Formal Languages and Automata Theory
Exercises Regular Languages
Unit 5

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* Several exercises are based on the ones proposed in the following books:

- Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón. *Teoría de autómatas y lenguajes formales.* McGraw-Hill (2007).
- Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga. *Teoría de lenguajes, gramáticas y autómatas.* Publicaciones R.A.E.C. (1997).
- Pedro Isasi, Paloma Martínez y Daniel Borrajo. *Lenguajes, Gramáticas y Autómatas. Un enfoque práctico.* Addison-Wesley (1997).

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1. **Obtain the minimum DFA equivalent to each one of the following grammars describing the intermediate steps: G→NFA→DFA→minimal DFA.**

a) G_A = ({a,b,c}, {S,A,B}, S, P_A) P_A = { S ::= aA | bB | c A ::= aB | b | cA B ::= a | bA | c }	**b)** G_B = ({a,b,c}, {S,B,C,E}, S, P_B) P_B = { S ::= a | aS | aB | cC C ::= c B ::= bE | b E ::= bB | b }
c) G_C = ({a,b,c}, {S,A,B,C,D}, S, P_C) P_C = { S ::= aA A ::= aA | bB | a B ::= bB | bC | b C ::= bC | cD | bB D ::= bC | bB |cC }	**d)** G_D = ({c,f,d}, {A,B,C,D,E,F}, A, P_D) P_D = { A ::= cB | cE | f | fC B ::= cB | fD | dE C ::= cA D ::= cD | fD E ::= cE | fF | dF F ::= cF | fF }

Solution:

Section a:

NFA	DFA	Minimal DFA
`NFA=({a,b,c},{Q0,Q1,Q2,Q3},f,Q0,Q3)` f(Q0, a) = Q1 f(Q0, b) = Q2 f(Q0, c) = Q3 f(Q1, a) = Q2 f(Q1, b) = Q3 f(Q1, c) = Q1 f(Q2, a) = Q3 f(Q2, b) = Q1 f(Q2, c) = Q3	`DFA=({a,b,c},{Q0,Q1,Q2,Q3,Q4},f,Q0,Q3)` f(Q0, a) = Q1 f(Q0, b) = Q2 f(Q0, c) = Q3 f(Q1, a) = Q2 f(Q1, b) = Q3 f(Q1, c) = Q1 f(Q2, a) = Q3 f(Q2, b) = Q1 f(Q2, c) = Q3 f(Q3, a) = Q4 f(Q3, b) = Q4 f(Q3, c) = Q4 f(Q4, a) = Q4 f(Q4, b) = Q4 f(Q4, c) = Q4	`Same DFA.`

Section b:

NFA	DFA	minimal DFA
NFA=({a,b,c},{Q0,Q1,Q2,Q3,Q7},f,Q0,Q3) f(Q0, a) = Q0,Q2,Q3 f(Q0, c) = Q1 f(Q1, c) = Q3 f(Q2, b) = Q2,Q3 f(Q7, b) = Q2,Q3	DFA=({a,b,c},{Q0,Q1,Q3,Q4,Q5,Q6,Q8},f,Q0,{Q3,Q4,Q6,Q8}) f(Q0, a) = Q4 f(Q0, b) = Q5 f(Q0, c) = Q1 f(Q1, a) = Q5 f(Q1, b) = Q5 f(Q1, c) = Q3 f(Q3, a) = Q5 f(Q3, b) = Q5 f(Q3, c) = Q5 f(Q4, a) = Q4 f(Q4, b) = Q8 f(Q4, c) = Q1 f(Q5, a) = Q5 f(Q5, b) = Q5 f(Q5, c) = Q5 f(Q6, a) = Q5 f(Q6, b) = Q8 f(Q6, c) = Q5 f(Q8, a) = Q5 f(Q8, b) = Q6 f(Q8, c) = Q5	DFAmin=({a,b,c},{Q0,Q1,Q3,Q4,Q5,Q9},f,Q0,{Q3,Q4,Q9}) f(Q0, a) = Q4 f(Q0, b) = Q5 f(Q0, c) = Q1 f(Q1, a) = Q5 f(Q1, b) = Q5 f(Q1, c) = Q3 f(Q3, a) = Q5 f(Q3, b) = Q5 f(Q3, c) = Q5 f(Q4, a) = Q4 f(Q4, b) = Q9 f(Q4, c) = Q1 f(Q5, a) = Q5 f(Q5, b) = Q5 f(Q5, c) = Q5 f(Q9, a) = Q5 f(Q9, b) = Q9 f(Q9, c) = Q5

