# Applications of the Integral

# Areas

- Area between the **graph of a function**, the $x$-axis, between $a$ and $b$:
$$A = \int_a^b |f| dx$$
- Area between the **graphs of two functions** $f, g$, between $a$ and $b$:
$$A = \int_a^b |f - g| dx$$
- Area using **parametric equations:** the area between the graph of $x = x(t),\ y = y(t)$ and the $x$-axis between $t = t_0$ and $t = t_1$ is:
$$A = \left| \int_{t_0}^{t_1} y(t)x'(t)dt \right|$$
- Area using **polar coordinates:** the area of the graph of $r = r(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ is
$$A = \int_\alpha^\beta \frac{1}{2} r^2(\theta) d\theta$$

- **Volume by parallel cross-sections:** if $A(x)$ is the area of parallel cross-sections over the entire length of a solid, the volume between $x = a$ and $x = b$ is
$$V = \int_a^b A(x)dx$$
- **The Disk method:** the volume of a solid of revolution obtained by rotating $|f(x)|$ about the $x$-axis between $x = a$ and $x = b$ is
$$V = \int_a^b \pi(f(x))^2 dx$$
- **The Shell method:** the volume of a solid of revolution obtained by rotating $f(x) \geq 0$, $x \in [a, b]$, $a \geq 0$, about the $y$-axis is
$$V = 2\pi \int_a^b xf(x)dx$$

# Lengths

- The **length of an arc of a curve** $f(x)$ between $x = a$ and $x = b$ is

$$L(f) = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

- If the curve is given in **parametric form**, the length is

$$L = \int_{t_0}^{t_1} \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

## Def.

$$\int_a^{\infty} f(x) = \lim_{N\to\infty} \int_a^N f(x).$$

If the limit is finite we say that the integral **converges** otherwise we say that the integral diverges.

## Theorem (Integral test for series)

*Consider $f \geq 0$ a monotone decreasing function defined for $x \geq 1$. Let $a_n = f(n)$, then*

$$\sum_{n=1}^{\infty} a_n \text{ and } \int_1^{\infty} f(x)\, dx,$$

*have the same behaviour, or both converge or both diverge.*