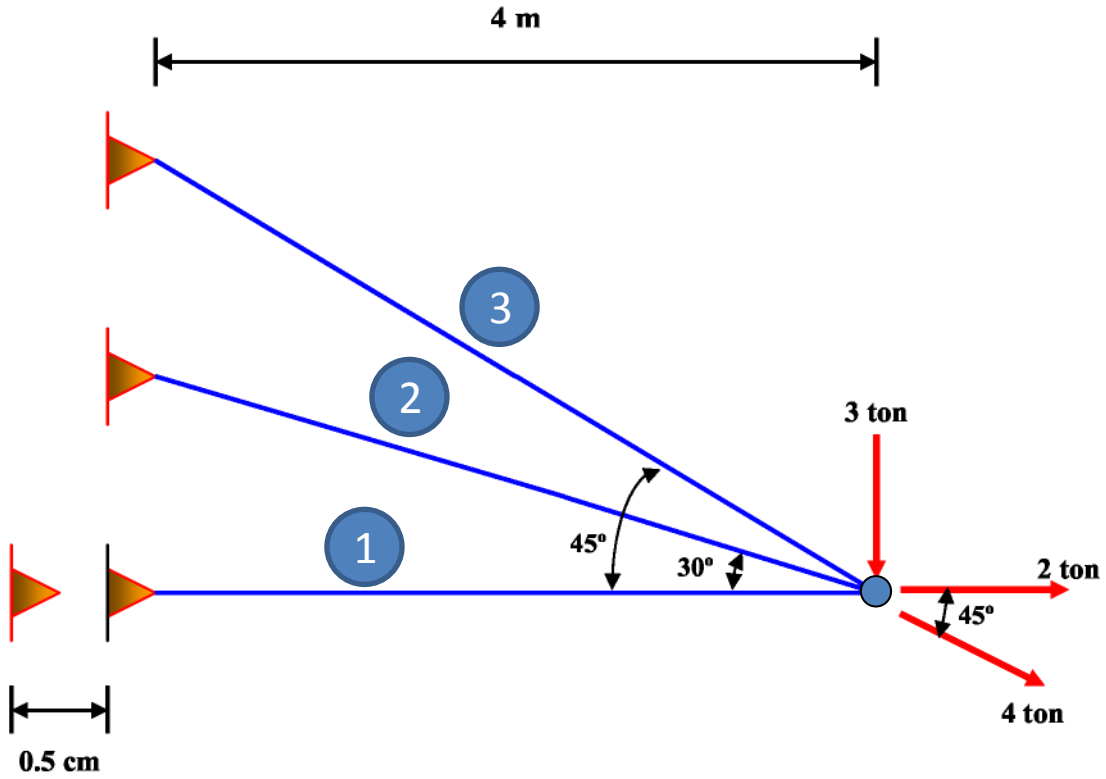
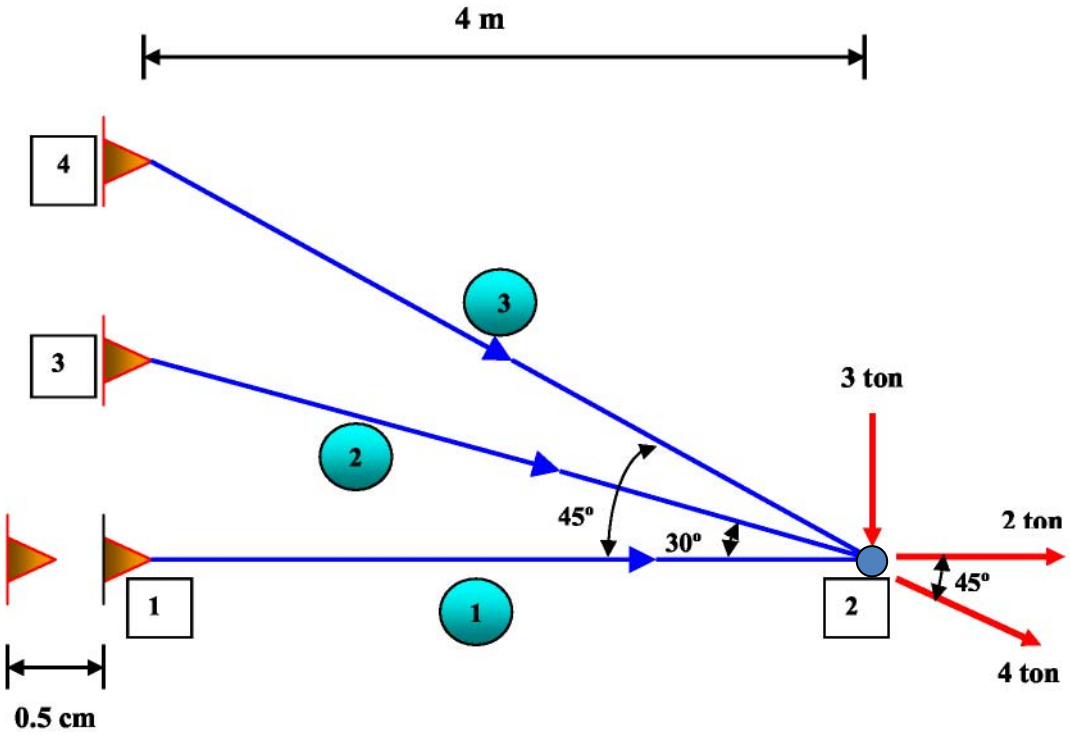