Formal Languages and Automata Theory

Section c:

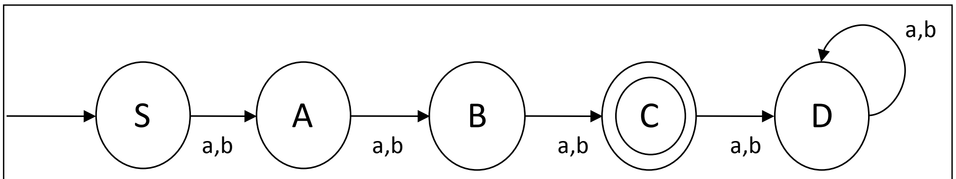
NFA	DFA	Minimal DFA
NFA=({a,b,c},{Q0,Q1,Q2,Q3,Q4,Q5},f,Q0,Q5) f(Q0, a) = Q1 f(Q1, a) = Q1,Q5 f(Q1, b) = Q2 f(Q2, b) = Q2,Q3,Q5 f(Q3, b) = Q2,Q3 f(Q3, c) = Q4 f(Q4, b) = Q2,Q3 f(Q4, c) = Q3	DFA=({a,b,c},{Q0,Q1,Q2,Q3,Q4,Q6,Q7,Q8,Q9},f,Q0,{Q7,Q8}) f(Q0, a) = Q1 f(Q0, b) = Q6 f(Q0, c) = Q6 f(Q1, a) = Q7 f(Q1, b) = Q2 f(Q1, c) = Q6 f(Q7, a) = Q7 f(Q7, b) = Q2 f(Q7, c) = Q6 f(Q2, a) = Q6 f(Q2, b) = Q8 f(Q2, c) = Q6 f(Q8, a) = Q6 f(Q8, b) = Q8 f(Q8, c) = Q4 f(Q4, a) = Q6 f(Q4, b) = Q9 f(Q4, c) = Q3 f(Q9, a) = Q6 f(Q9, b) = Q8 f(Q9, c) = Q4 f(Q3, a) = Q6 f(Q3, b) = Q9 f(Q3, c) = Q4 f(Q6, a) = Q6 f(Q6, b) = Q6 f(Q6, c) = Q6	DFAmin=({a,b,c},{Q0,Q1,Q2,Q6,Q7,Q8,Q9,Q10},f,Q0,{Q7,Q8}) f(Q0, a) = Q1 f(Q0, b) = Q6 f(Q0, c) = Q6 f(Q1, a) = Q7 f(Q1, b) = Q2 f(Q1, c) = Q6 f(Q7, a) = Q7 f(Q7, b) = Q2 f(Q7, c) = Q6 f(Q2, a) = Q6 f(Q2, b) = Q8 f(Q2, c) = Q6 f(Q8, a) = Q6 f(Q8, b) = Q8 f(Q8, c) = Q10 f(Q9, a) = Q6 f(Q9, b) = Q8 f(Q9, c) = Q10 f(Q6, a) = Q6 f(Q6, b) = Q6 f(Q6, c) = Q6 f(Q10, a) = Q6 f(Q10, b) = Q9 f(Q10, c) = Q10

Section d:

NFA	DFA	Minimal DFA
NFA=({c,f,d},{Q0,Q1,Q2,Q3,Q4,Q5,Q6},f,Q0,Q6) f(Q0, c) = Q1,Q4 f(Q0, f) = Q2,Q6 f(Q1, c) = Q1 f(Q1, f) = Q3 f(Q1, d) = Q4 f(Q2, c) = Q0 f(Q3, c) = Q3 f(Q3, f) = Q3 f(Q4, c) = Q4 f(Q4, f) = Q5 f(Q4, d) = Q5 f(Q5, c) = Q5 f(Q5, f) = Q5	DFA=({c,f,d},{Q0,Q5,Q7,Q8,Q9,Q10,Q11},f,Q0,Q8) f(Q0, c) = Q7 f(Q0, f) = Q8 f(Q0, d) = Q9 f(Q7, c) = Q7 f(Q7, f) = Q10 f(Q7, d) = Q11 f(Q8, c) = Q0 f(Q8, f) = Q9 f(Q8, d) = Q9 f(Q10, c) = Q10 f(Q10, f) = Q10 f(Q10, d) = Q9 f(Q11, c) = Q11 f(Q11, f) = Q5 f(Q11, d) = Q5 f(Q5, c) = Q5 f(Q5, f) = Q5 f(Q5, d) = Q9 f(Q9, c) = Q9 f(Q9, f) = Q9 f(Q9, d) = Q9	DFAmin=({c,f,d},{C1,C2,C3},f,C3,C2) f(C3, c) = C1 f(C3, f) = C2 f(C3, d) = C1 f(C1, c) = C1 f(C1, f) = C1 f(C1, d) = C1 f(C2, c) = C3 f(C2, f) = C1 f(C2, d) = C1

2. **Given the alphabet {a,b}, construct a DFA which recognizes string with length "3" of the universal language. Obtain the G3 corresponding to this automaton.**

<u>Solution</u>:



S::=aA | bA
A::=aB | bB
B::=aC | bC
B::=a | b
C::=aD | bD
D::=aD | bD

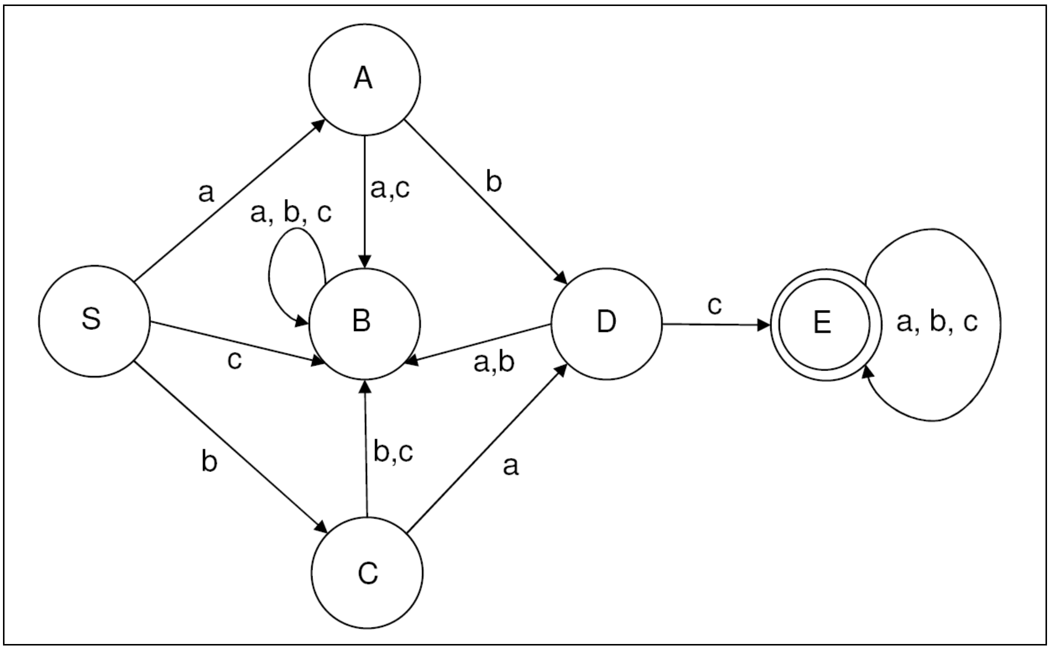
Gramática Bien Formada
S::=aA | bA
A::=aB | bB
B::=a | b

3. **We have a door with only one lock. To open it, it is necessary to use three different keys (called a,b, and c), in a predefined order, which is following described:**

   - **Key a, then key b, then key c, or**
   - **Key b, then key a, then key c.**

   If this order is not followed, then the lock is blocked (for instance, if the key a is used and following it is introduced again).
   Once the door is open, the introduction of keys in the lock (in every possible order) does not affect the closing device (i.e. the door remains open).
   Consider that the names of the different keys are symbols of an alphabet, over which a language L whose words are the valid sequences for the opening of the door is defined. For instance, abcbc is a word included in the language. It is required:
   a) **Design a finite automata FA which accepts L.**
   b) **Well-formed Grammar which generates words in L**

