Universidad Carlos III de Madrid
Escuela Politécnica Superior
Departamento de Matemáticas
First Course. Telecommunication Engineering

GRADE

Calculus II. Second Test, April 13th, 2010

Surname .... Name ....

D.N.I. .... Group ....

Time length: 80 min.

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1. Let D be the region bounded by the y axis and the parabola $x = -4y^2 + 1$. Compute

$$\iint_D y^2 dA.$$

[2 p.]

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2. Let $G = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq \pi^2, y \geq 0\}$, calculate the integral

$$\iint_G \sin(\sqrt{x^2 + y^2}) dA.$$

[2 p.]

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3. Compute

$$\iiint_B \frac{1}{\sqrt{x^2 + y^2 + z^2}}\, dV, \text{ where } B = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}.$$

[2 p.]

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4. Find the volume of the region bounded by the cylinders $x^2 + y^2 = 4$, $x^2 + y^2 = 16$ and the planes $z = -4$ and $z = 3$.

[2 p.]

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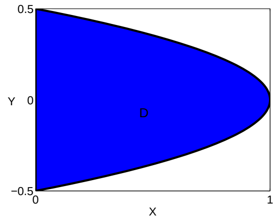
5. Solve the following initial value problem

$$y'' - 2y' + y = te^t,$$
$$y(0) = 0, \; y'(0) = 3.$$

[2 p.]

ANSWERS:

1. $$\iint_D y^2 dA = \int_{-1/2}^{1/2} dy \int_0^{-4y^2+1} y^2 dx = \int_{-1/2}^{1/2} y^2 x \Big|_{x=0}^{-4y^2+1} dy =$$
$$= \int_{-1/2}^{1/2} (-4y^2+1) y^2 dy = \int_{-1/2}^{1/2} (-4y^4+y^2) dy =$$
$$= \Big[\frac{1}{3}y^3 - \frac{4}{5}y^5\Big]_{y=-1/2}^{y=1/2} = 2\Big[\frac{1}{3\cdot 8} - \frac{1}{5\cdot 8}\Big] = \frac{1}{30}.$$



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2. $$\iint_G \sin(\sqrt{x^2+y^2}) dA \underset{polar}{=} \int_0^{\pi} d\theta \int_0^{\pi} r\sin r dr = \pi \int_0^{\pi} r \sin r dr \underset{IBP}{=}$$
$$= \pi\Big[-r\cos r\Big|_{r=0}^{\pi} + \int_0^{\pi} \cos r dr\Big] = \pi\Big[-\pi\cos\pi + \sin r\Big|_{r=0}^{\pi}\Big] = \pi^2.$$

With the following integration by parts (IBP): $u = r, dv = \sin r dr$.

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3. $$\iiint_B \frac{1}{\sqrt{x^2+y^2+z^2}} dV \underset{spherical}{=} \int_0^{2\pi} d\theta \int_0^{\pi} d\phi \int_0^1 \frac{r^2 \sin\phi}{r} dr =$$
$$= \int_0^{2\pi} d\theta \int_0^{\pi} \sin\phi \, d\phi \int_0^1 r dr = 2\pi \cdot 2 \cdot \frac{1}{2} = 2\pi.$$

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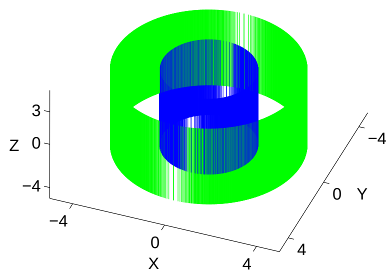
4. Calling W the region of integration, the volume between the cylinders is

$$V = \iiint_W dV \underset{cylindrical}{=} \int_0^{2\pi} d\theta \int_{-4}^{3} dz \int_2^4 r dr =$$
$$2\pi \cdot 7 \cdot \frac{12}{2} = 84\pi.$$



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5. Doing the Transform of Laplace to the equation:

$$s^2F(s) - sy(0) - y'(0) - 2\big(sF(s) - y(0)\big) + F(s) = L[te^t](s),$$
$$(s^2 - 2s + 1)F(s) - 3 = \frac{1}{(s-1)^2}$$
$$(s-1)^2 F(s) = 3 + \frac{1}{(s-1)^2}$$
$$F(s) = \frac{3}{(s-1)^2} + \frac{1}{(s-1)^4} = 3L[te^t](s) + \frac{L[t^3e^t](s)}{3!} \Rightarrow$$
$$y(t) = 3te^t + \frac{1}{3!}t^3e^t = e^t\Big(3t + \frac{t^3}{6}\Big).$$