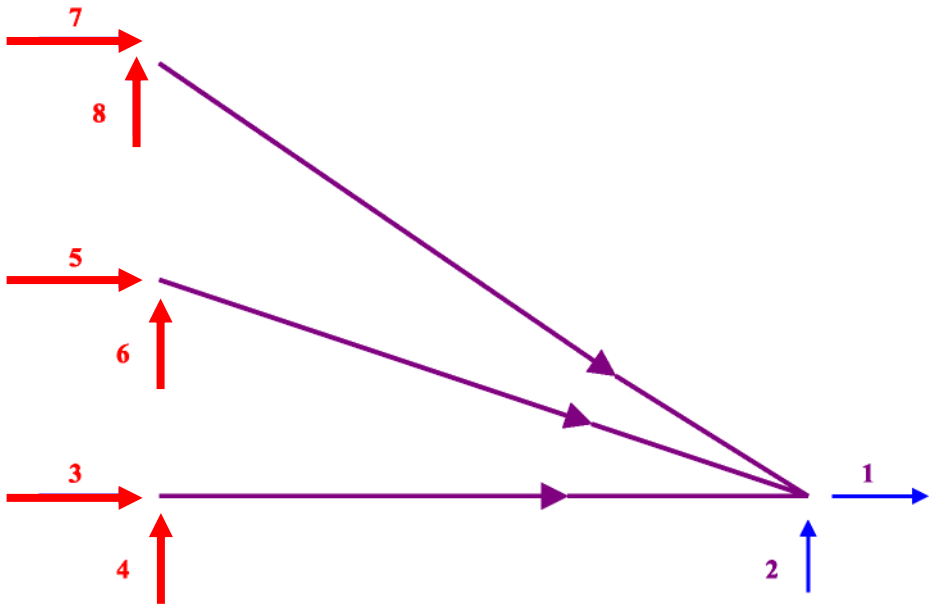
BARRA 1 $A = 10\ cm^2$
BARRA 2 $A = 9\ cm^2$
BARRA 3 $A = 6\ cm^2$

