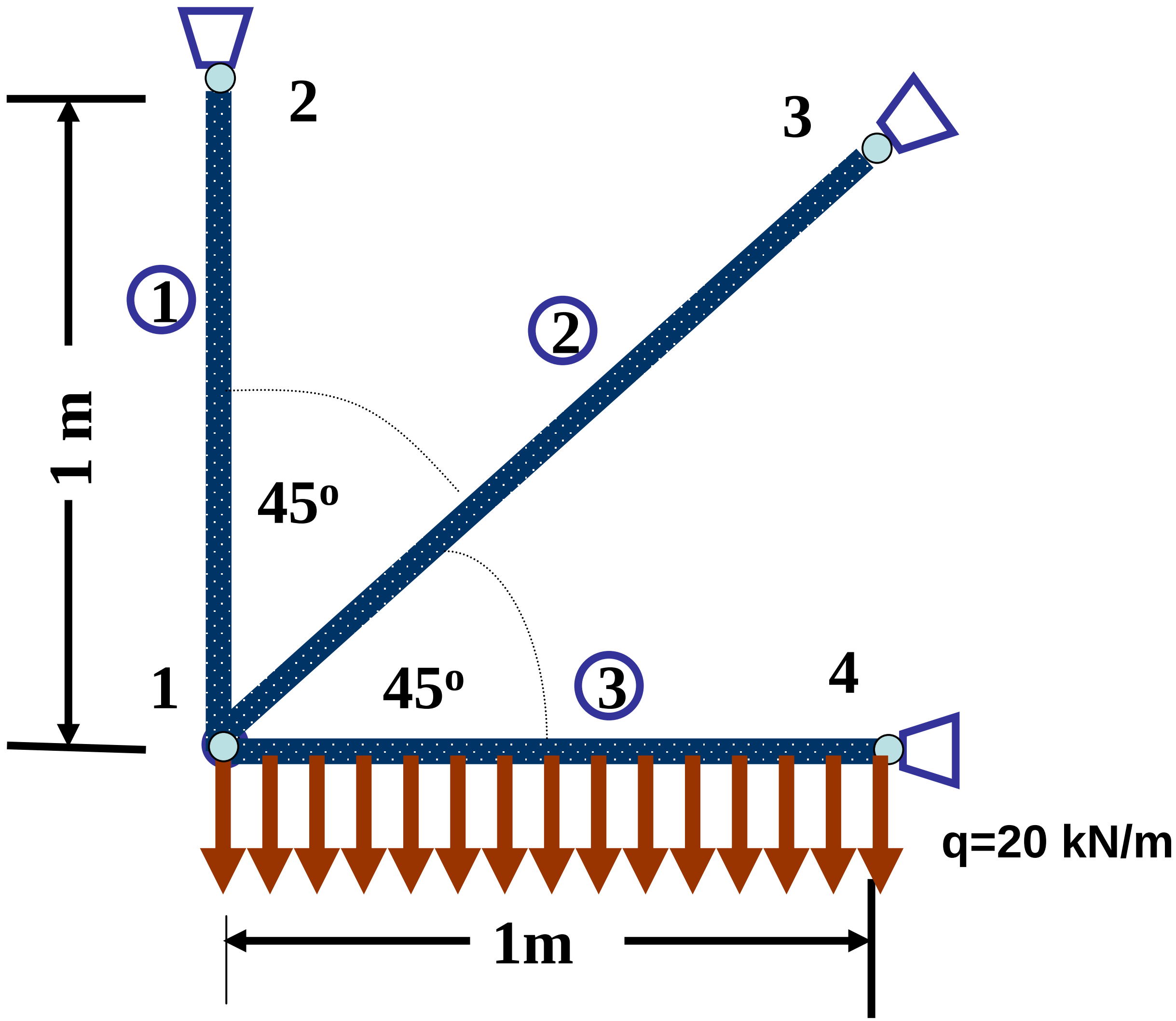
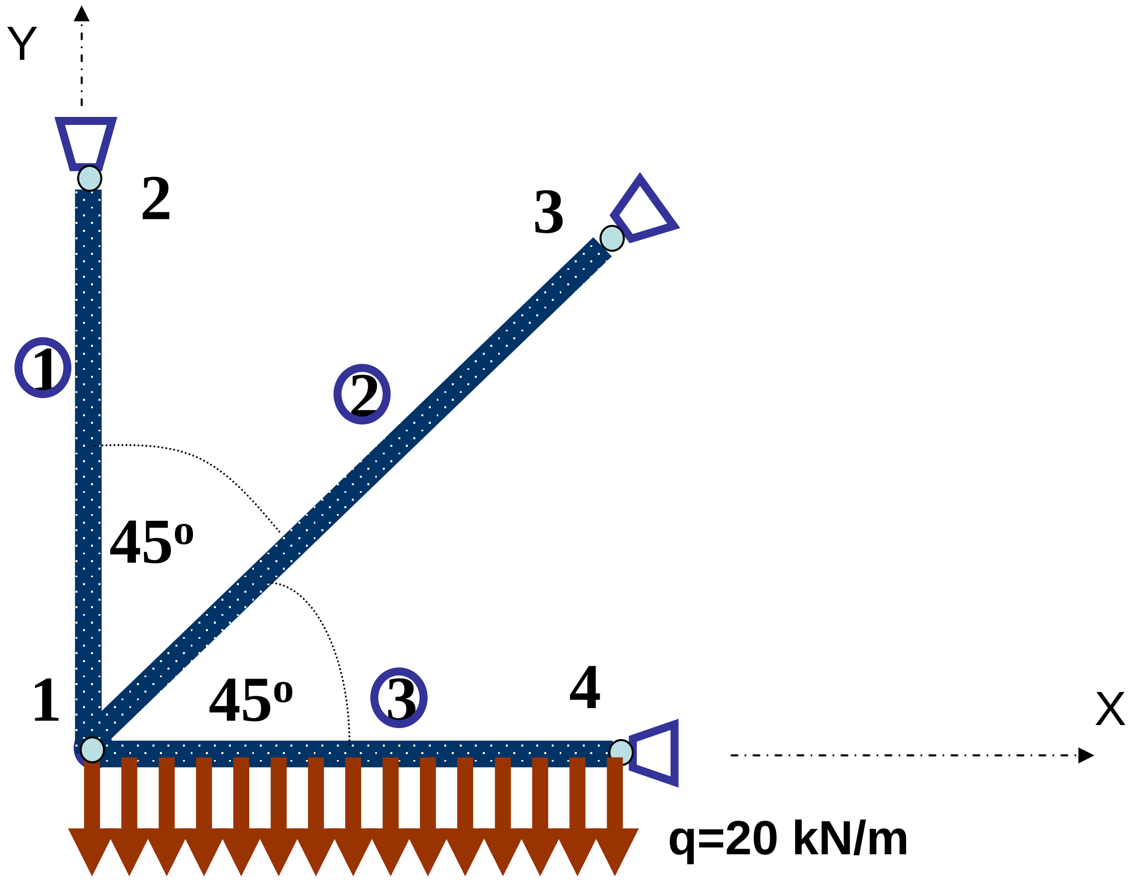
La estructura de la figura se encuentra sometida a la acción de un sobrecarga Uniforme en la barra nº 3. Determinar los desplazamientos del nudo 1 así como los esfuerzos axiles y tensiones en las barras.





