

Bachelor in Aerospace Engineering



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Department of Continuum Mechanics and Structural Engineering

Aerospace Structures

Chapter 4. Laminate and sandwich structures

Failure criteria



Chapter 4. Composite beams and plates

Failure criteria

- 1. Failure mechanisms**
- 2. Global failure criteria**
- 3. Intralaminar failure criteria**
- 4. Delamination criteria**
- 5. References**



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Failure mechanisms

Matrix cracking



**Intralaminar
failure
mechanisms**

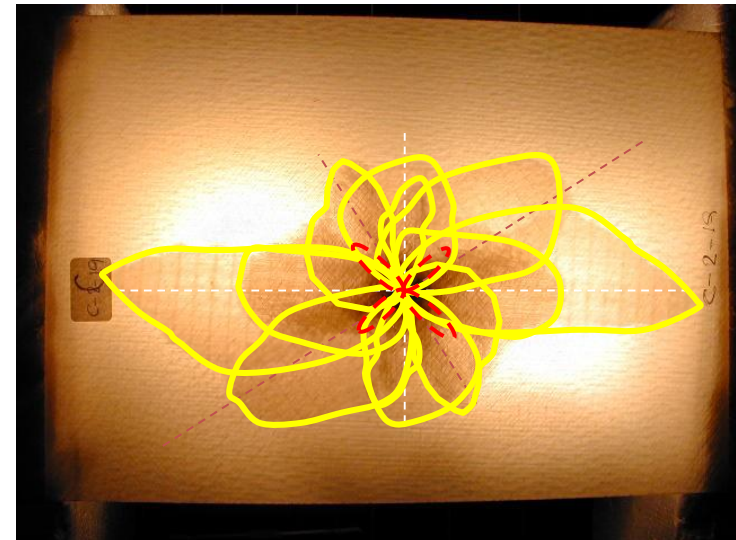


Fibre failure

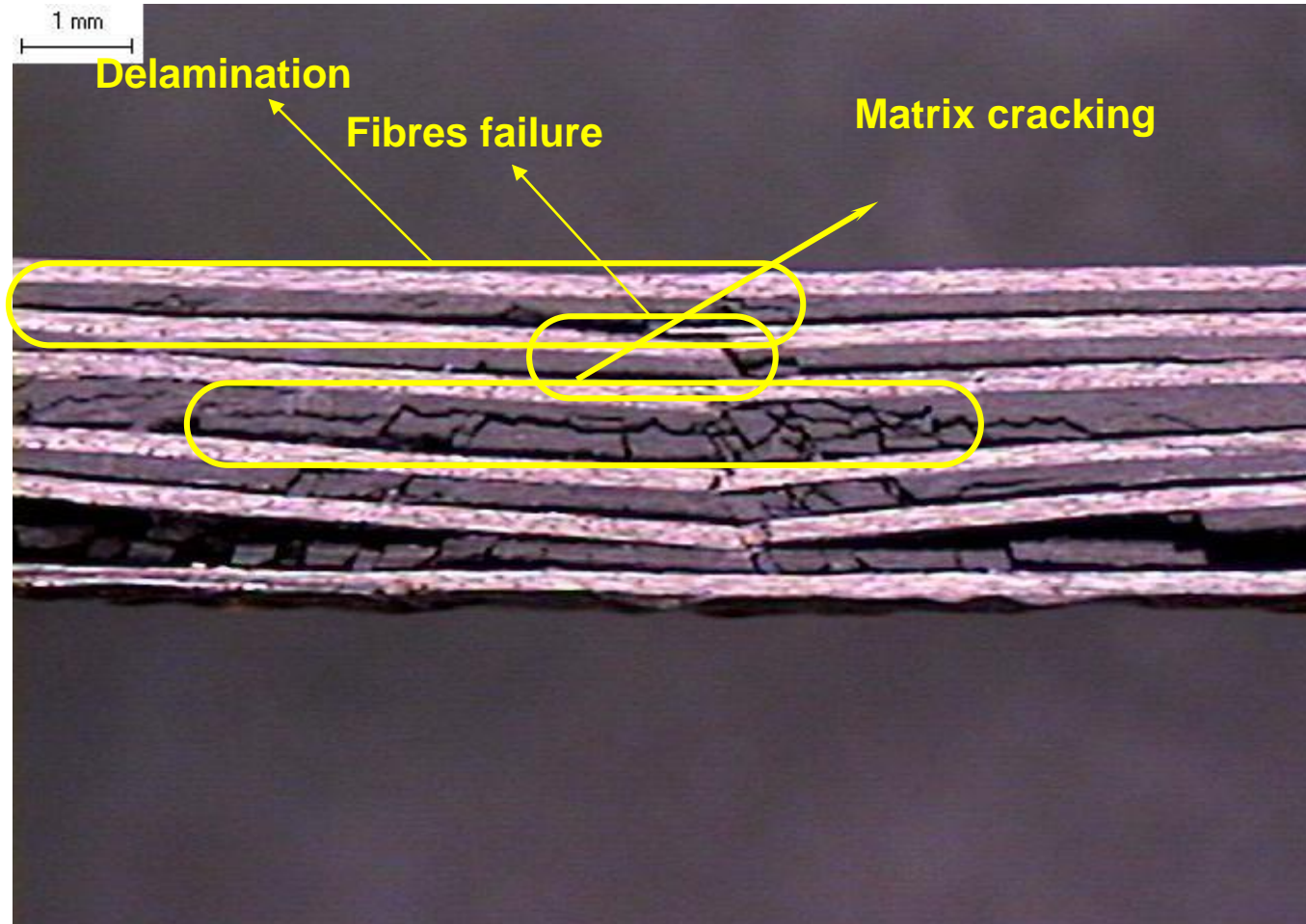
**Interlaminar
failure
mechanisms**



Delamination



Failure mechanisms





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Failure criteria for isotropic materials