$E=2\cdot10^7\ t/m^2$

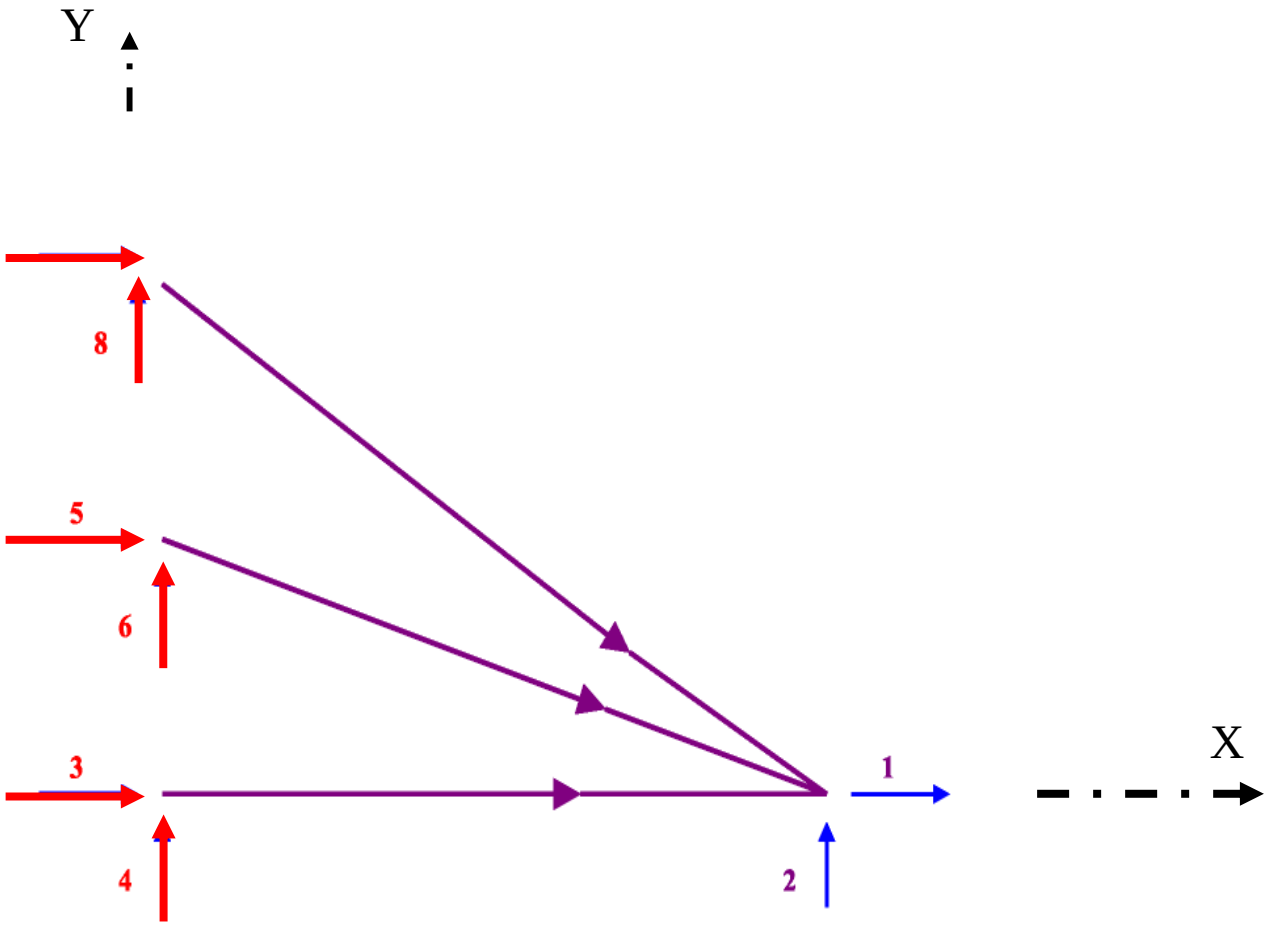
PASO 1 Enumeración de nudos, elementos y sentido de análisis de las barras.



PASO 2 Enumeración de los grados de libertad. Color rojo, los restringidos.



EJES GLOBALES DE LA ESTRUCTURA



MATRIZ DE CAMBIO DE EJES GLOBALES A LOCALES

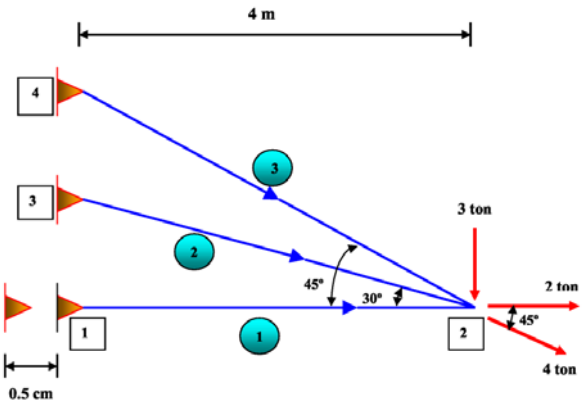
$$[T]=\begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$$

