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Aerospace Structures

Chapter 4. Laminate and sandwich structures

Sandwich structures

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Chapter 4. Composite beams and plates

Sandwich structures

Authors: Enrique Barbero Pozuelo, José Fernández Sáez, Carlos Santiuste Romero

Chapter 4. Composite beams and plates

Sandwich structures

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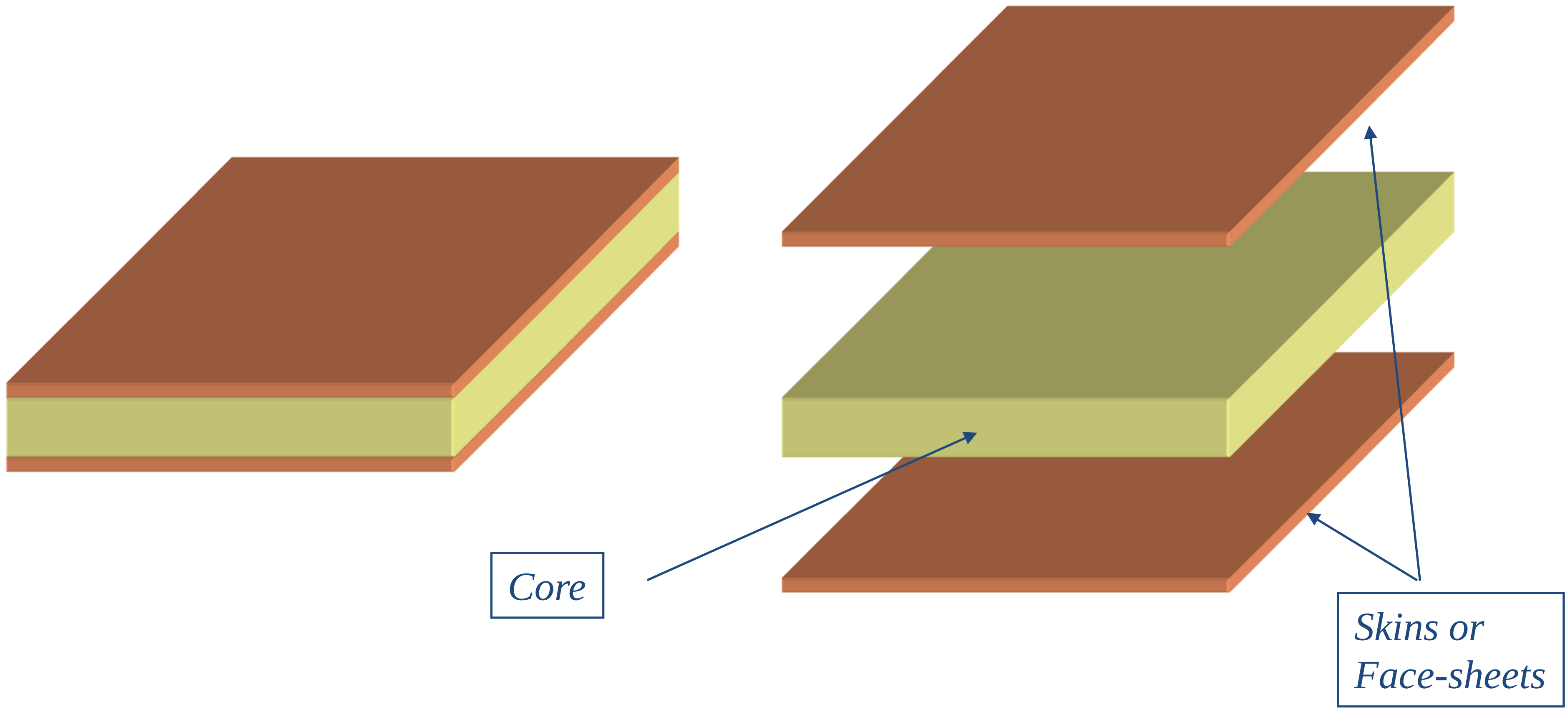
Definition of a sandwich structure



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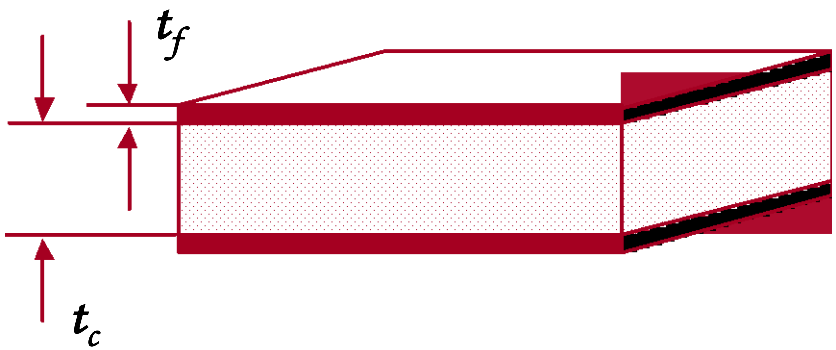
Introduction

Sandwich parameters



Geometry

$$10 \leq \frac{t_c}{t_f} \leq 100$$

$$0.25 \ mm \leq e_f \leq 12.7 \ mm$$

Core density

$$20 \ \frac{kg}{m^3} \leq \rho_c \leq 1000 \ \frac{kg}{m^3}$$

Adhesive thickness

$$0.025 \ mm \leq t_a \leq 0.2 \ mm$$

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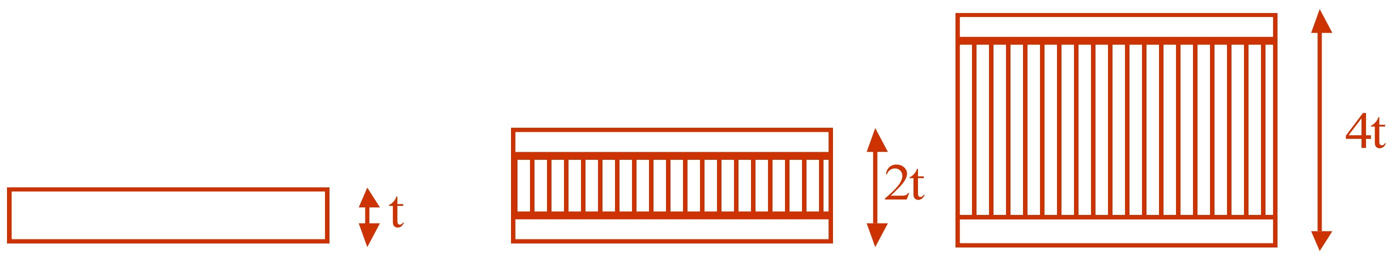
Introduction

