Formal Languages and Automata Theory
Exercises Regular Languages
Unit 5 – Part 2

Authors:

Araceli Sanchis de Miguel
Agapito Ledezma Espino
Jose A. Iglesias Martínez
Beatriz García Jiménez
Juan Manuel Alonso Weber

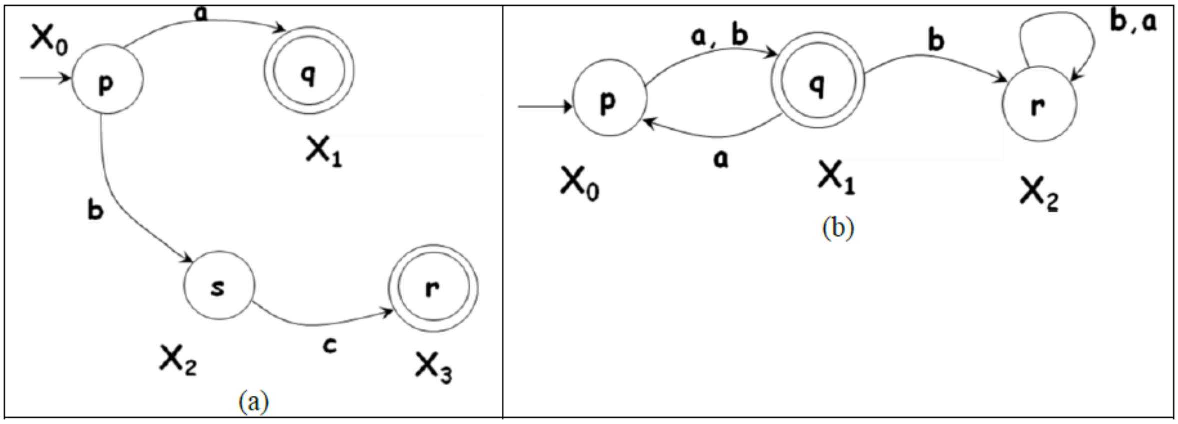
* Several exercises are based on the ones proposed in the following books:

- Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón. *Teoría de autómatas y lenguajes formales.* McGraw-Hill (2007).
- Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga. *Teoría de lenguajes, gramáticas y autómatas.* Publicaciones R.A.E.C. (1997).
- Pedro Isasi, Paloma Martínez y Daniel Borrajo. *Lenguajes, Gramáticas y Autómatas. Un enfoque práctico.* Addison-Wesley (1997).

Universidad Carlos III de Madrid
www.uc3m.es

1. Given $\Sigma = \{a,b,c\}$ and $\alpha_1 = a\,(a|b|c)^*$ and $\alpha_2 = a|bc|b^2a$. Describe in detail $L(\alpha_1)$ and $L(\alpha_2)$.

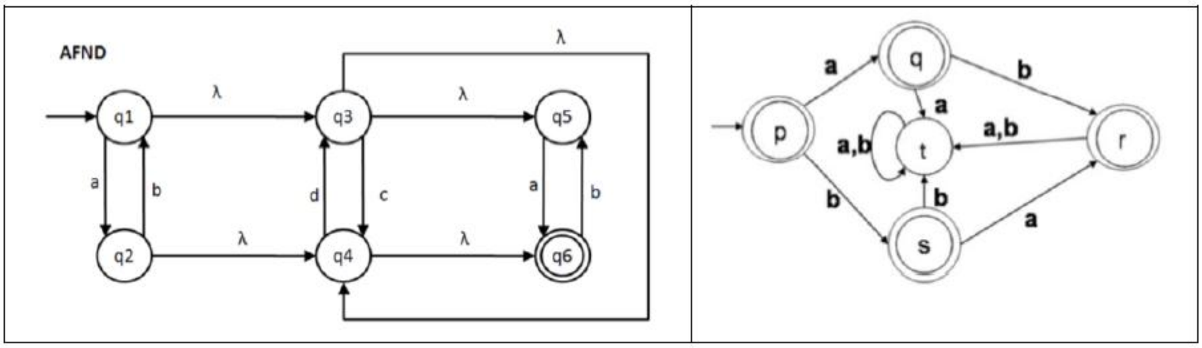
2. Obtain the characteristic equations of the FA defined by the following transition diagrams.