$$\{F\}_{ejes\ locales} = [T]\{F\}_{ejes\ globales}$$

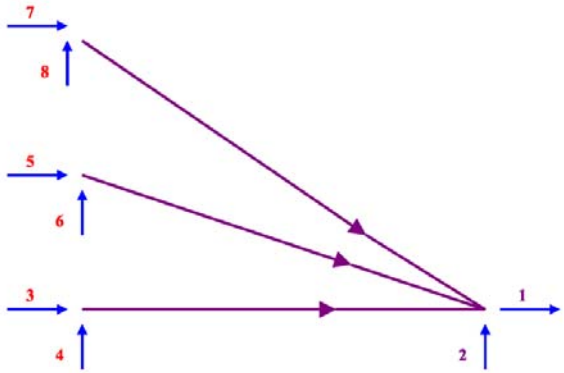
$$\{u\}_{ejes\ locales} = [T]\{u\}_{ejes\ globales}$$

$$\{F\}_{ejes\ globales} = [T]^{-1}[K^e][T]\{u\}_{ejes\ globales}$$

OBTENCIÓN DE LAS MATRICES DE RIGIDEZ DE LOS ELEMENTOS DE LA ESTRUCTURA EN EJES GLOBALES



PASO 2 Enumeración de los grados de libertad. Color rojo, los restringidos.



PASO 3 Cálculo matrices elementales

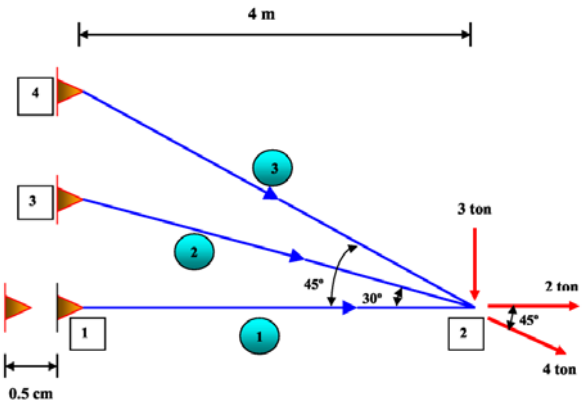
Elemento 1

Xi = 0 Xf = 4 m
Yi = 0 Yf = 0
L = 4 m Cos α = 1 Sen α = 0
A = 10 cm² = 0.001 m² E = 2E7 Ton/m²

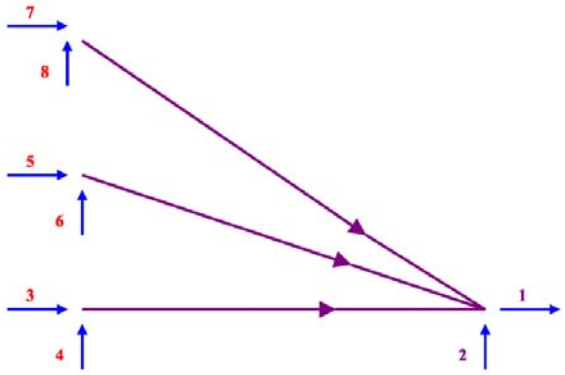
$$\alpha = 0^{o}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$[K]_1 = \begin{matrix} \\ \\ \\ \\ \end{matrix} \begin{matrix} (3) & (4) & (1) & (2) \end{matrix} \\ \begin{bmatrix} 5000 & 0 & -5000 & 0 \\ 0 & 0 & 0 & 0 \\ -5000 & 0 & 5000 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} (3) \\ (4) \\ (1) \\ (2) \end{matrix}$$



PASO 2 Enumeración de los grados de libertad. Color rojo, los restringidos.