Advantages of sandwich

- **High strength and stiffness under bending moments**



Core thickness	**0**	**t**	**3·t**
Bending stiffness	1	7	37
Bending strength	1	3,5	9,25
Weight	1	1,03	1,06

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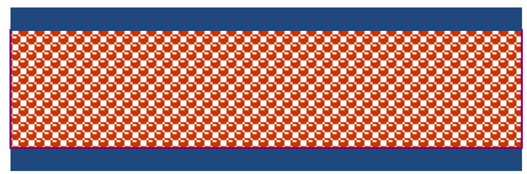
Advantages of sandwich

- **Thermal isolation**

$$\overset{o}{Q} = 1 \ \text{MW}$$



$$\overset{o}{Q} = 300 \ \text{W}$$

Metal: $k = 100 \dfrac{W}{m\,^{o}C}$

Face-sheet: $k = 100 \dfrac{W}{m\,^{o}C}$

PUR core: $k = 0{,}07 \dfrac{W}{m\,^{o}C}$

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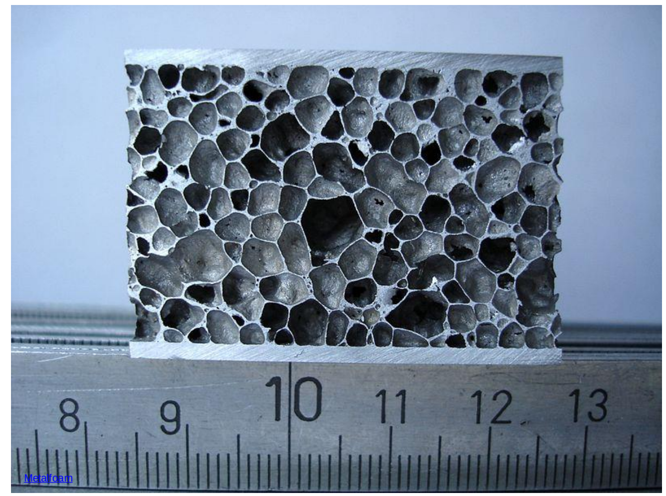
Core typologies

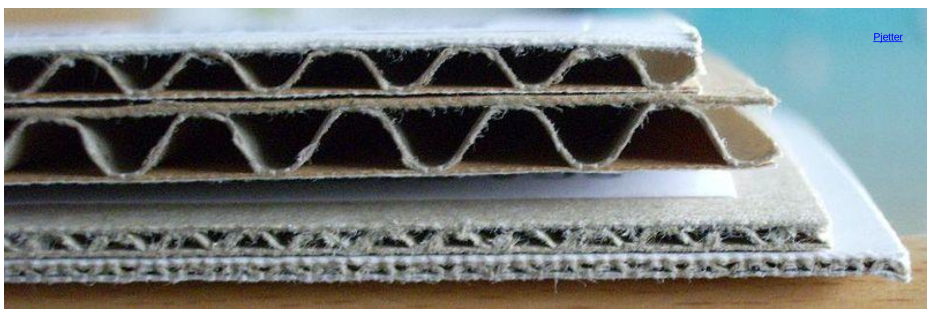
•http://commons.wikimedia.org/wiki/File:Steinbichler_Shearography_Honeycomb_with_CFRP_Top_Layer_Artificial_failures_that_simulate_layer-core_delaminations_Material.jpg?uselang=es

Honeycomb panel



http://commons.wikimedia.org/wiki/File:Aluminium_foam_sandwich.jpg?uselang=es

Foams



•http://commons.wikimedia.org/wiki/File:Corrugated_board_B_C_E_and_F_flute.JPG?uselang=es

Corrugated core

Introduction

Core typologies

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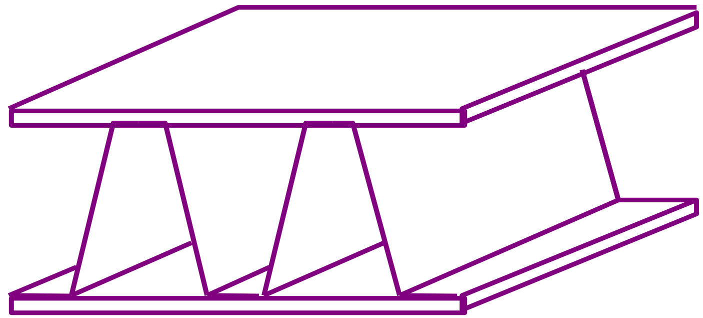
Corrugated core

Face-sheets

Metals
Laminates

Core

Metals
Laminates



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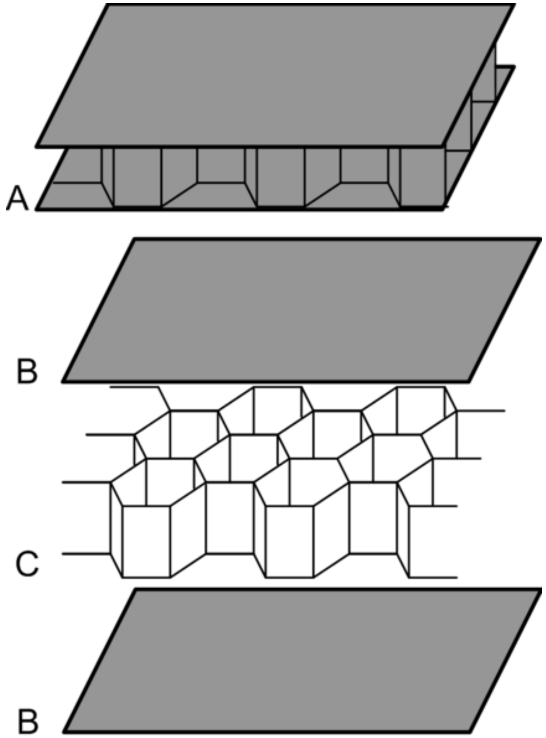
Honeycomb panel

George William Herbert
http://commons.wikimedia.org/wiki/File:CompositeSandwich.png?uselang=es



Face-sheets

Metals
Laminates

Core

Aluminium alloy (Al5052, Al5056, Al2024)
Nomex® (aramid fibres/ Phenol formaldehyde resin)
Kraft Cardboard
Carbon-epoxy
Kevlar-epoxy

