Universidad Carlos III de Madrid

Escuela Politécnica Superior

Departamento de Matemáticas

First Course. Bachelor's Degree in
Telematics, Communication System and Audiovisual System Engineering

GRADE

Calculus I. First Test, November 3, 2008

Surname..... Name.....
D.N.I. ..... Group .....

Time length: 80 min.

1. (Problem 1.1.9) Find the set H of $x \in \mathbb{R}$ that verify:

$$H = \{\, |x^2 - 2x| < 1 \,\}.$$

[2 p.]

2. (Problem 1.3.6) Compute the limit

$$\lim_{x \to -\infty} \Big(\frac{2x+7}{2x-6}\Big)^{\sqrt{4x^2+x-3}}.$$

[3 p.]

3. (Problem 2.3.3) Represent graphically on $\mathbb{R}$ the function

$$f(x) = \frac{\mathrm{e}^{2x}}{\mathrm{e}^x - 1}.$$

Show the domain, asymptotes, local extrema, increasing and decreasing intervals and the convex and concave intervals.
Does this function have absolute maximum or minimum?
Is this function injective? and surjective?

[5 p.]

ANSWERS:

1. $$-1 < x^2 - 2x < 1 \Rightarrow \ H = (1-\sqrt{2}, 1) \cup (1, 1+\sqrt{2})$$

2.
$$\lim_{x\to-\infty} \left(\frac{2x+7}{2x-6}\right)^{\sqrt{4x^2+x-3}} = \lim_{x\to-\infty} \left(1+\frac{13}{2x-6}\right)^{\frac{2x-6}{13}\cdot\frac{13}{2x-6}\sqrt{4x^2+x-3}} =$$
$$= \exp\left(\lim_{x\to-\infty} \frac{13\sqrt{4x^2+x-3}}{2x-6}\right) = e^{-13}.$$

3.

- $\mathrm{Dom}(f) = \mathbb{R} - 0$.
  No intersection with axes.

- $\lim\limits_{x\to 0^+} f(x) = \infty$, $\lim\limits_{x\to 0^-} f(x) = -\infty \Rightarrow$ vertical asymptote at $x = 0 \Rightarrow$
  there is no global maximum neither minimum.
  $\lim\limits_{x\to\infty} f(x) = \infty$, $\lim\limits_{x\to-\infty} f(x) = 0 \Rightarrow$ horizontal asymptote at $x \to -\infty$

- $f'(x) = \dfrac{e^{3x} - 2e^{2x}}{(e^x-1)^2}$, $\quad f'(x) = 0 \Rightarrow x = \ln 2$.
  Decreasing intervals: $(-\infty, 0) \cup (0, \ln 2)$, points with $f'(x) < 0$.
  Increasing interval: $(\ln 2, \infty)$, points with $f'(x) > 0$.
  Hence, $f(\ln 2) = 4$ is a local minimum.

- $f''(x) = \dfrac{e^{4x} - 3e^{3x} + 4e^{2x}}{(e^x-1)^3}$
  Convex interval: $(0, \infty)$ points with $f''(x) > 0$.
  Concave interval: $(-\infty, 0)$ points with $f''(x) < 0$.

- The function is not surjective as $f(x) \neq 0$ and is not injective because there are points with the same image.