Tresca plasticity criterion

$$\sigma_1 - \sigma_3 = 2k$$

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

$$k = \frac{\sigma_y}{2}$$

Von Mises plasticity criterion

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6(k')^2$$

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) = 6(k')^2$$

$$J_2 = (k')^2$$

Failure criteria for composite materials

Tsai-Hill criterion

$$I_f = \frac{\sigma_{11}^2}{X^2} - \frac{\sigma_{11}\sigma_{22}}{X^2} + \frac{\sigma_{22}^2}{Y^2} + \frac{\sigma_{12}^2}{S^2}$$

Tsai-Wu criterion

$$I_f = \left(\frac{1}{X_t} - \frac{1}{X_c} \right) \sigma_{11} + \left(\frac{1}{Y_t} - \frac{1}{Y_c} \right) \sigma_{22} + \frac{\sigma_{11}^2}{X_t X_c} + \frac{\sigma_{22}^2}{Y_t Y_c} + \left(\frac{\sigma_{12}}{S_{12}} \right)^2 + 2F_{12} \sigma_{11} \sigma_{22}$$

$$F_{12} = -\frac{1}{2} \sqrt{\frac{1}{X_t X_c Y_t Y_c}}$$



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Intralaminar failure criteria for composite materials

Failure criterion	Hou 3D	Hashin 3D
Fibres tensile	$d_{ft}^2 = \left(\frac{\sigma_{11}}{X_t} \right)^2 + \left(\frac{\sigma_{12}^2 + \sigma_{13}^2}{S_L^2} \right)$	$d_f^2 = \left(\frac{\sigma_{11}}{X_t} \right)^2 + \alpha \left(\frac{\sigma_{12}^2 + \sigma_{13}^2}{S_L^2} \right)$
Fibres compressive		$d_{fc}^2 = \left(\frac{\sigma_{11}}{X_c} \right)^2$
Matrix cracking	$d_{mt}^2 = \left(\frac{\sigma_{22}}{Y_t} \right)^2 + \left(\frac{\sigma_{12}}{S_L} \right)^2 + \left(\frac{\sigma_{23}}{S_T} \right)^2$	$d_{mt}^2 = \left(\frac{\sigma_{22} + \sigma_{33}}{Y_t} \right)^2 + \left(\frac{\sigma_{12}^2 + \sigma_{13}^2}{S_L^2} \right) + \left(\frac{\sigma_{23}^2 - \sigma_{22}\sigma_{33}}{S_T^2} \right)^2$
Matrix crushing	$d_{mc}^2 = \frac{1}{4} \left(\frac{-\sigma_{22}}{S_T} \right)^2 + \frac{Y_c^2 \sigma_{22}}{4S_T^2 Y_c} - \frac{\sigma_{22}}{Y_c} + \left(\frac{\sigma_{12}}{S_L} \right)^2$	$d_{mc}^2 = \frac{1}{Y_c} \left[\left(\frac{Y_c}{2S_T} \right)^2 - 1 \right] (\sigma_{22} + \sigma_{33}) + \left(\frac{\sigma_{22} + \sigma_{33}}{2S_T} \right)^2 + \frac{(\sigma_{23}^2 + \sigma_{22}\sigma_{33})}{S_T^2} + \frac{(\sigma_{12}^2 + \sigma_{13}^2)}{S_L^2}$



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Delamination criteria

Delamination onset criteria

Maximum stress

$$\sigma_{33} \geq Z_t \quad \text{ó} \quad \sigma_{31} \geq S_{31} \quad \text{ó} \quad \sigma_{23} \geq S_{23}$$

Lee

$$\sigma_{33} \geq Z_t \quad \text{ó} \quad \sqrt{(\sigma_{12}^2 + \sigma_{13}^2)} \geq S_{23}$$

Hashin

$$\left(\frac{\sigma_{33}}{Z_t}\right)^2 + \left(\frac{\sigma_{23}}{S_{23}}\right)^2 + \left(\frac{\sigma_{13}}{S_{31}}\right)^2 \geq 1$$

Goyal

$$\left(\frac{\sigma_{33}}{Z_t}\right)^2 + \left(\frac{\sigma_{23}}{S_{23}}\right)^\gamma + \left(\frac{\sigma_{13}}{S_{31}}\right)^\gamma \geq 1$$

Wisnom

$$2.6\sigma_e^2 = (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 + 0.6\sigma_e (\sigma_I + \sigma_{II} + \sigma_{III})$$



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Impact on Composite Structures

S. Abrate

Cambridge University Press, 1998

Damage Mechanics of Composite Materials

R. Talreja

Elsevier, 1994

Analysis and performance of fiber composites

B.D. Argawal and L.J. Broutman

John Wiley & Sons, Inc., 1990

World wide failure exercise

Several authors

Issues 58 (1998), 62 (2002) and 64 (2004) in the journal “Composite Science and Technology”

Review of methodologies for composite material modelling incorporating failure

A.C. Orifici, I. Herszberg, R.S. Thomson

Composite Structures 86 (2008) 194–210