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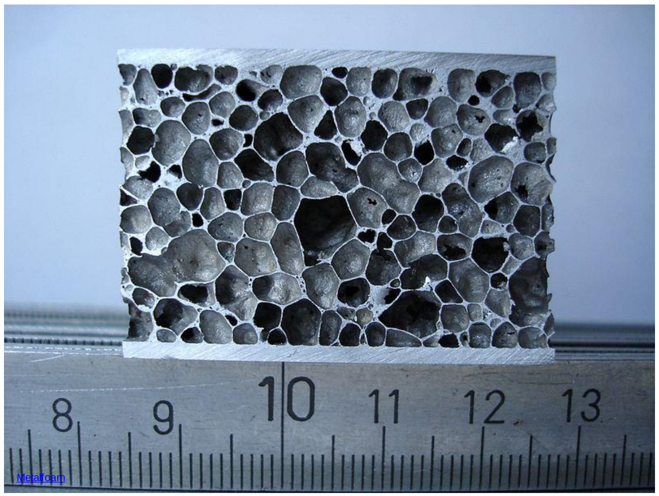
Core typologies



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Foams

Face-sheets

Metals
Laminates
Plywood

Core

Phenol formaldehyde resin
PVC
Metallic foams

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Chapter 4. Composite beams and plates

Sandwich structures

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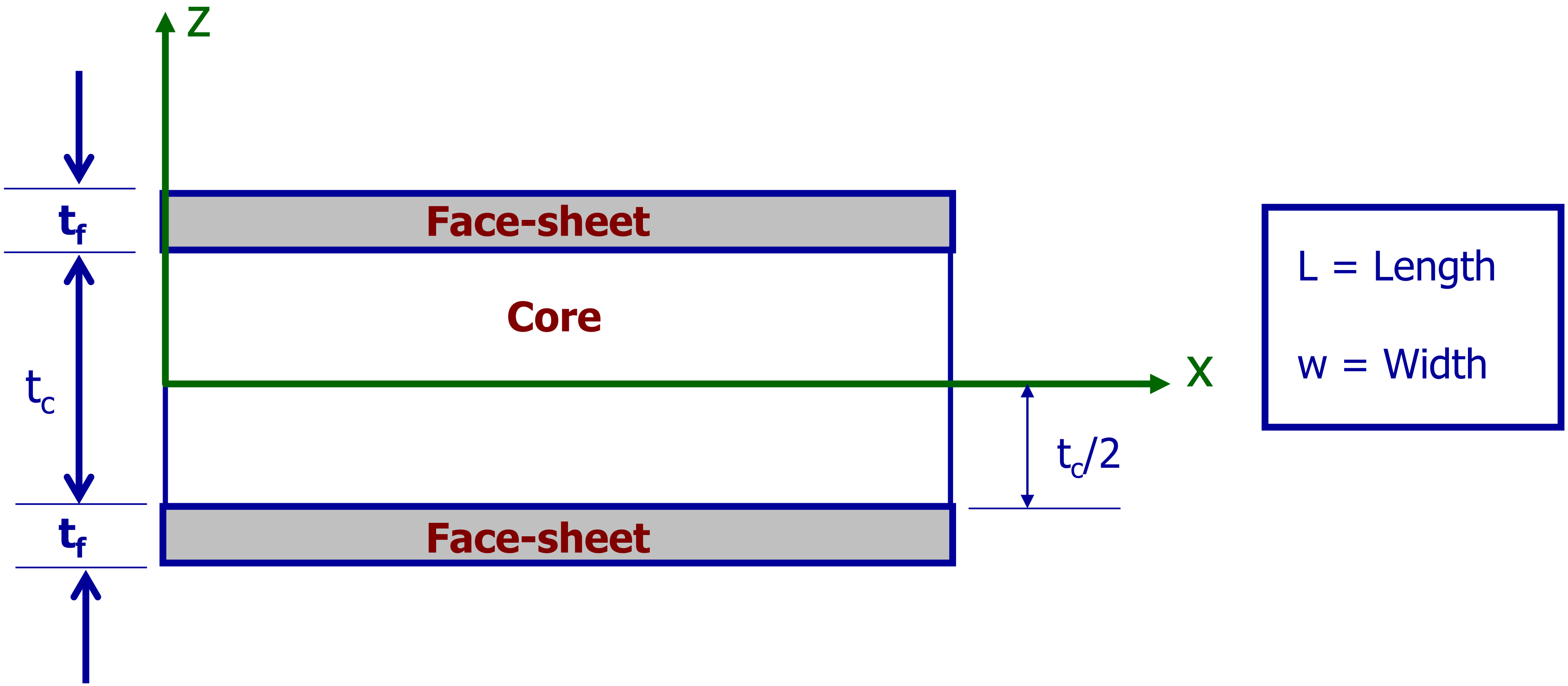
Hypotheses



- Hypotheses:
  - $t_c >> t_f$
  - $E_c << E_f$

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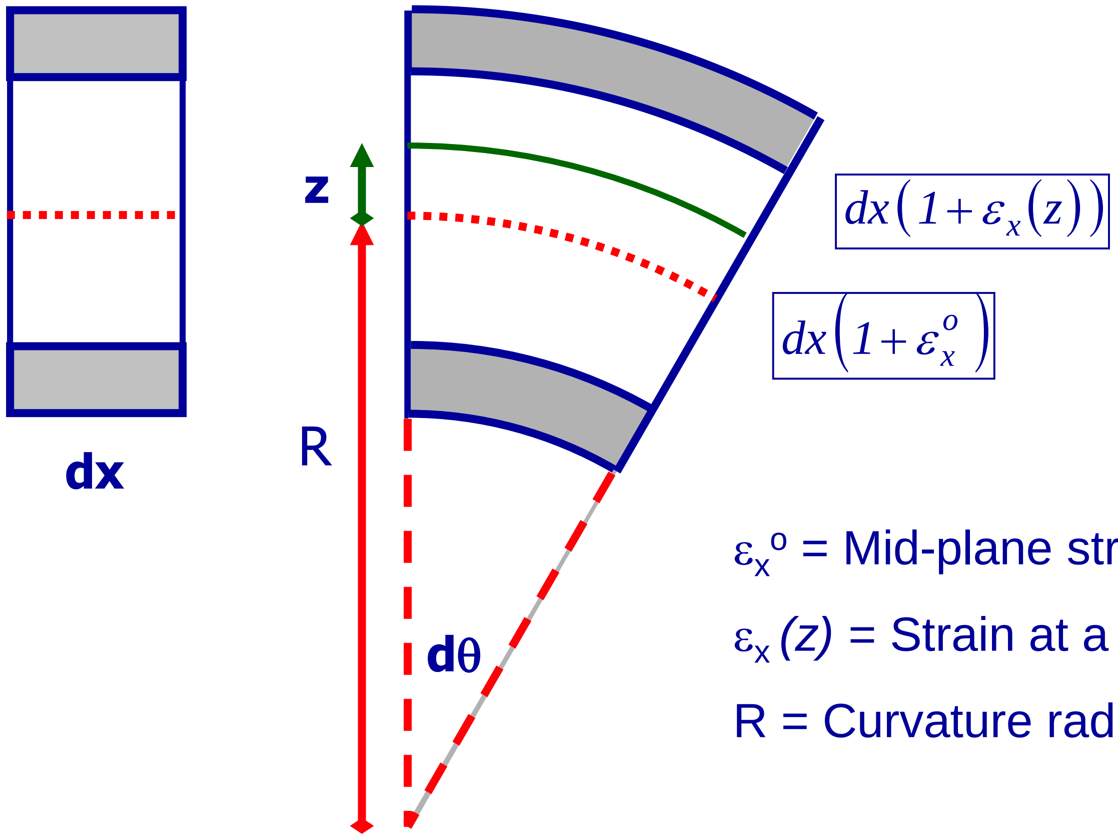
Hypotheses



