Formal Languages and Automata Theory
Exercises Push-Down Automata
Unit 6

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* Several exercises are based on the ones proposed in the following books:

- Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón. *Teoría de autómatas y lenguajes formales.* McGraw-Hill (2007).
- Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga. *Teoría de lenguajes, gramáticas y autómatas.* Publicaciones R.A.E.C. (1997).
- Pedro Isasi, Paloma Martínez y Daniel Borrajo. *Lenguajes, Gramáticas y Autómatas. Un enfoque práctico.* Addison-Wesley (1997).

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1. Design a Push-Down Automaton for each one of the following languages:
   a. $L = \{ a^n \bullet b^n \mid n \geq 0\}$
   b. $L = \{ a^n \bullet b^{2n} \mid n \geq 0\}$
   c. $L = \{ a^{2n} \bullet b^n \mid n \geq 0\}$

2. Design a Push-Down Automaton for each one of the following languages:
   a. $L = \{ a^{n+1} \bullet b^n \mid n > 0\}$
   b. $L = \{ a^{2n+1} \bullet b^n \mid n > 0\}$

3. Design a Push-Down Automaton for the language $L = \{a^{n+m} \cdot b^{m+t} \cdot a^t \cdot b^n \mid n, t > 0\ m \geq 0\}$

4. Design a Push-Down Automaton for the language $L = \{ a^{n+m} \cdot b^{m+t} \cdot a^t \cdot b^n \mid n, t, m > 0\}$

5. Design, directly and without calculating the PDA, a grammar to generate each one of the following languages.
   a. $L = \{ a^n \bullet b^n \mid n \geq 0\}$
   b. $L = \{ a^n \bullet b^{2n} \mid n \geq 0\}$
   c. $L = \{ a^{2n} \bullet b^n \mid n \geq 0\}$
   d. $L = \{ a^{2n} \bullet b^n \mid n \geq 0\}$
   e. $L = \{ a^{n+1} \bullet b^n \mid n > 0\}$
   f. $L = \{ a^{2n+1} \bullet b^n \mid n > 0\}$
   g. $L = \{a^{n+m} \cdot b^{m+t} \cdot a^t \cdot b^n \mid n, t > 0\ m \geq 0\}$
   h. $L = \{ a^{n+m} \cdot b^{m+t} \cdot a^t \cdot b^n \mid n, t, m > 0\}$

   - Compare these grammars with the ones obtained for the previous exercise.
   - Transform each grammar into a PDA, and compare them with the ones obtained ones for the previous exercise.
   - Select a word included in the language $L(G_i)$ and verify this by showing the complete derivation. Verify that this word is also recognized by the equivalent automaton by showing the movements included in the instantaneous descriptions.

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6. Design a Push-Down Automaton which recognizes the language of arithmetical expressions with the following alphabet Σ={ 0, 1 , + , * , (,) }

7. Obtain the PDA_E corresponding to the grammar

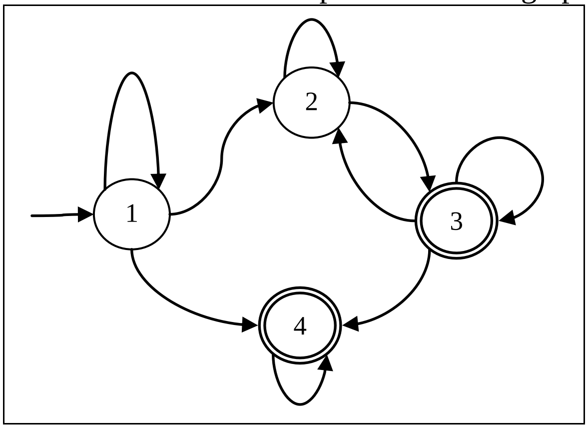
   G_{FNG}=({a,b,c,d}, {S,A,B}, S, P), with the following production rules:

   S ::= a S B | b A | b | d
   A ::= b A | b
   B ::= c

8. Obtain formally the PDAE equivalent to the following PDA_F:

   a. $PDAF_a$=(Σ, {0,1,A0}, {1,2,3,4}, A0, 1, f, {3,4}), where f is given by:

      Note: Additional information not provided in the graph is not required.



   b. [Exam Problem Feb 1999] PDA_{Fb}=({a,b}, {A,B}, {q1,q2,q3,q4}, A, q1, f, {q4}), where f is given by:

      f(q1,a,A) = {(q2,BA), (q4,A)}
      f(q1,λ,A) = {(q4, λ)}
      f(q2,a,B) = {(q2,BB)}
      f(q2,b,B) = {(q3, λ)}
      f(q3,λ,A) = {(q4,A)}
      f(q3,b,B) = {(q3, λ)}

9. Describe the transition functions which generate the following movements:

   (p,1001, A) |- (p, 001,1A) |- (p, 01, 01A) |- (q, 1, 1A) |- (q, λ, A) |- (q, λ, λ)

10. Obtain the PDA corresponding to the following grammar.

   G = (Σ_T, Σ_N, A, P), P= {A ::= a B A | b , B ::= b A B | a}

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