Elemento 2

Xi = 0 Xf = 4 m

$Yi = \frac{4*\sqrt{3}}{3} \approx 2.3094\,m$ Yf = 0

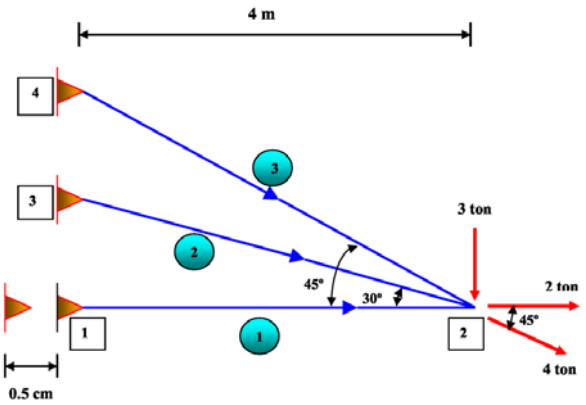
$L = \sqrt{\frac{192}{9}} \approx 4.619\,m$ $Cos\ \alpha = \frac{\sqrt{3}}{2} = 0.866$ Sen α = - 0.5

A = 9 cm² = 0.0009 m² E = 2E7 Ton/m²

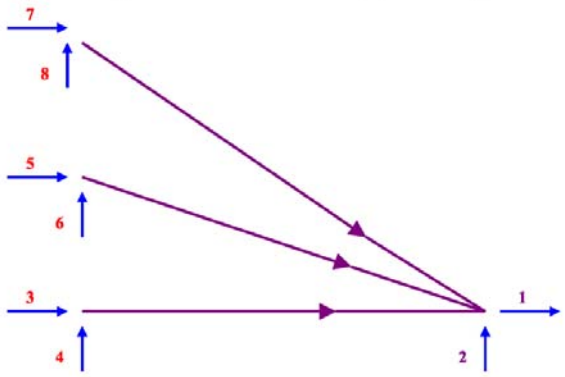
$$\alpha := \frac{-\pi}{6}$$

$$T = \begin{pmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 0.866 & -0.5 \\ 0 & 0 & 0.5 & 0.866 \end{pmatrix}$$

$$[K]_2 = \begin{array}{c} \begin{matrix} (5) & (6) & (1) & (2) \end{matrix} \\ \begin{bmatrix} 2922836 & -1687.5 & -2922.836 & 1687.5 \\ -1687.5 & 974.279 & 1687.5 & -974.279 \\ -2922.836 & 1687.5 & 2922.836 & -1687.5 \\ 1687.5 & -974.279 & -1687.5 & 974.279 \end{bmatrix} \end{array} \begin{matrix} (5) \\ (6) \\ (1) \\ (2) \end{matrix}$$



PASO 2 Enumeración de los grados de libertad. Color rojo, los restringidos.



Elemento 3

$Xi = 0 \qquad Xf = 4\ m$

$Yi = 4\ m \qquad Yf = 0$

$L = 4\sqrt{2}\ m \cong 5.657\ m \qquad \cos\alpha = \frac{\sqrt{2}}{2} \approx 0.707 \qquad \operatorname{sen}\alpha = \frac{-\sqrt{2}}{2} \cong -0.707$

$A = 6\ cm^2 = 0.0006\ m^2 \qquad E = 2E7\ Ton/m^2$

$$\alpha := \frac{-\pi}{4}$$

$$T = \begin{pmatrix} 0.707 & -0.707 & 0 & 0 \\ 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 0.707 & -0.707 \\ 0 & 0 & 0.707 & 0.707 \end{pmatrix}$$

$$[K]_3 = \begin{array}{c} \begin{matrix} \text{(7)} & \text{(8)} & \text{(1)} & \text{(2)} \end{matrix} \\ \begin{bmatrix} 1060.66 & -1060.66 & -1060.66 & 1060.66 \\ -1060.66 & 1060.66 & 1060.66 & -1060.66 \\ -1060.66 & 1060.66 & 1060.66 & -1060.66 \\ 1060.66 & -1060.66 & -1060.66 & 1060.66 \end{bmatrix} \end{array} \begin{matrix} \text{(7)} \\ \text{(8)} \\ \text{(1)} \\ \text{(2)} \end{matrix}$$

PASO 4 Cálculo matriz total de la estructura

$$[K] = \begin{array}{c} \begin{matrix} (1) & (2) & (3) & (4) & (5) & (6) & (7) & (8) \end{matrix} \\ \left[\begin{array}{cc:cc:cccc} 8983496 & -2748.16 & -5000 & 0 & -2922.836 & 1687.5 & -1060.66 & 1060.66 \\ -2748.496 & 2034.939 & 0 & 0 & 1687.5 & -974.279 & 1060.66 & -1060.66 \\ \hdashline -5000 & 0 & 5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2922.836 & 1687.5 & 0 & 0 & 2922.836 & -1687.5 & 0 & 0 \\ 1687.5 & -974.279 & 0 & 0 & -1687.5 & 974.279 & 0 & 0 \\ -1060.66 & 1060.66 & 0 & 0 & 0 & 0 & 1060.66 & -1060.66 \\ 1060.66 & -1060.66 & 0 & 0 & 0 & 0 & -1060.66 & 1060.66 \end{array}\right] \end{array} \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \\ (8) \end{matrix}$$