ε_x^o = Mid-plane strain

ε_x *(z)* = Strain at a distance *z* from mid-plane

R = Curvature radius

$$R \cdot d\theta = dx\left(1+\varepsilon_x^o\right)$$
$$\left(R+z\right)d\theta = dx\left(1+\varepsilon_x\left(z\right)\right)$$

$\Rightarrow$

$$\varepsilon_x\left(z\right) = \varepsilon_x^o + \frac{z}{R}\left(1+\varepsilon_x^o\right)$$

$\Rightarrow$

$$\varepsilon_x\left(z\right) \cong \varepsilon_x^o + \frac{z}{R}$$

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Chapter 4. Composite beams and plates

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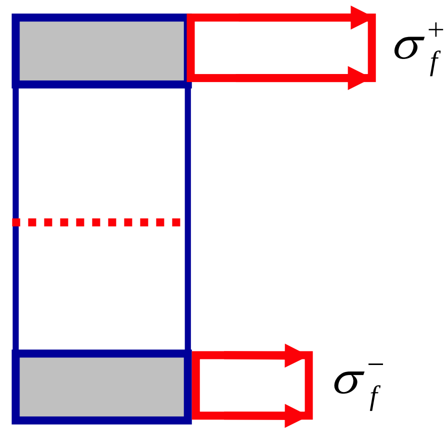
Stress field in sandwich beams

Normal stresses

Considering only normal stresses in x direction:

$$\sigma_x(z) \approx E(z)\cdot\left(\varepsilon_x^o + \frac{z}{R}\right)$$

Different Young modulus in core and face-sheets



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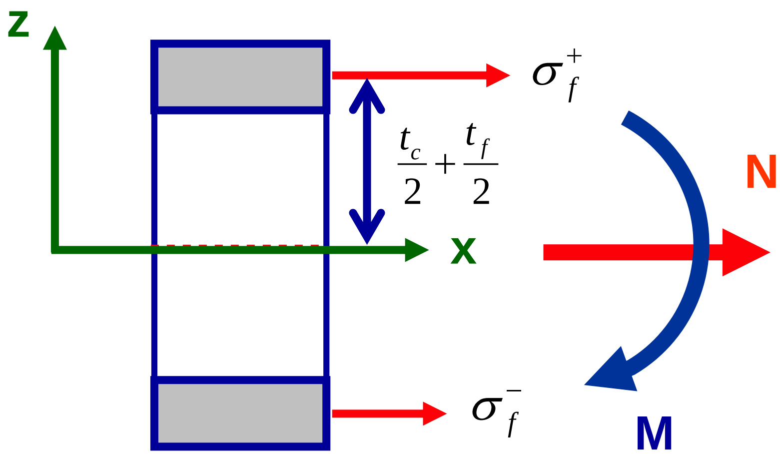
Stress field in sandwich beams

Normal stresses

- Axial forces and bending moments



Internal forces can be found integrating normal stresses through the section

$$N = \sigma_f^+ t_f w + \sigma_f^- t_f w$$

$$M = \sigma_f^+ t_f w\left(\frac{t_f + t_c}{2}\right) - \sigma_f^- t_f w\left(\frac{t_f + t_c}{2}\right)$$

$$\sigma_f^+ = \frac{1}{2}\left(\frac{N}{t_f w} + \frac{2M}{t_f w(t_f + t_c)}\right)$$

$$\sigma_f^- = \frac{1}{2}\left(\frac{N}{t_f w} - \frac{2M}{t_f w(t_f + t_c)}\right)$$

The units of these internal forces are:

N(N) and M(N·m)

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Stress field in sandwich beams

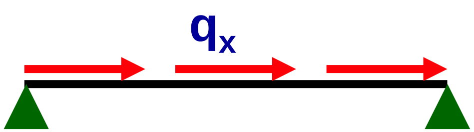
Shear stresses

Equilibrium equation in direction x:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + X = 0 \quad \Rightarrow \quad \frac{\partial \tau_{xz}}{\partial z} = -X - \frac{\partial \sigma_x}{\partial x}$$

X = Forces per unit volume

q_x= Forces per unit length in direction X



$$X = \frac{q_x}{w \cdot 2t_f}$$

Force per unit length, q_x; is uniformly distributed in the face-sheets section

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Stress field in sandwich beams

Shear stresses

Force per unit length, q_x; is uniformly distributed in the face-sheets section

- In the core:

$$\begin{matrix} X = 0 \\ \sigma_x = 0 \end{matrix} \rightarrow \frac{\partial \tau_{xz}}{\partial z} = 0 \rightarrow \tau_{xz} = \tau_c = cte$$

- In the skins:

$$\begin{matrix} X = \dfrac{q_x(x)}{2wt_f} \\ \sigma_x = \sigma_f(x) \end{matrix} \rightarrow \frac{\partial \tau_{xz}}{\partial z} = f(x) \rightarrow \tau_{xz} \text{ directly related to z}$$

In external faces: $\tau_{xz} = 0$

To verify the equilibrium equations there must be a discontinuity the shear stresses $\tau_{xz} = \tau_c$

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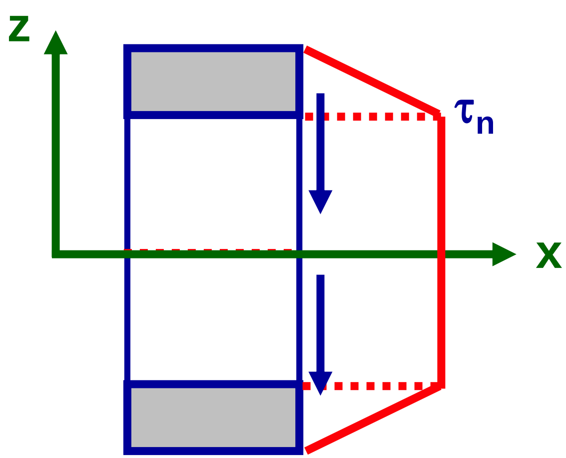
Stress field in sandwich beams