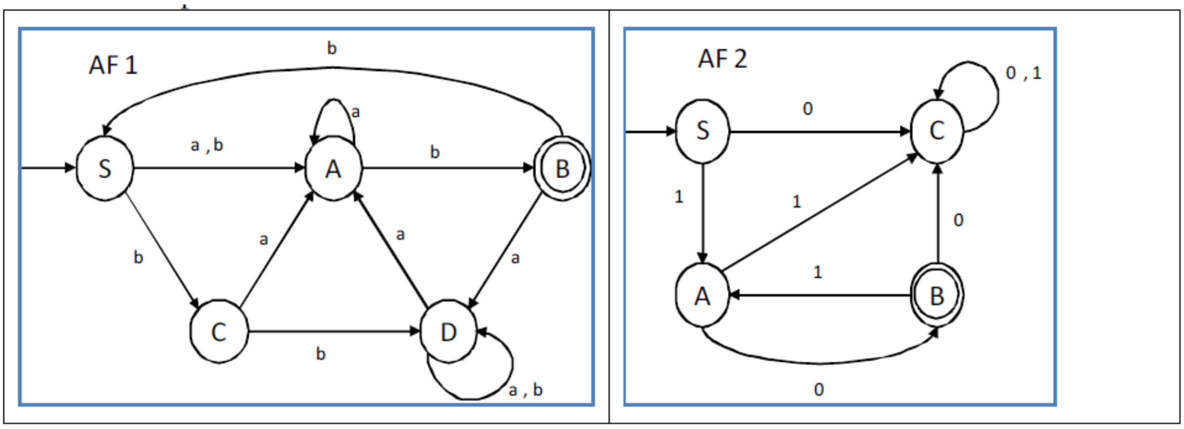
3. Given the RE (b•a*)*, which represents a regular language, construct a FA accepting this regular language.

4. Given the RE (b•a*)* obtain a G3 right-linear grammar that generates the same language defined by the RE. Verify whether the G3 right-linear grammar obtained from the RE would be the same that the obtained from the FA designed in the previous problem. That is to say, exercise 4 (RE→G3RL$_1$), exercise 3 (RE→FA), verify if (FA→G3RL$_2$) both grammars are exactly equal or simply equivalent.

5. Given the FA represented in the following figures, obtain the REs which define the languages recognized by each FA.

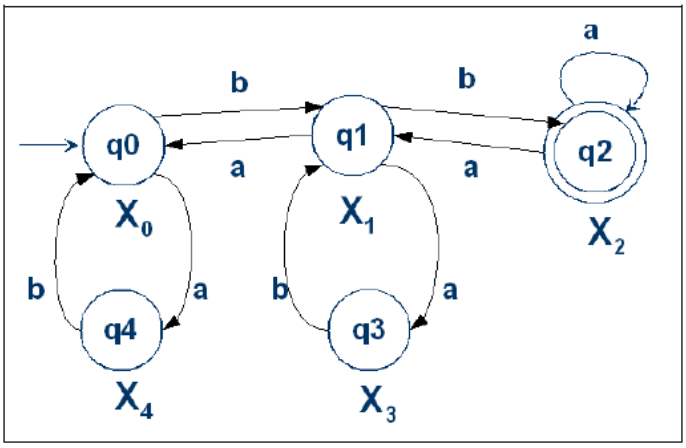


6. Indicate the RE corresponding to the languages recognized by the following Finite Automata.



7. Given the following grammar: G =({0,1}, {A,B,C}, A, P), P={A::=0B, A::=λ ,B::=1C , B::=1 , C::=0B)}. Obtain the RE which represents the same language generated by the grammar.