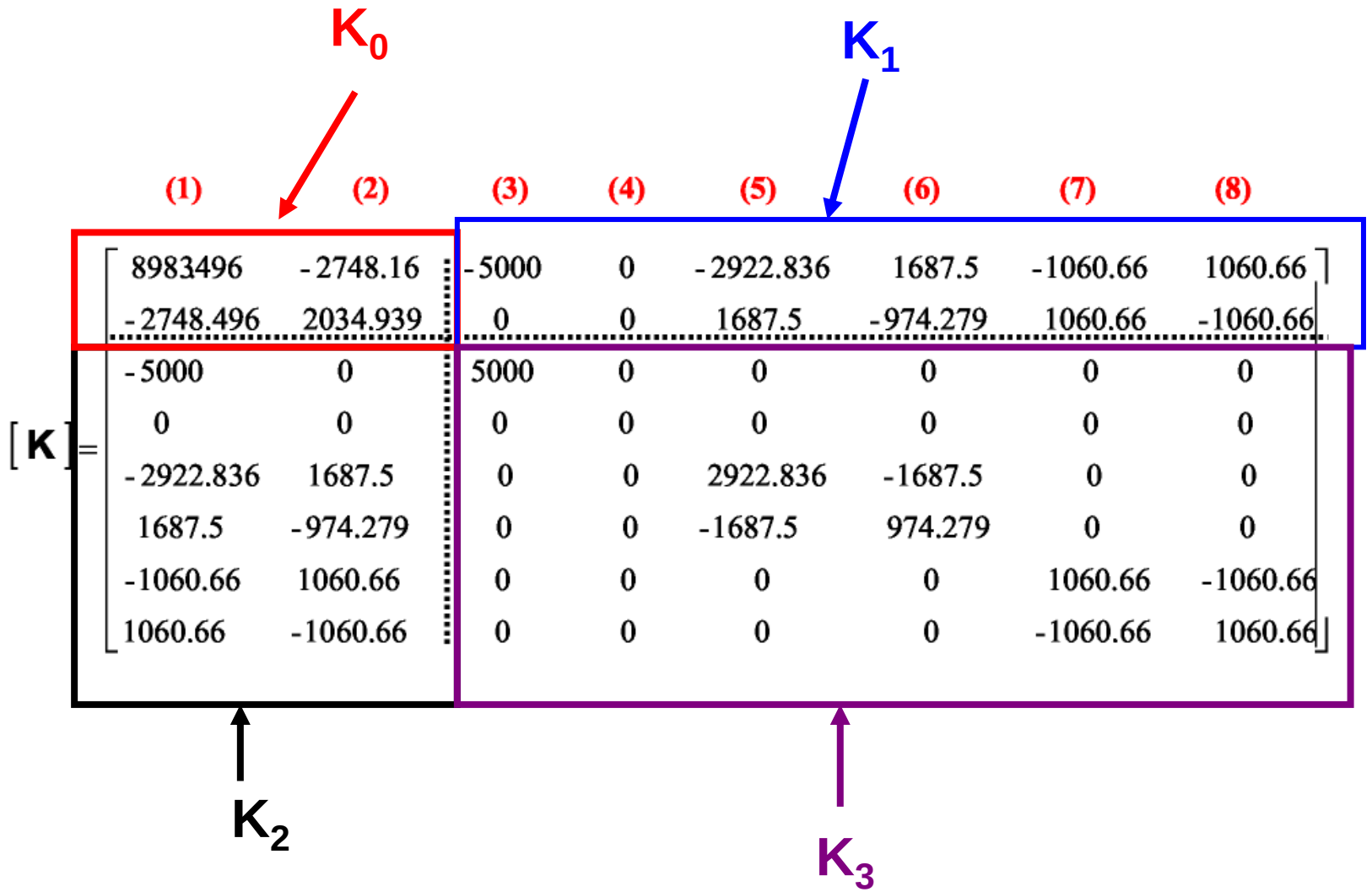
PASO 5 Cálculo vector de cargas nodales.