Shear stresses



Equilibrium equation for bottom face-sheet:

$$\frac{\partial \tau_{xz}}{\partial z} = -\frac{\partial \sigma_f^-}{\partial x} - \frac{q_x}{2wt_f}$$

Boundary conditions:

$$z = -\left(\frac{t_c}{2} + t_f\right) \rightarrow \tau_{xz} = 0$$

$$z = -\frac{t_c}{2} \qquad \tau_{xz} = \tau_c$$

$$\Rightarrow \quad \tau_c = -t_f\left(\frac{\partial \sigma_f^-}{\partial x} + \frac{q_x}{2wt_f}\right)$$

$$\tau_c = \frac{1}{2w}\left(\frac{\partial N}{\partial x} + \frac{2}{(t_f + t_c)}\frac{\partial M}{\partial x} - q_x\right)$$

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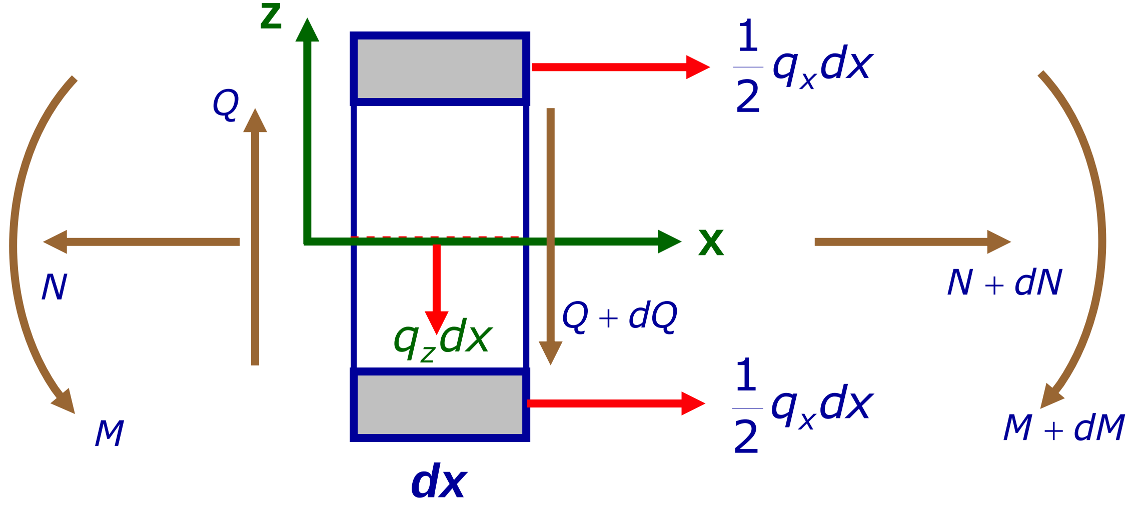
Shear force

Equilibrium equations of a beam element of differential length *dx*



- Longitudinal forces equilibrium

$$-N+(N+dN)+q_x dx=0$$

$$\frac{dN}{dx}=-q_x$$

- Transverse forces equilibrium

$$-Q+(Q+dQ)+q_Z dx=0$$

$$\frac{dQ}{dx}=-q_Z$$

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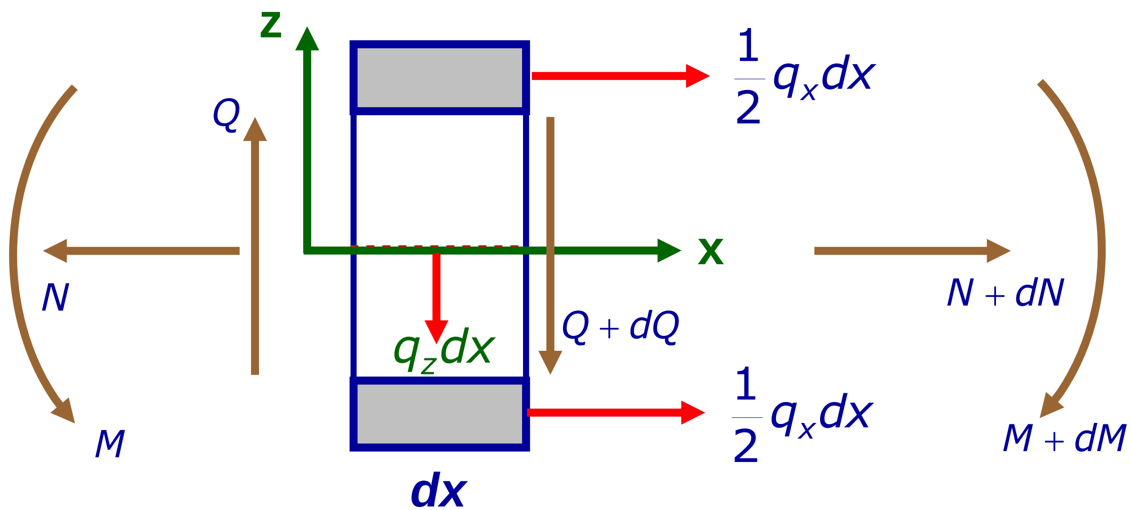
Stress field in sandwich beams

Shear force

Equilibrium equations of a beam element of differential length *dx*



- Moments equilibrium

$$-M + (M + dM) + (Q + dQ)dx + q_z dx \frac{dz}{2} = 0$$

$$dM + Qdx = 0$$

$$\frac{dM}{dx} = -Q$$

Shear force

Shear stress as a function of shear force:

$$\tau_c = -\frac{Q}{w\left(t_f + t_c\right)}$$

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Chapter 4. Composite beams and plates

Session 10. Sandwich structures

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Strains in sandwich beams

Normal stress in the face-sheets mid-plane:

$$\sigma_x(z) \approx E(z)\cdot\left(\varepsilon_x^o + \frac{z}{R}\right) \quad \begin{cases} z = \dfrac{t_f + t_c}{2} & \sigma_f^+ = E_f\left(\varepsilon_x^o + \dfrac{1}{R}\dfrac{t_f + t_c}{2}\right) \\ z = -\dfrac{t_f + t_c}{2} & \sigma_f^- = E_f\left(\varepsilon_x^o - \dfrac{1}{R}\dfrac{t_f + t_c}{2}\right) \end{cases}$$