8. Determine the language recognized by the following automaton. To do this, use the characteristic equations.



9. Given the following right-linear grammar, G= ({0,1},{S,A,B,C},S,P), where P={ S::= 1A | 1B, A ::= 0A | 0C | 1C | 1 , B ::= 1A | 1C | 1 , C ::= 1}. Calculate formally the RE of the language associated to this grammar.

10. Given the grammar: G=({0,1},{S,A,B,C,D},S,P) where P={ S ::= ABCD , A ::= 0A | 1A | λ , B ::= 0 ,C ::= 0 | 1 , D ::= 0 | 1 }. Construct formally, from a regular expression of the language generated by it, an equivalent regular grammar. It is not required to formally generate the RE of the language generated by the grammar specified in the exercise.

11. Find a RE equivalent to the following grammar:

$$
\begin{array}{ll}
S \rightarrow & AS \\
S \rightarrow & \lambda \\
A \rightarrow & xB \\
B \rightarrow & xB \\
B \rightarrow & yB \\
A \rightarrow & x
\end{array}
$$

12. Given the language generated for the following context-free grammar

$$
\begin{array}{ll}
S \rightarrow & xSy \\
S \rightarrow & ySx \\
S \rightarrow & \lambda
\end{array}
$$

and the language L_{RE} corresponding to the regular expression (*x*|*y*)*.

Indicate which of the following relationships between L_G and L_{RE} are true and justify formally your answer.

1. $L_G \subset L_{RE}$
2. $L_{RE} \subset L_G$
3. $L_G = L_{RE}$

13. Given a language L which consists of every string which starts by zero or more x's followed by at least one, which is/are (at the same time) also followed by an even number of z's. For instance, xxyyzz, xyyzzzz, and yzzzz are included in L.

   a. Write the regular expression whose language is L.

   b. Calculate the minimal automaton whose language is L.

14. Indicate the results for the following operations: δ(a), δ(a*), δ(aa*), δ(a*|a) δ(a*b), δ(a*b*), δ(a*b*|b*a*), δ(a*b*|b*a*|a).

15. Calculate the following derivatives:

    a. Da(aa*bb*)

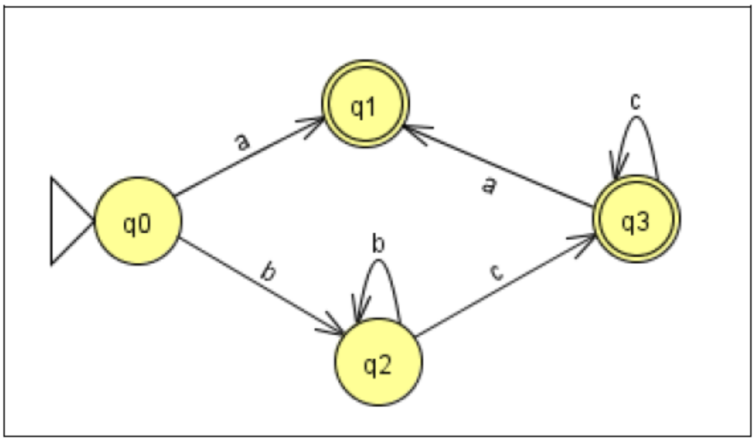
    b. Da(a*abb*)

    c. Da(abb*)

16. Simplify the following regular expression: *α = a | a(b | aa)(b*aa)*b* | a(aa | b)** by using the equivalence properties of the regular expressions.

17. Calculate the derivative $D_{ab}(\alpha)$ where α = a*ab, using the definitions of the derivatives of regular expressions.

18. Given the following automaton, calculate the associated language by using the algorithm of the characteristic equations.



19. Obtain the grammar for the regular expression a(aa | b)*.

20. Given the following regular expression a*c*(a|b) (cb)*, construct formally an equivalent regular grammar.

21. Given the language denoted by the RE *(ab)*(ba)*(abba)**, obtain the DFA which recognizes the same language. Follow these two processes:

    a. Design a Finite Automaton to recognize the sentences in the language. If the Finite Automaton is a NFA, transform it into an equivalent DFA.
    b. Obtain the corresponding grammar from the RE (using the concept of derivatives). Obtain the minimal DFA from the grammar.