$$\{F\} = \begin{Bmatrix} F0 \\ \hline F1 \end{Bmatrix} = \begin{Bmatrix} 4.8284 \\ -5.8284 \\ \hline F3 \\ F4 \\ F5 \\ F6 \\ F7 \\ F8 \end{Bmatrix}$$

= 4*cos(45)+2
= -4*sen(45)-3

PASO 5 Cálculo vector de desplazamientos.

$$\{U\} = \begin{Bmatrix} U0 \\ \hline U1 \end{Bmatrix} = \begin{Bmatrix} U1 \\ U2 \\ \hline -0.005 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$


K0
K1
(1)
(2)
(3)
(4)
(5)
(6)
(7)
(8)
[K] =
8983496 -2748.16 -5000 0 -2922.836 1687.5 -1060.66 1060.66
-2748.496 2034.939 0 0 1687.5 -974.279 1060.66 -1060.66
-5000 0 5000 0 0 0 0 0
0 0 0 0 0 0 0 0
-2922.836 1687.5 0 0 2922.836 -1687.5 0 0
1687.5 -974.279 0 0 -1687.5 974.279 0 0
-1060.66 1060.66 0 0 0 0 1060.66 -1060.66
1060.66 -1060.66 0 0 0 0 -1060.66 1060.66
K2
K3

PASO 6 Planteamiento sistema general de ecuaciones.

$$\begin{Bmatrix} Fo \\ F1 \end{Bmatrix} = \left[\begin{array}{c|c} Ko & K1 \\ \hline K2 & K3 \end{array}\right] * \begin{Bmatrix} Uo \\ U1 \end{Bmatrix}$$

Ecuación 1: Cálculo de desplazamientos

$$\{Fo\} = [Ko] * \{Uo\} + [K1] * \{U1\}$$

$$\{Fo\} - [K1] * \{U1\} = [Ko] * \{Uo\}$$

$$\{Uo\} = [Ko]^{-1} * (\{Fo\} - [K1] * \{U1\})$$

Ecuación 2: Cálculo de reacciones

$$\{F1\} = [K2] * \{Uo\} + [K3] * \{U1\}$$

$$\{F1\} = [K2] * [Ko]^{-1} * (\{Fo\} - [K1] * \{U1\}) + [K3] * \{U1\}$$

PASO 7 Cálculo de desplazamientos

$$\begin{Bmatrix} 4.8284 \\ -5.8284 \end{Bmatrix} = \begin{bmatrix} 8983.496 & -2748.16 \\ -2748.16 & 2034.939 \end{bmatrix} * \begin{Bmatrix} UI \\ U2 \end{Bmatrix} +$$

$$\begin{bmatrix} -5000 & 0 & -2922.836 & 1687.5 & -1060.66 & 1060.66 \\ 0 & 0 & 1687.5 & -974.279 & 1060.66 & -1060.66 \end{bmatrix} * \begin{Bmatrix} -0.005 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 4.8284 \\ -5.8284 \end{Bmatrix} = \begin{bmatrix} 8983.496 & -2748.16 \\ -2748.16 & 2034.939 \end{bmatrix} * \begin{Bmatrix} U1 \\ U2 \end{Bmatrix} + \begin{Bmatrix} 25 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} -20.1716 \\ -5.8284 \end{Bmatrix} = \begin{bmatrix} 8983.496 & -2748.16 \\ -2748.16 & 2034.939 \end{bmatrix} * \begin{Bmatrix} U1 \\ U2 \end{Bmatrix}$$