Replacing the values of σ_f^+ *and* σ_f^-

$$\frac{1}{2}\left(\frac{N}{t_f w} + \frac{2M}{t_f w(t_f + t_c)}\right) = E_f\left(\varepsilon_x^o + \frac{1}{R}\frac{t_f + t_c}{2}\right)$$

$$\frac{1}{2}\left(\frac{N}{t_f w} - \frac{2M}{t_f w(t_f + t_c)}\right) = E_f\left(\varepsilon_x^o - \frac{1}{R}\frac{t_f + t_c}{2}\right)$$

$\Rightarrow$

$$\varepsilon_x^o = \frac{N}{2E_f w t_f}$$

$$\frac{1}{R} = \frac{2M}{E_f w t_f (t_f + t_c)^2}$$

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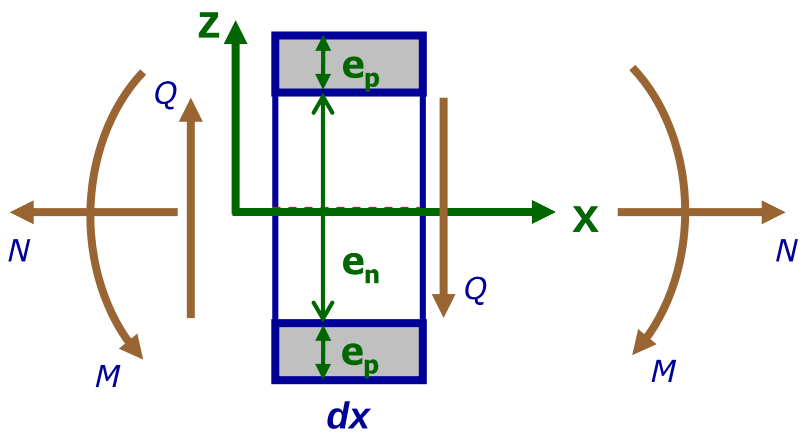
Strains in sandwich beams

Summary



- w = beam width
- tf = face-sheet thickness
- t_c = core thickness
- N = Axial force
- Q = Shear force
- M = Bending moment

Stresses:

$$\sigma_f^+ = \frac{1}{2}\left(\frac{N}{t_f w} + \frac{2M}{t_f w(t_f + t_c)}\right)$$

$$\sigma_p^- = \frac{1}{2}\left(\frac{N}{t_f w} - \frac{2M}{t_f w(t_f + t_c)}\right)$$

$$\tau_c = -\frac{Q}{B(t_f + t_c)}$$

Strains:

$$\varepsilon_x(z) = \varepsilon_x^o + \frac{1}{R} z$$

$$\varepsilon_x^o = \frac{N}{2E_f w t_f}$$

$$\frac{1}{R} = \frac{2M}{E_f w t_f (t_f + t_f)^2}$$

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Chapter 4. Composite beams and plates

Sandwich structures

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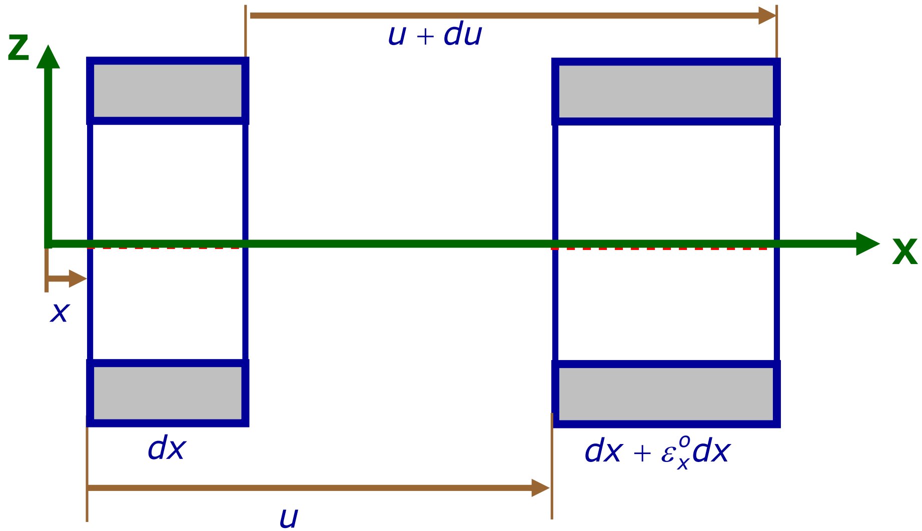
Displacements

Longitudinal displacements:



$$dx + (u + du) = u + dx + \varepsilon_x^o dx$$

$$\frac{du}{dx} = \varepsilon_x^o$$

Integrating from the initial section to a generic section:

$$u = u_A + \int_{x_A}^{x} \varepsilon_x^o \, dx$$

Substituting the value of ε_x^o

$$u = u_A + \int_{x_A}^{x} \frac{N}{2E_f w t_f} dx \quad \rightarrow$$

$$u = u_A + C_n \int_{x_A}^{x} N \, dx$$

$$C_n = \frac{1}{E_f w t_f}$$

$$\langle EA \rangle = 2 \cdot E_f w t_f$$

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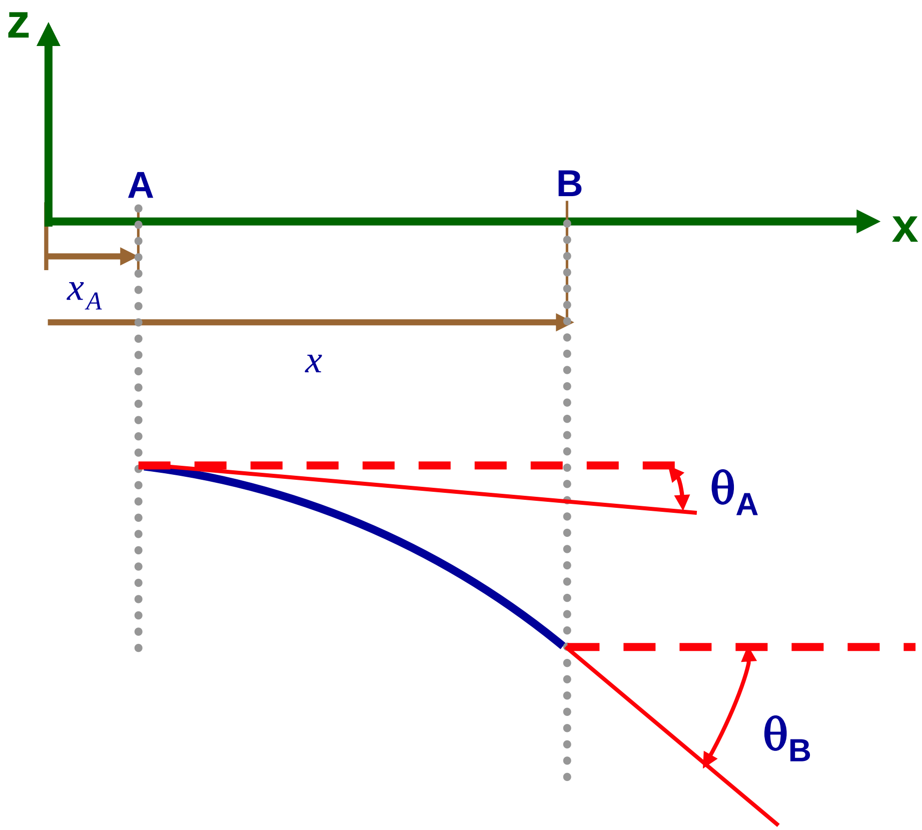
Displacements

