

uc3m

Universidad **Carlos III** de Madrid

Departamento de Matemáticas

MATEMÁTICAS II. Resolución problemas

Curso preparatorio para el acceso a la universidad
para mayores de 25 años

Tema 2

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<http://ocw.uc3m.es/matematicas>



2. Cálculo Integral

2.1. Cálculo de primitivas

Problema 2.1.1 *i)* $I = \int (\frac{1}{x} + 7x) dx = \log|x| + \frac{7}{2}x^2$. *ii)* $I = \frac{2}{3}x^{3/2} + e^x = \frac{2}{3}\sqrt{x^3} + e^x$.

iii) $I = \int \operatorname{tg}^2 x dx = \int (\sec^2 x - 1) dx = \operatorname{tg} x - x$. *iv)* $I = -\frac{1}{3} \cos 3x + \frac{1}{5} \operatorname{sen} 5x$.

v) $I = 2 \int (x+1) dx = x^2 + 2x$. *vi)* $I = \int (x^{-1/4} + x^{1/3}) dx = \frac{4}{3}x^{3/4} + \frac{3}{4}x^{4/3} = \frac{4}{3}\sqrt[4]{x^3} + \frac{3}{4}\sqrt[3]{x^4}$.

vii) $I = \int (e^{3x} + 1 + e^{-x}) dx = \frac{1}{3}e^{3x} + x - e^{-x}$.

viii) $I = \int \frac{1 + \operatorname{sen} x}{\cos^2 x} dx = \int (\frac{1}{\cos^2 x} + \frac{\operatorname{sen} x}{\cos^2 x}) dx = \operatorname{tg} x + \frac{1}{\cos x}$.

ix) $I = \frac{1}{4} \int \frac{1}{(x/2)^2 + 1} dx = \frac{1}{2} \operatorname{arc} \operatorname{tg}(x/2)$. *x)* $I = \frac{1}{3} \int \frac{1}{\sqrt{1 - (x/3)^2}} dx = \operatorname{arc} \operatorname{sen}(x/3)$.

xi) $I = \frac{1}{8}(1 + x^4)^8$. *xii)* $I = \frac{1}{2} \frac{2}{3}(3 + x^2)^{3/2} = \frac{1}{3}\sqrt{(3 + x^2)^3}$.

Problema 2.1.2 *i)* $I = xe^x - \int e^x dx = xe^x - e^x = e^x(x - 1)$. *ii)* $I = x^2e^x - 2 \int xe^x dx = e^x(x^2 - 2x - 2)$.

iii) $I = -x \cos x + \int \cos x dx = -x \cos x + \operatorname{sen} x$.

iv) $I = -e^x \cos x + \int e^x \cos x dx = -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x dx = \frac{1}{2}e^x(\operatorname{sen} x - \cos x)$.

v) $I = \frac{1}{2}x^2 \log^2 x - \int x \log x dx = \frac{1}{2}x^2 \log^2 x - \frac{1}{2}x^2 \log x + \frac{1}{2} \int x dx = x^2(\frac{1}{2} \log^2 x - \frac{1}{2} \log x + \frac{1}{4})$.

vi) $I = x \operatorname{sen}(\log x) - \int \cos(\log x) dx = x \operatorname{sen}(\log x) - x \cos(\log x) - \int \operatorname{sen}(\log x) dx = \frac{1}{2}x(\operatorname{sen}(\log x) - \cos(\log x))$.

Problema 2.1.3 *i)* $\sqrt[6]{x} = t \Rightarrow x = t^6 \Rightarrow dx = 6t^5 dt \Rightarrow I = \int \frac{6t^5}{t^2 + t^3} dt = 6 \int \frac{t^3}{t+1} dt = 6 \int (t^2 - 1 + \frac{1}{t+1}) dt = 2t^3 - 6t + 6 \log|t+1| = 2\sqrt{x} - 6\sqrt[6]{x} + 6 \log|\sqrt[6]{x} + 1|$.

ii) $\sqrt{x} = t \Rightarrow \frac{dx}{2\sqrt{x}} = dt \Rightarrow I = \int 2e^t dt = 2e^t = 2e^{\sqrt{x}}$.

iii) $x = 2 \operatorname{sen} t \Rightarrow dx = 2 \cos t dt \Rightarrow I = \int \cos^2 t dt = \frac{1}{2} \int (1 + \cos 2t) dt = \frac{t}{2} + \frac{1}{4} \operatorname{sen} 2t = \frac{1}{2} \operatorname{arc} \operatorname{sen}(x/2) + \frac{1}{4}x\sqrt{1 - (x/2)^2} = \frac{1}{2} \operatorname{arc} \operatorname{sen}(x/2) + \frac{1}{8}x\sqrt{4 - x^2}$.

iv) $e^x = t \Rightarrow x = \log t \Rightarrow dx = \frac{dt}{t} \Rightarrow I = \int \frac{dt}{t(t+1)} = \int (\frac{1}{t} - \frac{1}{t+1}) dt = \log|t| - \log|t+1| = x - \log|e^x + 1|$.

v) $x = \operatorname{sen} t \Rightarrow I = \int \frac{1}{\operatorname{sen}^2 t} dt = -\operatorname{cotg} t = -\frac{\cos t}{\operatorname{sen} t} = -\frac{\sqrt{1-x^2}}{x}$.