<u>Solution</u>:



G = ({a, b, c}, {S, A, B, C, D, E}, S, P)

P = {
    S ::= aA | bC
    A ::= bD
    C ::= aD
    D ::= cE | c
    E ::= aE | bE | cE | a | b | c
}

4. Given the RE (b•a*)*, which represents a regular language, construct a FA accepting this regular language

<u>Solution</u>:

R = (ba*)*

$D_a(R)$ = $D_a((ba^*)^*)$ = $D_a(ba^*)(ba^*)^*$

= (~~$D_a(b)$~~a^* + ~~$\delta(b)$~~$D_a(a^*)$) $(ba^*)^*$

= $(\phi a^* + \phi D_a(a^*))\ (ba^*)^*$ = ϕ

$D_b(R)$ = $D_b((ba^*)^*)$ = $D_b(ba^*)(ba^*)^*$

= ($D_b(b)a^*$ + ~~$\delta(b)$~~$D_b(a^*)$) $(ba^*)^*$

= $(\lambda a^* + \phi D_b(a^*))\ (ba^*)^*$ = **a*(ba*)*** = **R1**

$D_a(R1)$ = $D_a(a^*(ba^*)^*)$

= $D_a(a^*)(ba^*)^* + \delta(a^*)D_a((ba^*)^*)$

= $D_a(a)a^*(ba^*)^* + \lambda$~~$D_a(R)$~~

= $\lambda a^*(ba^*)^* + \lambda\phi$ = $a^*(ba^*)^*$ = R1

$D_b(R1)$ = $D_b(a^*(ba^*)^*)$

= ~~$D_b(a^*)$~~$(ba^*)^* + \delta(a^*)D_b((ba^*)^*)$

= $\phi(ba^*)^* + \lambda D_b(R)$ = ϕ + R1 = R1

R = (ba*)* **R1 = a*(ba*)***

$D_a(R)$	=	ϕ	$\delta(D_a(R))$	$= \delta(\phi)$		=	ϕ
$D_b(R)$	=	R1	$\delta(D_b(R))$	$= \delta(R1) =$	$\delta(a^*(ba^*)^*))$	=	λ
$D_a(R1)$	=	R1	$\delta(D_a(R1))$	$= \delta(R1) =$	$\delta(a^*(ba^*)^*))$	=	λ
$D_b(R1)$	=	R1	$\delta(D_b(R1))$	$= \delta(R1) =$	$\delta(a^*(ba^*)^*))$	=	λ

P = {

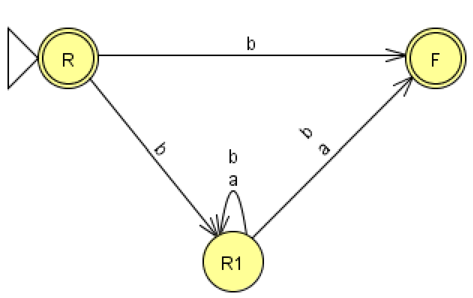
R → bR1 R→ b
R1→ aR1 R1→ a
R1→ bR1 R1→ b

R → λ

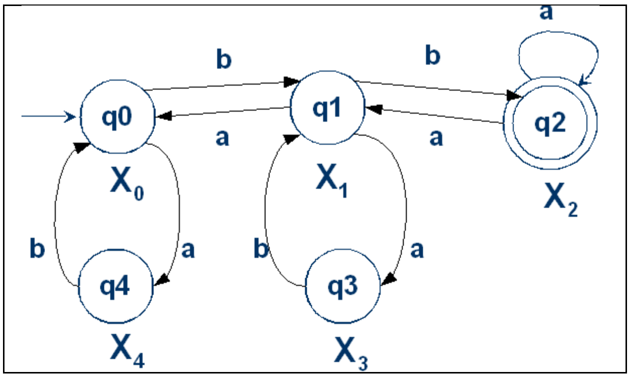
}

G = ({a,b}, {R, R1}, R, P)

AF = ({a,b}, {R, R1, F}, f, R, {F})