Rotation



$$R \cdot d\theta = dx\left(1 + \varepsilon_x^o\right)$$

$$\frac{d\theta}{dx} = \frac{1 + \varepsilon_x^o}{R} \approx \frac{1}{R}$$

$$\theta - \theta_A = \int_{x_A}^{x} \frac{1}{R} dx$$

Substituting R by its value:

$$\theta = \theta_A + C_m \int_{x_A}^{x} M \, dx$$

$$C_m = \frac{2}{E_f w t_f \left(t_f + t_c\right)^2}$$

$$\langle EI \rangle = \frac{1}{2} \cdot E_f \cdot w \cdot t_f \cdot \left(t_f + t_c\right)^2$$

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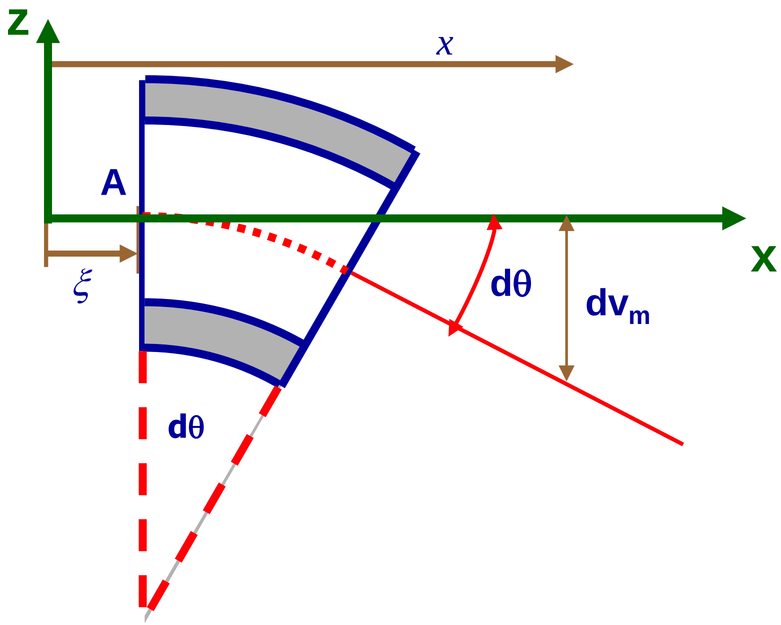
Displacements

Transverse displacements

- Due to rotations



dv_m= section transverse displacement at x due to the relative rotation, dθ, between section ξ y other section infinitely near

$$d\theta = \frac{dv_m}{(x-\xi)}$$

$$d\theta = MC_m dx \quad \rightarrow \quad dv_m = (x-\xi) MC_m dx$$

The relative displacement between A and x can be found integrating the transverse displacements produced by the rotation of the sections between A and x. It is necessary to add the displacement due to the rotation of section A, θ_A

$$v_m = v_A + \theta_A (x - x_A) + C_m \int_{\xi = x_A}^{\xi = x} M (x - \xi) d\xi$$

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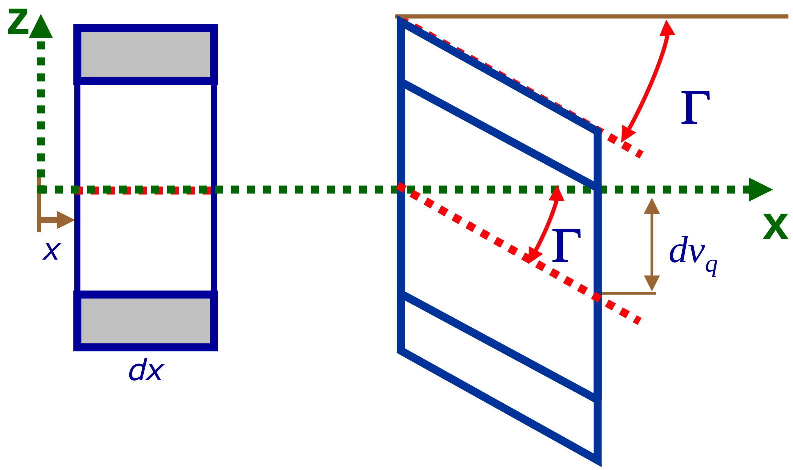
Transverse displacements

- Due to shear strains



There are relative transverse displacements in a section produced by shear strains

$$dv_q = \Gamma\, dx \quad \rightarrow \quad v_q = v_q^A + \int_{x_A}^{x} \Gamma\, dx$$

Shear strains are associated to shear stresses.

$$\gamma = \frac{\tau}{G}$$

G is a material property, thus it has different values in face-sheets and core, moreover τ is not constant through face-sheet thickness. Γ can not be identify with a shear strain γ because this is not constant.