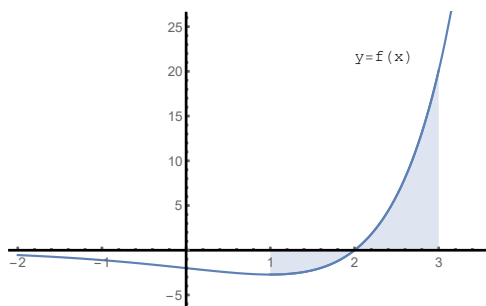
vi) $1 + \sqrt{x+1} = t \Rightarrow x = t^2 - 2t - 2 \Rightarrow dx = (2t - 2) dt \Rightarrow I = \int \frac{2t-2}{t} dt = 2t - 2 \log|t| = 2(1 + \sqrt{x+1}) - 2 \log|1 + \sqrt{x+1}|$.

Problema 2.1.4 *i)* $I = \frac{1}{2} \log|x^2 + 3|$. *ii)* $I = \frac{1}{2} \log|x^2 + 9| + \operatorname{arc} \operatorname{tg}(x/3)$.

$$\begin{aligned}
 \text{iii) } I &= \int \frac{2x+4-3}{x^2+4x+5} dx = \int \frac{2x+4}{x^2+4x+5} - \frac{3}{(x+2)^2+1} dx = \log|x^2+4x+5| - 3 \operatorname{arc} \operatorname{tg}(x+2). \\
 \text{iv) } I &= \int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \right) dx = \int \left(\frac{3/2}{x-1} - \frac{1/2}{x+1} - \frac{1}{(x+1)^2} \right) dx = \frac{3}{2} \log|x-1| - \\
 &\frac{1}{2} \log|x+1| + \frac{1}{x+1}. \text{ v) } I = \int \left(\frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \right) dx = \int \left(\frac{1}{x+1} - \frac{2}{x+2} - \frac{1}{(x+2)^2} \right) dx = \\
 &\log|x+1| - \log|x+2| + \frac{1}{x+2}. \text{ vi) } I = \int \left(x^2 + \frac{x}{x^3-3x+2} \right) dx = \int \left(x^2 + \frac{A}{x-1} + \frac{B}{(x-1)^2} + \right. \\
 &\left. \frac{C}{x+2} \right) dx; A(x-1)(x+2) + B(x+2) + C(x-1)^2 = x \Rightarrow A = \frac{4}{9}, B = \frac{1}{3}, C = -\frac{2}{9} \Rightarrow I = \\
 &\int \left(x^2 + \frac{4/9}{x-1} + \frac{1/3}{(x-1)^2} - \frac{2/9}{x+2} \right) dx = \frac{1}{3}x^3 + \frac{4}{9} \log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9} \log|x+2|.
 \end{aligned}$$

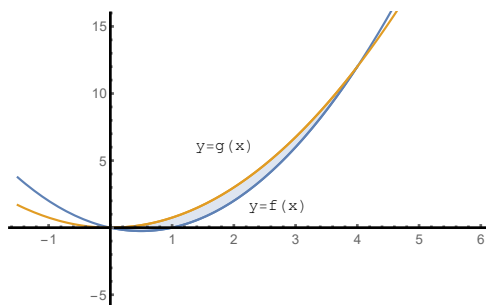
2.2. Aplicaciones de la integral

Problema 2.2.1 a)



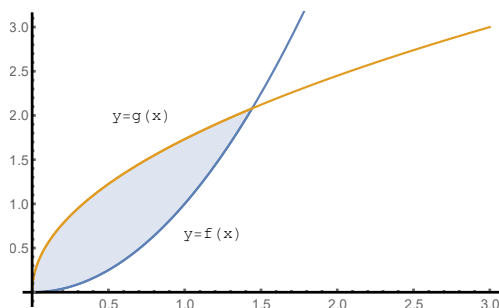
$$b) A = - \int_1^2 (x-2)e^x dx + \int_2^3 (x-2)e^x dx = -e^x(x-3) \Big|_1^2 + e^x(x-3) \Big|_2^3 = 2e^2 + 2e.$$

Problema 2.2.2



$$A = \int_0^4 \left(\frac{3}{4}x^2 - (x^2 - 4) \right) dx = \left(-\frac{1}{18}x^3 + \frac{1}{2}x^2 \right) \Big|_0^4 = \frac{8}{3}.$$

Problema 2.2.3



$$x^2 = b\sqrt{x} \Rightarrow x = b^{2/3}; \quad 1 = \int_0^{b^{2/3}} (b\sqrt{x} - x^2) dx = \frac{1}{3}b^2 \Rightarrow b = \sqrt{3}.$$

- A₃P -
- ERC -