U1 = - 0.00531905 metros = - 5.23905 milímetros

U2 = - 0.0100475 metros = - 10.0475 milímetros

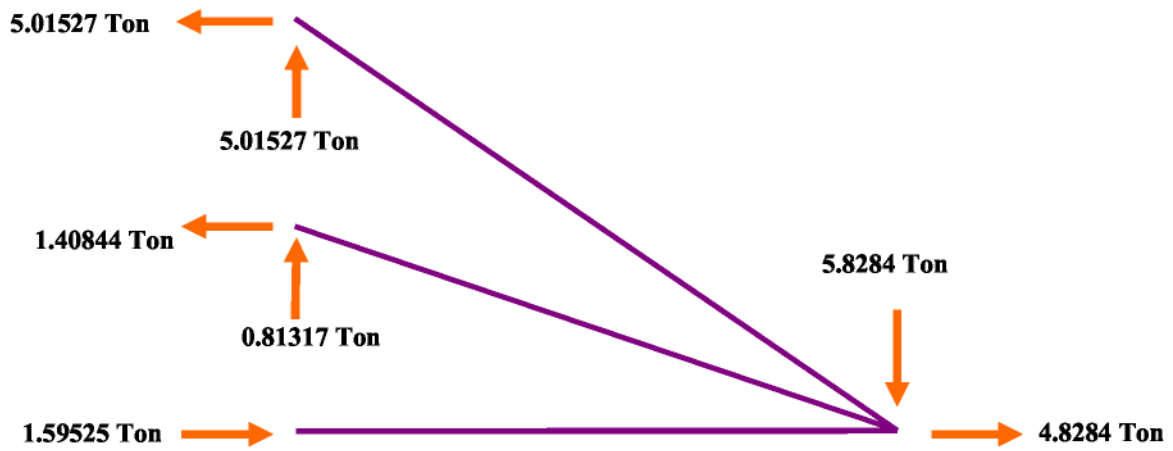
PASO 8 Cálculo de reacciones

$$\begin{Bmatrix} F3 \\ F4 \\ F5 \\ F6 \\ F7 \\ F8 \end{Bmatrix} = \begin{bmatrix} -5000 & 0 \\ 0 & 0 \\ -2922.836 & 1687.5 \\ 1687.5 & -974.279 \\ -1060.66 & 1060.66 \\ 1060.66 & -1060.66 \end{bmatrix} * \begin{Bmatrix} -0.00531905 \\ -0.0100475 \end{Bmatrix} +$$

$$\begin{bmatrix} 5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2922.836 & -1687.5 & 0 & 0 \\ 0 & 0 & -1687.5 & 974.279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1060.660 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1060.66 \end{bmatrix} * \begin{Bmatrix} -0.005 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} F3 \\ F4 \\ F5 \\ F6 \\ F7 \\ F8 \end{Bmatrix} = \begin{Bmatrix} 26.59525 \\ 0 \\ -1.408445 \\ 0.81317 \\ -5.01527 \\ 5.01527 \end{Bmatrix} + \begin{Bmatrix} -25 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1.59525 \\ 0 \\ -1.40844 \\ 0.81317 \\ -5.01527 \\ 5.01527 \end{Bmatrix}$$

PASO 9 Dibujo de las reacciones



PASO 10 Cálculo de fuerzas axiales

$$Fi = \frac{Ai \quad Ei}{Li} * [\ COS \quad \alpha \qquad SEN \quad \alpha\ ] * \begin{Bmatrix} Uxf & - Uxi \\ Uyf & - Uyi \end{Bmatrix}$$

Barra 1:

$$F1 = \frac{0.001\,m^2 * 2E7\ Ton}{4\,m * m^2} * [\ 1 \quad 0\ ] * \begin{Bmatrix} -0.00531905 - (0.005) \\ -0.0100475 - 0 \end{Bmatrix}$$

F1 = -1.5925 Ton compresión

Barra 2:

$$F2 = \frac{0.0009\,m^2 * 2E7\ Ton}{\sqrt{\frac{192}{9}}\,m * m^2} * \begin{bmatrix} \frac{\sqrt{3}}{2} & -0.5 \end{bmatrix} * \begin{Bmatrix} -0.00531905 - 0 \\ -0.0100475 - 0 \end{Bmatrix}$$

F2 = 1.626 Ton tracción

Barra 3:

$$F3 = \frac{0.0006\,m^2 * 2E7\ Ton}{4\sqrt{2}\,m * m^2} * \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} * \begin{Bmatrix} -0.00531905 - 0 \\ -0.0100475 - 0 \end{Bmatrix}$$

F3 = 7.09267 Ton tracción