Displacements

Transverse displacements

- Due to shear strains

Using the principle of virtual works in a section subjected to internal forces N, Q, M.

$$M\cdot d\theta + N\cdot du + Q\cdot\Gamma\, dx = \int_{Area}\left(\sigma_x\varepsilon_x + \tau_{xx}\gamma_{xz}\right)dA\cdot dx = \int_{Area}\left(\frac{\sigma_x^2}{E}+\frac{\tau_{xz}^2}{G}\right)dA\cdot dx$$

σ_x and τ_{xy} can be replaced by their values

$$\sigma_x\begin{cases}\text{In face-sheets} & \sigma_x=\sigma_f^+ \text{ or } \sigma_f^- \\ \text{In core} & \sigma_x = 0\end{cases}$$

$$\tau_{xy}\begin{cases}\text{In face-sheets} & \begin{cases} z=\pm\left(t_f+t_c/2\right)\rightarrow\tau_{xy}=0 \\ z=\pm\left(t_c/2\right)\rightarrow\tau_{xy}=\tau_n\end{cases} \\ \text{In core} & \tau_{xy}=\tau_n\end{cases}$$

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Displacements

Transverse displacements

- Due to shear strains

$$M\cdot\frac{d\theta}{dx}+N\cdot\frac{du}{dx}+Q\cdot\Gamma=\frac{\sigma_f^{+2}}{E_f}wt_f+\frac{\sigma_f^{-2}}{E_f}wt_f+\frac{\tau_c^2}{G_c}wt_c+\frac{2\tau_c^2}{3G_f}wt_f$$

$$\frac{du}{dx}=C_n\cdot N \qquad \frac{d\theta}{dx}=C_m\cdot M$$

$$M^2\cdot C_m+N^2\cdot C_n+Q\cdot\Gamma=M^2\cdot C_m+N^2\cdot C_n+Q^2\frac{t_c}{w(t_c+t_f)^2G_c}\cdot\left(1+\frac{2t_fG_c}{3t_cG_f}\right)$$

$$\Gamma=C_q\cdot Q$$

$$C_q=\frac{t_c}{w(t_c+t_f)^2G_c}\cdot\left(1+\frac{2t_fG_c}{3t_cG_f}\right)$$

$$\langle GA_s\rangle=\frac{w(t_c+t_f)^2G_c}{t_c\cdot\left(1+\frac{2t_fG_c}{3t_cG_f}\right)}$$

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Displacements

Transverse displacements

Thus global transverse displacements yields:

$$v = v_A + \theta_A (x - x_A) + C_m \int_{\xi = x_A}^{\xi = x} M(x - \xi)\, d\xi + C_q \int_{x_A}^{x} Q\, d\xi$$

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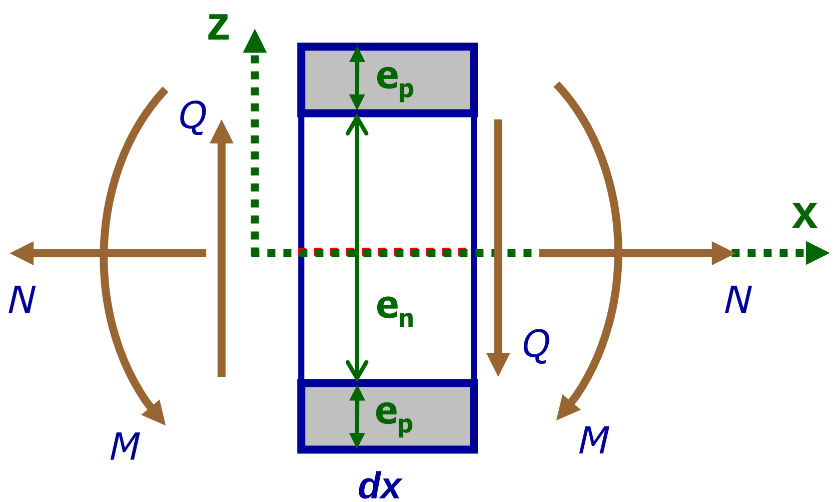
Displacements

Summary



$$u = u_A + C_n \int_{x_A}^{x} N\, dx$$

$$\theta = \theta_A + C_m \int_{x_A}^{x} M\, dx$$

$$v = v_A + \theta_A (x - x_A) + C_m \int_{\xi = x_A}^{\xi = x} M (x - \xi)\, d\xi + C_q \int_{x_A}^{x} Q\, d\xi$$

$$C_n = \frac{1}{2E_f w t_f} \qquad C_m = \frac{2}{E_f w t_f (t_f + t_c)^2} \qquad C_q = \frac{t_c}{w (t_c + t_f)^2 G_c} \cdot \left(1 + \frac{2 t_f G_c}{3 t_c G_f}\right)$$

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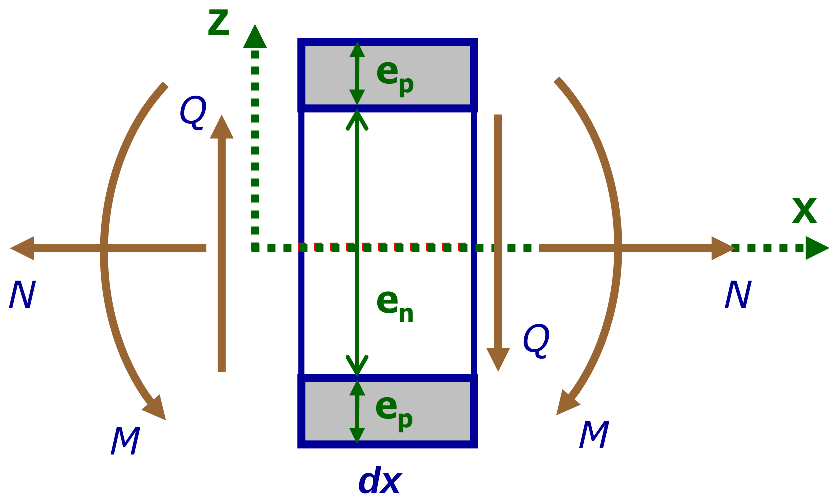
Displacements

Summary



$$u = u_A + \int_{x_A}^{x} \frac{N}{\langle EA \rangle} dx$$

$$\theta = \theta_A + \int_{x_A}^{x} \frac{M}{\langle EI \rangle} dx$$

$$v = v_A + \theta_A (x - x_A) + \int_{\xi = x_A}^{\xi = x} \frac{M}{\langle EI \rangle} (x - \xi) d\xi + \int_{x_A}^{x} \frac{Q}{\langle GA_c \rangle} d\xi$$

$$\langle EA \rangle = 2E_f w t_f \qquad \langle EI \rangle = \frac{1}{2} E_f w t_f (t_f + t_c)^2 \qquad \langle GA_s \rangle = \frac{w(t_c + t_f)^2 G_c}{t_c \cdot \left(1 + \frac{2t_f G_c}{3t_c G_f}\right)}$$

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Chapter 4. Composite beams and plates

Sandwich structures

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References

The Behaviour of Structures Composed of Composite Materials

J.R. Vinson y R.L. Sierakowski

Kluwer Academic Publishers. 2002.

Cap. 4: Beams, columns and Rods of Composite Materials

Mechanics of laminated composite plates. Theory and analysis

J.N. Reddy

CRC Press. 1997.

Cap 6: One- Dimensional Analysis of Laminated Plates

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