Formal Languages and Automata Theory
Exercises Push-Down Automata
Unit 6

Authors:

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* Several exercises are based on the ones proposed in the following books:

- Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón. *Teoría de autómatas y lenguajes formales.* McGraw-Hill (2007).
- Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga. *Teoría de lenguajes, gramáticas y autómatas.* Publicaciones R.A.E.C. (1997).
- Pedro Isasi, Paloma Martínez y Daniel Borrajo. *Lenguajes, Gramáticas y Autómatas. Un enfoque práctico.* Addison-Wesley (1997).

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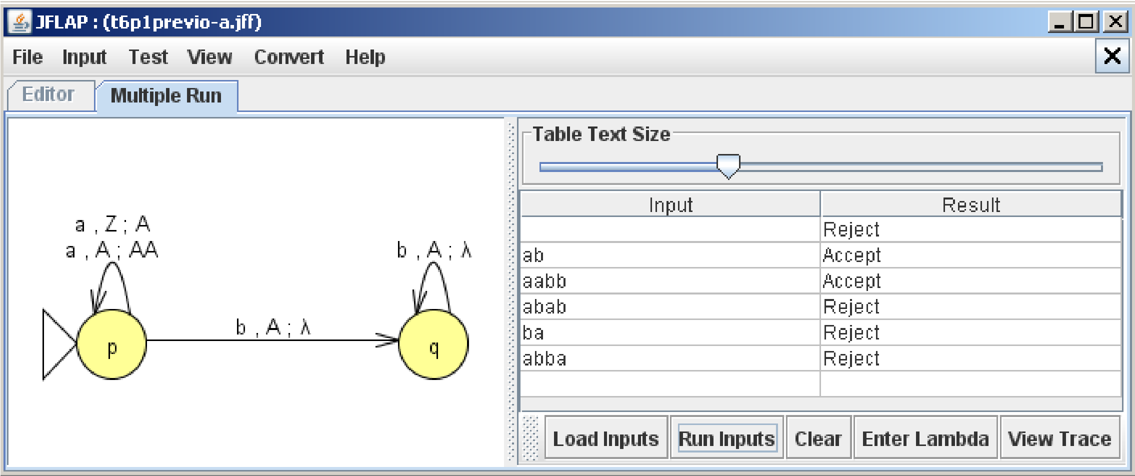
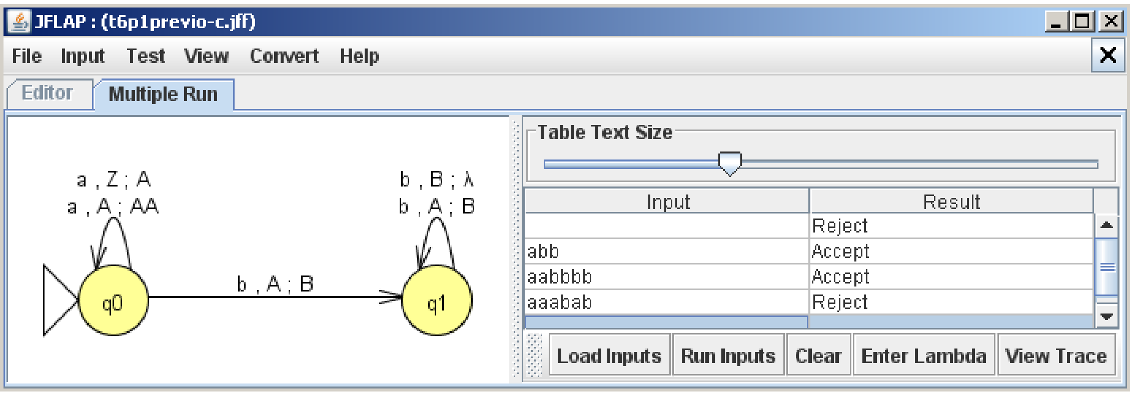
1. **Design a Push-Down Automaton for each one of the following languages:**
   - **a.** $L = \{ a^n \bullet b^n \mid n \geq 0\}$
   - **b.** $L = \{ a^n \bullet b^{2n} \mid n > 0\}$
   - **c.** $L = \{ a^{2n} \bullet b^n \mid n \geq 0\}$
   - **d.** $L = \{ a^{2n} \bullet b^n \mid n > 0\}$

Solution:

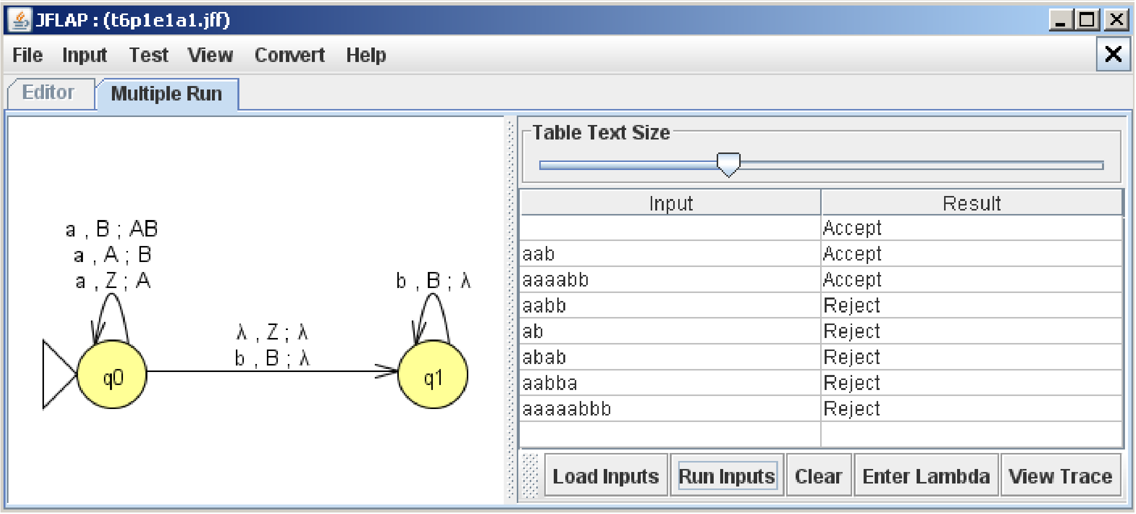
- **a.** $L = \{ a^n \bullet b^n \mid n \geq 0\}$



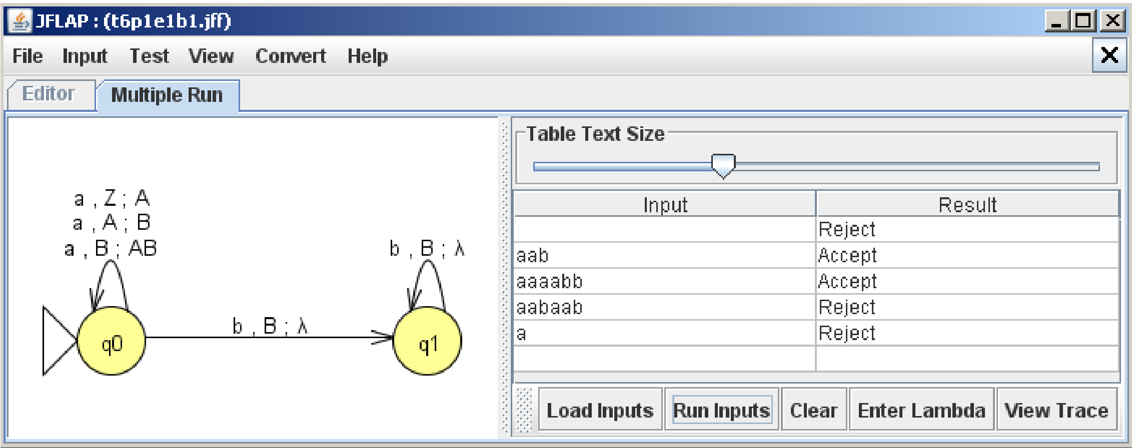
- **b.** $L = \{ a^n \bullet b^{2n} \mid n > 0\}$