E = 200 x 10^9 Pa para todas las barras

Elemento	Nudo i	Nudo j	L (m)	A (cm^2)	θ	C	S	C^2	S^2	CS
1	1	2	1,000	2,25	90º	0	1	0	1	0
2	1	3	1,000	2,25	45º	0,7071	0,7071	0,5	0,5	0,5
3	1	4	1,414	2,25	0	1	0	1	0	0

$$\left[\boldsymbol{k}^{(1)}\right] = \frac{(200 \cdot 10^{9})\,(2{,}25 \cdot 10^{-4})}{1} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$4{,}5 \times 10^{7}$ N/m

$$\left[\boldsymbol{k}^{(2)}\right]=\frac{(200\cdot 10^{9})\,(2{,}25\cdot 10^{-4})}{\sqrt{2}}\begin{bmatrix} 0{,}5 & 0{,}5 & -0{,}5 & -0{,}5 \\ 0{,}5 & 0{,}5 & -0{,}5 & -0{,}5 \\ -0{,}5 & -0{,}5 & 0{,}5 & 0{,}5 \\ -0{,}5 & -0{,}5 & 0{,}5 & 0{,}5 \end{bmatrix}$$

$$\left[\boldsymbol{k}^{(3)}\right]=\frac{(200\cdot 10^{9})\,(2{,}25\cdot 10^{-4})}{1}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
\left[\mathbf{K}\right]=\begin{bmatrix}
\mathbf{k}_{11}^{(1)} & \mathbf{k}_{12}^{(1)} & \mathbf{k}_{13}^{(1)} & \mathbf{k}_{14}^{(1)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{k}_{21}^{(1)} & \mathbf{k}_{22}^{(1)} & \mathbf{k}_{23}^{(1)} & \mathbf{k}_{24}^{(1)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{k}_{31}^{(1)} & \mathbf{k}_{32}^{(1)} & \mathbf{k}_{33}^{(1)} & \mathbf{k}_{34}^{(1)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{k}_{41}^{(1)} & \mathbf{k}_{42}^{(1)} & \mathbf{k}_{43}^{(1)} & \mathbf{k}_{44}^{(1)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{bmatrix}+\cdots
$$

$$\cdots\begin{bmatrix} k_{11}^{(2)} & k_{12}^{(2)} & 0 & 0 & k_{13}^{(2)} & k_{14}^{(2)} & 0 & 0 \\ k_{21}^{(2)} & k_{22}^{(2)} & 0 & 0 & k_{23}^{(2)} & k_{24}^{(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_{31}^{(2)} & k_{32}^{(2)} & 0 & 0 & k_{33}^{(2)} & k_{34}^{(2)} & 0 & 0 \\ k_{41}^{(2)} & k_{42}^{(2)} & 0 & 0 & k_{43}^{(2)} & k_{44}^{(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \cdots$$

$$\cdots\begin{bmatrix} k_{11}^{(3)} & k_{12}^{(3)} & 0 & 0 & 0 & 0 & k_{13}^{(3)} & k_{14}^{(3)} \\ k_{21}^{(3)} & k_{22}^{(3)} & 0 & 0 & 0 & 0 & k_{23}^{(3)} & k_{24}^{(3)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_{31}^{(3)} & k_{32}^{(3)} & 0 & 0 & 0 & 0 & k_{33}^{(3)} & k_{34}^{(3)} \\ k_{41}^{(3)} & k_{42}^{(3)} & 0 & 0 & 0 & 0 & k_{43}^{(3)} & k_{44}^{(3)} \end{bmatrix}$$

