety factor of 3.
- Only the cylindrical part of the module will be studied

Ans. W = 279 Kg

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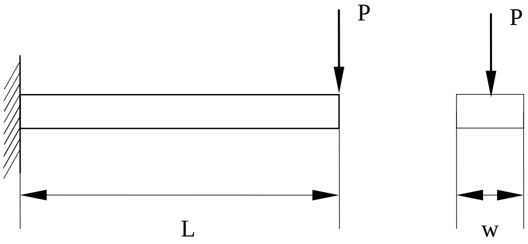
4.2. A point load P is applied on the free end of a cantilever beam (see figure). The beam is composed of a glass/epoxy laminate with the stacking sequence $[\,0/90\,]_{3S}$. The angles are measured with respect to the longitudinal axis of the beam.



1. Determine the bending stiffness matrix of the laminate.
2. Determine the $\{k\}$ vector at any point of the beam.
3. Indicate the point of the beam which reaches the maximum stress, and its value as a function of the applied load, P.
4. Calculate the load P which causes the failure of the beam.

DATA:

E_1 = 38.60 GPa
E_2 = 8.27 GPa
ν_{21} = 0.26
G_{12} = 4.14 GPa

X = 1062 MPa
Y = 31 MPa
S = 72 MPa

L=0.5 m
w = 0.04 m
t_i=1.25 mm

NOTE:

- It will be assumed that a lamina fails if the Tsai-Hill criterion is verified:

$$\left(\frac{\sigma_{11}}{X}\right)^2+\left(\frac{\sigma_{22}}{Y}\right)^2+\left(\frac{\tau_{12}}{S}\right)^2-\frac{\sigma_{11}\cdot\sigma_{22}}{X^2}=1$$

Ans. P_{max} = 375 N

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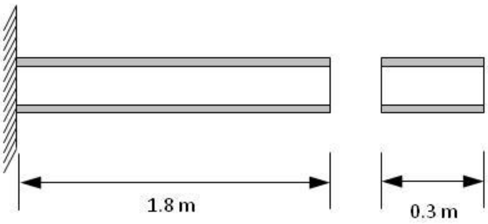
4.3 The cantilever beam in the figure is made from a sandwich composed of a 20 mm core and two 4 mm face-sheets.



The core is made of an isotropic foam with the following properties:
E = 500 MPa
ν = 0.4

The skins are made from a laminate with the following engineering elastic constants:
E_1 = 70 GPa
E_2 = 30 GPa
G_{12} = 15 GPa
ν_{21} = 0.25

If the beam is subjected to a load per unit length on the upper side equal to 300 N / m; determine the stress distribution at the fixed end and the maximum deflection at the free end, using two procedures:

- Applying the theory of the laminate
- Applying the sandwich theory.

Ans. Assuming laminate theory: $\sigma_{1,max}$ = 6.596 MPa, $\sigma_{2,max}$ = 0.014 MPa,
Assuming sandwich theory: $\sigma_{p,max}$ = 6.33 MPa, $\tau_{c,max}$ = -28125 Pa,

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4.4. The structure of a small satellite is composed by several beams of length **L** as can be seen in the figure 1a. Each elements can be modelled as simply supported beams subjected to a point moment **M** applied at both supports, Fig. 1b.

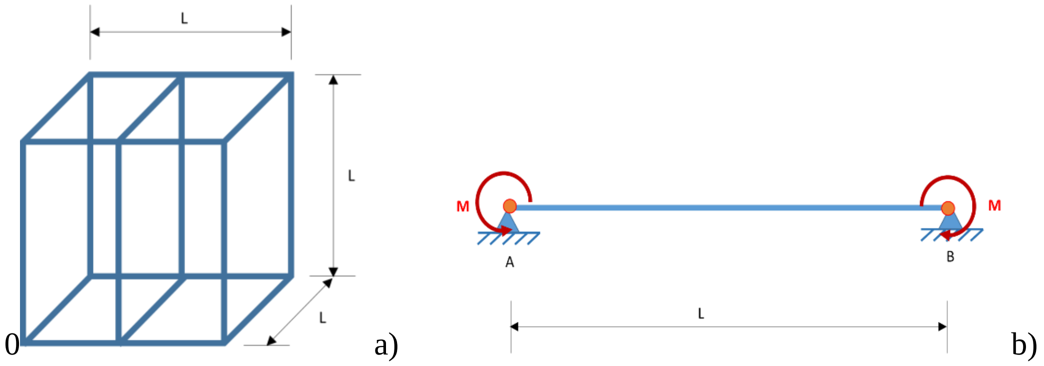

Figure 1

The cross section of the beam is showed in figure 2. The thickness **t** of the section is smaller than the dimension **b**.

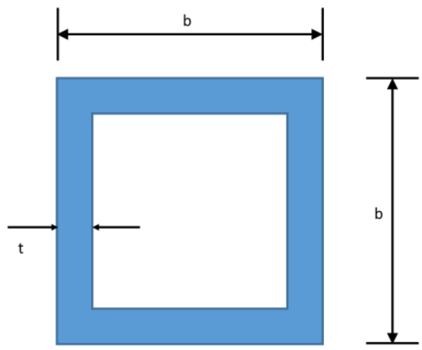

Figure 2

Each beam is made from a laminate **[±45/0]$_{\mathbf{2OS}}$** of the material **AS/H3501**.

a) Determine, as a function on the moment **M** and the length **L**, maximum internal forces and cross-sections of the bar in which they appear.
b) The maximum stress and strain in the top flange of the cross-section where the maximum internal forces appear.

Assuming L=1.5 m and b =150 mm, determine:

c) The moment **M** that produce the failure of the beam according to Tsai-Hill criterion with a safety factor of 1.5.

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Data:

E1, (GPa)	138.00
E2, (GPa)	8.96
ν21	0.30
G12, (GPa)	7.10
v/f	0.66
ρ (kg/m3)	1600.00
h0, (mm)	0, 125
X (MPa)	1447
X' (MPa)	1447
Y (MPa)	52
Y' (MPa)	206
S (MPa)	93

Sol. a) The internal forces is equal to M. b) $=\frac{M}{b^2}$, $\{\varepsilon\} = \begin{Bmatrix} 1.08 \cdot 10^{-9} N \\ -7.05 \cdot 10^{-10} N \\ 0 \end{Bmatrix}$, $\{\sigma\} = \begin{Bmatrix} 27.2\ N \\ 18.5 N \\ 12.3 N \end{Bmatrix}$, c) M = 519.84 kN

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