Graphs

GRAPHS

1. Domain

2. Intersection with x-axis $\rightarrow f(x) = 0$
   Intersection with y-axis $\rightarrow f(0) = y$

3. Symmetries

   $$f(-x) = +f(x) \rightarrow \text{even}$$
   $$f(-x) = -f(x) \rightarrow \text{odd}$$

   Periodicity $\rightarrow f(x + T) = f(x)$

4. Asymptotes:

   $$\text{Vertical} \rightarrow \lim_{x \to x_0} f(x) = \pm\infty$$
   $$\text{Horizontal} \rightarrow \lim_{x \to \pm\infty} f(x) = H$$
   $$\text{Oblique} \rightarrow \lim_{x \to \pm\infty} f(x) - (mx + b) = 0 \rightarrow m = \lim_{x \to \infty} \frac{f(x)}{x}$$
   $$b = \lim_{x \to \infty} (f(x) - mx)$$

5. Continuity: $\lim_{x \to x_0} f(x) = f(x_0)$

GRAPHS

6. Derivative: monotonicity and critical points

   $f'(x) > 0$ increasing
   $f'(x) < 0$ decreasing
   $f'(x) = 0$ or $f'(x)$ does not exist $\rightarrow$ **critical points**

7. Local maxima and minima: $x_0 \rightarrow$ critical point

   $f'(x_0) = 0,\ f''(x_0) > 0$ local minimum
   $f'(x_0) = 0,\ f''(x_0) < 0$ local maximum
   $f'(x) : - \mapsto +$ local minimum
   $f'(x) : + \mapsto -$ local maximum

8. Concavity

   $f''(x) > 0$ convex
   $f''(x) < 0$ concave

9. Inflection points. Concavity changes. $f''(x_0) = 0$ or $\nexists f''(x_0)$

10. Global maxima and minima