Matriz de rigidez reducida:

$$\boldsymbol{K_r} = \begin{bmatrix} \boldsymbol{k}_{11}^1 + \boldsymbol{k}_{11}^2 + \boldsymbol{k}_{11}^3 & \boldsymbol{k}_{12}^1 + \boldsymbol{k}_{12}^2 + \boldsymbol{k}_{12}^3 \\ \boldsymbol{k}_{21}^1 + \boldsymbol{k}_{21}^2 + \boldsymbol{k}_{21}^3 & \boldsymbol{k}_{22}^1 + \boldsymbol{k}_{22}^2 + \boldsymbol{k}_{22}^3 \end{bmatrix}$$

$$\mathbf{Kr} = \begin{pmatrix} 6.091\times 10^7 & 1.591\times 10^7 \\ 1.591\times 10^7 & 6.091\times 10^7 \end{pmatrix}$$

$$\{F\} = \begin{Bmatrix} 0 \\ -10000 \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} - 10000 \end{Bmatrix} \qquad \{d\} = \begin{Bmatrix} u_{1X} \\ u_{1Y} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ -10000 \end{Bmatrix} = (4{,}5 \cdot 10^{7}) \begin{bmatrix} 1{,}354 & 0{,}354 \\ 0{,}354 & 1{,}354 \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_{1\boldsymbol{X}} \\ \boldsymbol{u}_{1\boldsymbol{Y}} \end{Bmatrix}$$

$$\begin{Bmatrix} \boldsymbol{u}_{1\boldsymbol{X}} \\ \boldsymbol{u}_{1\boldsymbol{Y}} \end{Bmatrix} = \begin{Bmatrix} 4{,}606\ \boldsymbol{x}\, 10^{-5} \\ -1{,}762\ \boldsymbol{x}\, 10^{-4} \end{Bmatrix} \boldsymbol{m}$$

Tensiones: $$\sigma = E\varepsilon = \frac{E}{L}\left(u'_{x2} - u'_{x1}\right) = \frac{E}{L}\begin{bmatrix}-c & -s & c & s\end{bmatrix}\{u\}_{ejes\ globales}$$

$$\sigma_1 := \frac{200\cdot 10^9}{1}\begin{pmatrix}0 & -1 & 0 & 1\end{pmatrix}\begin{pmatrix}4.606\cdot 10^{-5}\\ -1.762\cdot 10^{-4}\\ 0\\ 0\end{pmatrix}$$

$\sigma_1 = 3.524\times 10^{7}$ Pa

$$\sigma_2 := \frac{200\cdot 10^9}{\sqrt{2}}\begin{pmatrix}\frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}\begin{pmatrix}4.606\cdot 10^{-5}\\ -1.762\cdot 10^{-4}\\ 0\\ 0\end{pmatrix}$$

$\sigma_2 = 1.301\times 10^{7}$ Pa

$$\sigma_3 := \frac{200\cdot 10^9}{1}\begin{pmatrix}-1 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}4.606\cdot 10^{-5}\\ -1.762\cdot 10^{-4}\\ 0\\ 0\end{pmatrix}$$

$\sigma_3 = -9.212\times 10^{6}$ Pa

Axiles:

$$\boldsymbol{N}_1 = \sigma_1 \boldsymbol{A} = 7{,}929\ \boldsymbol{kN}\ \left(\boldsymbol{Tracción}\right)$$
$$\boldsymbol{N}_2 = \sigma_2 \boldsymbol{A} = 2{,}927\ \boldsymbol{kN}\ \left(\boldsymbol{Tracción}\right)$$
$$\boldsymbol{N}_3 = \sigma_3 \boldsymbol{A} = 2{,}072\ \boldsymbol{kN}\ \left(\boldsymbol{Compresión}\right)$$

Nota: Alternativamente, podíamos haber calculado primero los esfuerzos axiles utilizando:

$$N = \mathrm{EA}\varepsilon = \frac{\mathrm{EA}}{\mathrm{L}}\left(\mathrm{u'}_{\mathrm{X2}} - \mathrm{u'}_{\mathrm{X1}}\right) = \frac{\mathrm{EA}}{\mathrm{L}}\begin{bmatrix}-c & -s & c & s\end{bmatrix}\{u\}_{ejes\ globales}$$

y, posteriormente, dividiendo dichos esfuerzos por el área de cada barra, las tensiones.