5. **Determine the language recognized by the following automaton. To do this, use the characteristic equations.**



<u>**Solution**</u>:

X0 = aX4 + bX1
X1 = aX0 + bX2 + b + aX3
X2 = aX1+ aX2 + a
X3 = bX1
X4 = bX0

X0 = (ab)* b (ab)* b (ab)* a*

6. **Given the following right-linear grammar, G= ({0,1},{S,A,B,C},S,P),where P={ S::= 1A | 1B, A ::= 0A | 0C | 1C | 1 , B ::= 1A | 1C | 1 , C ::= 1}. Calculate formally the RE of the language associated to this grammar.**

<u>**Solution**</u>:

ER= (λ+1)(10*(01+11+1))+111+11

7. Simplify the following regular expression: $\alpha = a + a(b+aa)(b^*aa)^*b^* + a(aa+b)^*$ by using the equivalence properties of the regular expressions.

Solution:

$$\alpha = a(aa + b)^* + a(aa + b)^* = a(aa + b)^*$$

8. Calculate the derivative $D_{ab}(\alpha)$ where $\alpha = a^*ab$, using the definitions of the derivatives of regular expressions.

Solution:

$D_{ab}(a^*ab) = \lambda$

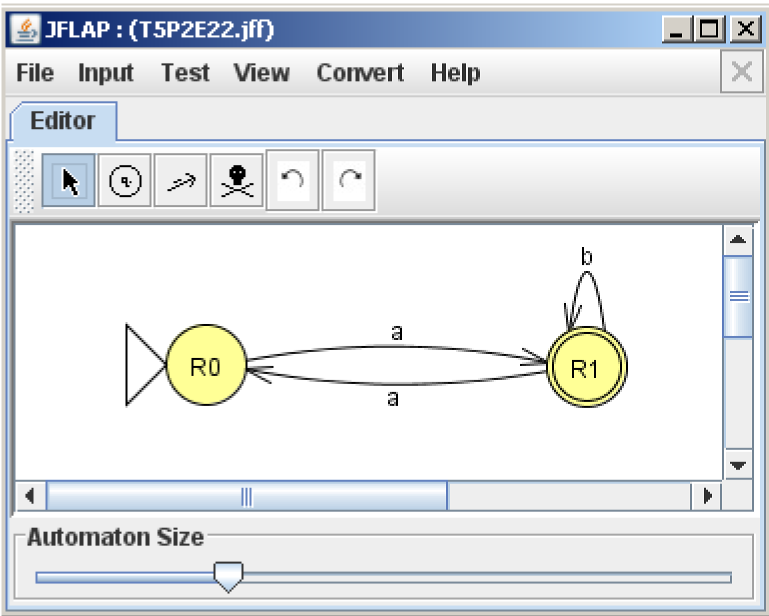
9. Obtain the grammar for the regular expression *a(aa + b).**

Solution:

$R_0 = a(aa+b)^*$
$R_1 = (aa+b)^*$

$D_a(R_0) = R_1$	$R_0 ::= aR_1$	$\delta(D_a(R_0)) = \lambda$	$R_0 ::= a$
$D_b(R_0) = \phi$			
$D_a(R_1) = R0$	$R_1 ::= aR_0$	$\delta(D_a(R_1)) = \phi$	
$D_b(R_1) = R_1$	$R_1 ::= bR_1$	$\delta(D_b(R_1)) = \lambda$	$R_1 ::= b$

$G = (\{a, b\}, \{R_0, R_1\}, R_0 , P)$
P = {
$R_0 ::= aR_1 \mid a$
$R_1 ::= aR_0$
$R_1 ::= bR_1 \mid R_1 ::= b$
}



10. Given the following regular expression *a*c*(a+b) (cb), construct formally an equivalent regular grammar.**

R_0 = a*c*(a+b)(cb)*
R_1 = a*c*(a+b)(cb)* + (cb)*
R_2 = (cb)*
R_3 = c*(a+b)(cb)*
R_4 = b(cb)*

R_0 -> aR_1 | a | bR_2 | b | cR_3
R_1 -> aR_1 | a | bR_2 | b | cR_3
R_2 -> cR_4
R_3 -> aR_2 | a | bR_2 | b | cR_3
R_4 -> bR_2 | b