c. $L = \{ a^{2n} \cdot b^{n} \mid n \geq 0\}$



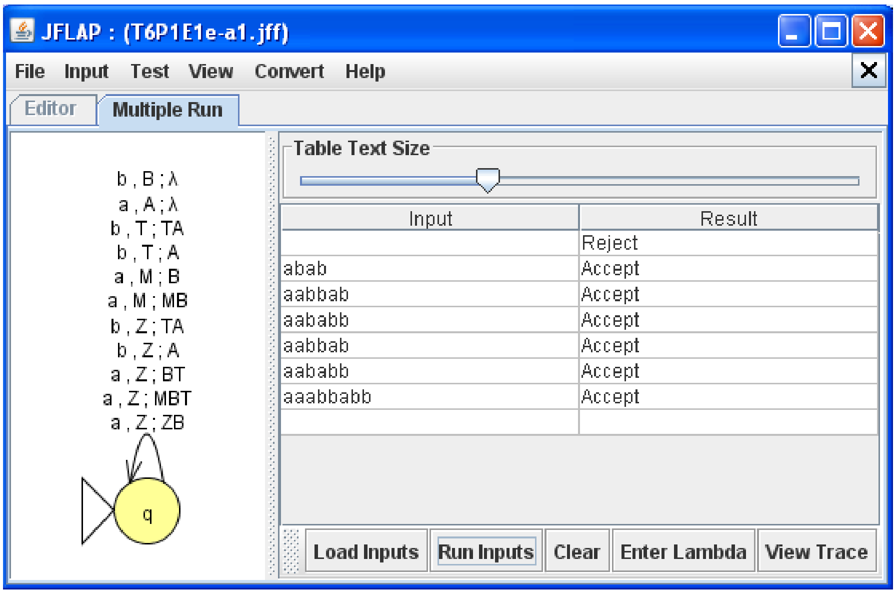
d. $L = \{ a^{2n} \cdot b^{n} \mid n > 0\}$



2. Design a Push-Down Automaton for the language: $L= \{ a^{n+m} \cdot b^{m+t} \cdot a^{t} \cdot b^{n} \mid n, t > 0, m \geq 0\}$

<u>Solution</u>:

$L = \{a^n(a^m b^m)(b^t a^t)b^n \mid n, t > 0, m \geq 0\}$



S → aSb	(q, aSb) ∈ f(q, λ, S)	For each terminal symbol, add:
S → MT	(q, MT) ∈ f(q, λ, S)	f(q, a, a) = (q, λ)
S → ba	(q, ba) ∈ f(q, λ, S)	f(q, b, b) = (q, λ)
S → bTa	(q, bTa) ∈ f(q, λ, S)	
M → aMb	(q, aMb) ∈ f(q, λ, M)	
M → ab	(q, ab) ∈ f(q, λ, M)	
T → ba	(q, ba) ∈ f(q, λ, T)	
T → bTa	(q, bTa) ∈ f(q, λ, T)	

3. Obtain the PDA$_E$ corresponding to the grammar

G$_{FNG}$ = ({a,b,c,d}, {S,A,B}, S, P), with the following production rules:

S ::= a S B | b A | b | d
A ::= b A | b
B ::= c

<u>Solution</u>:

G = ({a,b,c,d}, {S,A,B}, S, P), P={

S → aSB | bA | b | d
A → bA | b
B → c

}

G = ({a,b,c,d}, {S,A,B}, {q}, S, q, f, { }), f={

S → aSB ➔ f(q,a,S) = (q, SB)
S → bA ➔ f(q,b,S) = (q, A)
S → b ➔ f(q,b,S) = (q, λ)
S → d ➔ f(q,d,S) = (q, λ)
A → bA ➔ f(q,b,A) = (q, A)
A → b ➔ f(q,b,A) = (q, λ)
B → c ➔ f(q,c,B) = (q, λ)

}

4. Obtain formally the PDA_F equivalent to the following PDA_E:

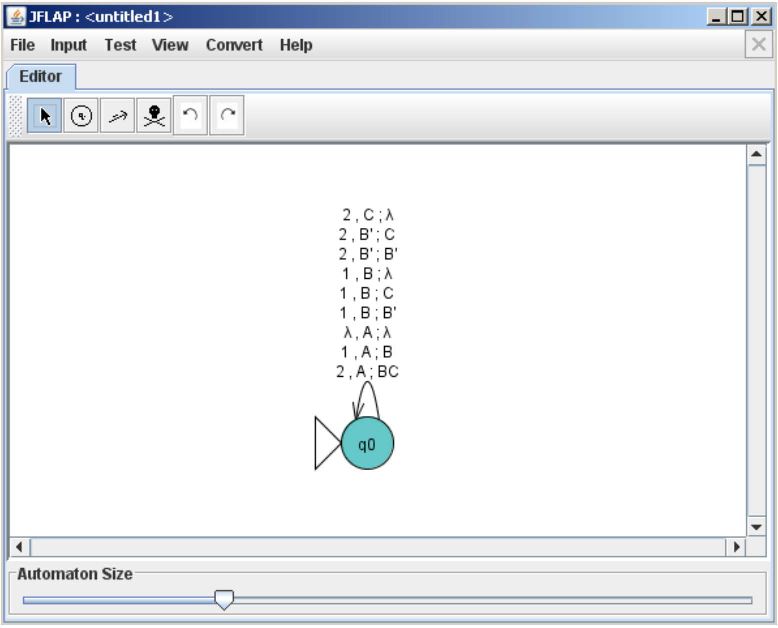
PDA_E=({1,2}, {A,B,B',C}, {q}, A, q, f, {Φ}), where f is given by:

f(q,2,A) = (q, BC)
f(q,1,A) = (q,B)
f(q,λ,A) = (q, λ)
f(q,1,B) = {(q,B'), (q,C), (q, λ)}
f(q,2,B') = {(q,B'), (q,C)}
f(q,2,C) = (q, λ)

<u>Solution</u>:

APf_a=({1,2}, {A,B,B',C,Z}, {q,p,r}, Z, p, f', {r}), donde f viene dada por:
f(p,λ,Z) = (q,AZ)
f(q,2,A) = (q, BC)
f(q,1,A) = (q,B)
f(q,λ,A) = (q, λ)
f(q,1,B) = {(q,B'), (q,C), (q, λ)}
f(q,2,B') = {(q,B'), (q,C)}
f(q,2,C) = (q, λ)
f(q, λ, Z) = (r, λ)

<u>Alternative solution</u>:



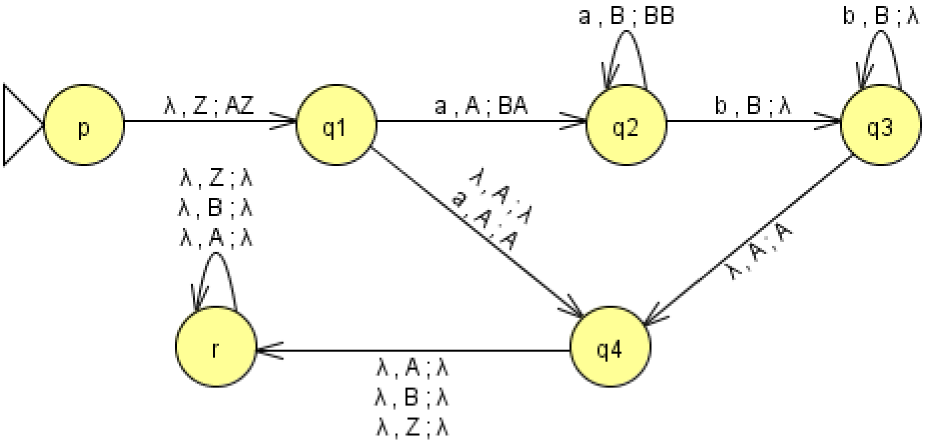
5. Obtain formally the PDA_E equivalent to the following PDA_F:

PDA_F=({a,b}, {A,B}, {q1,q2,q3,q4}, A, q1, f, {q4}), donde f viene dada por:

f(q1,a,A) = {(q2,BA), (q4,A)}
f(q1,λ,A) = {(q4, λ)}
f(q2,a,B) = {(q2,BB)}
f(q2,b,B) = {(q3, λ)}
f(q3,λ,A) = {(q4,A)}
f(q3,b,B) = {(q3, λ)}

Solution:

APf_b=({a,b}, {A,B,Z}, {q1,q2,q3,q4,p,r}, Z, p, f', {Φ}), where f':



6. Describe the transition functions which generate the following movements:

(p,1001, A) |- (p, 001,1A) |- (p, 01, 01A) |- (q, 1, 1A) |- (q, λ, A) |- (q, λ, λ)

Solution:

(p,1001, A) |- (p, 001,1A)	f(p,1,A) = (p, 1A)
(p, 001,1A) |- (p, 01, 01A)	f(p,0,1) = (p, 01)
(p, 01, 01A) |- (q, 1, 1A)	f(p,0,0) = (q, λ)
(q, 1, 1A) |- (q, λ, A)	f(q,1,1) = (q, λ)
(q, λ, A) |- (q, λ, λ)	f(q,λ,A) = (q,λ)