1. Given the following boolean functions:

$$f_1 = \sum_4 (0,2,3,6,7,8,15) + \Delta_4 (1,5,10)$$
$$f_2 = ac + \bar{b}cd + ab\bar{c}d$$

   a) Implement f_1 with the minimum possible number of NOR logic gates.
   b) Obtain a simplified expression of f_2 as a product of sums.
   c) Implement f_1 with a 4:16 decoder with active-high outputs and with active-low enable.
   d) Implement f_1 with a multiplexer with 8 data inputs and NAND gates.

2. Given the following logic function:

$$f(a,b,c,d) = \sum_4 (0,4,6,7,8,10,12) + \Delta_4 (2,13)$$

   a) Obtain the most simplified expression in a 2-level representation (SOP or POS)
   b) Implement the logic function f using only 2-input NOR gates.
   c) Implement f with a 4:16 decoder with active-low outputs.
   d) Implement f using a MUX2 and additional logic
   e) Implement f using only MUX4.

3. Given the following logic function:

$$f = \sum_4 (0,4,5,6,10) + \Delta_4 (2,7,8,13,15)$$

   a) Obtain the most simplified expression as product of sums
   b) Obtain the most simplified expression as sum of products
   c) Implement the Boolean function f using only 2-input NAND gates
   d) Implement f using a 4:16 decoder with active-low outputs and active low enable, and additional logic
   e) Implement f using a MUX8 and additional logic

4. Given the following functions:

$$F1(a,b,c,d) = (a+b)\cdot(a+b+\bar{c}+d)\cdot(\bar{a}+b+\bar{c}+d)$$
$$F2(a,b,c,d) = \sum_4 (1,7,11,13,14,15) + \Delta_4 (3,5,6,9,10,12)$$

Find:

   a) A simplified logic expression for F1 as sum of products.
   b) A simplified logic expression for F2 as product of sums.
   c) An implementation of F1 using only 2-input NOR gates

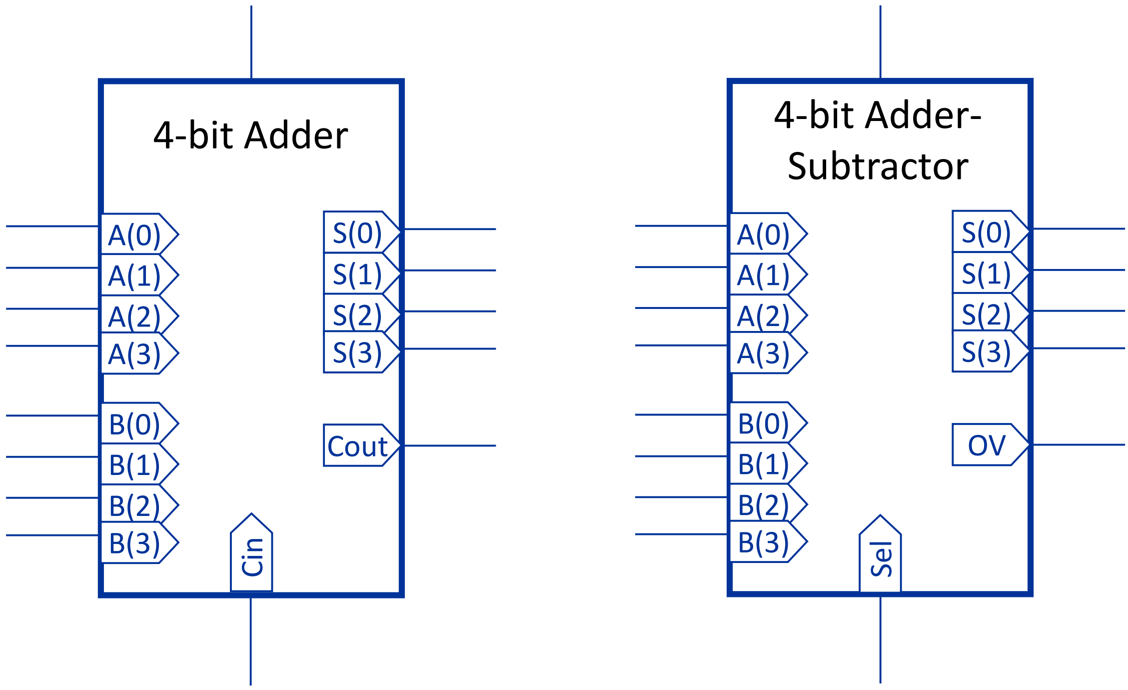
d) An implementation of F2 with only one active-high outputs decoder and additional logic gates if necessary.

5. Given the following boolean function:

$$f = \sum_4 (0,2,3,6,7,8,15) + \Delta_4 (1,5,10)$$

a) Find a simplified expression for f as a product of sums.
b) Find a simplified expression for f as a sum of products.
c) Implement f with a 4:16 decoder with active-low outputs and active-low enable.
d) Implement f with a multiplexer with 8 data inputs and NAND gates.
e) Implement f with a 3:8 decoder with active-high outputs.

6. Obtain the implementation of a 4-bit adder-subtractor (right figure) using a 4-bit adder (left figure), considering that the operations are performed in 2s-complement:



Cin and Cout are the carry-in and carry-out of the 4-bit adder.

Sel and OV are the selection input (add/subtract) and the overflow output of the 4-bit adder-subtractor.

Hint: Add the additional necessary gates to the circuit on the left to obtain the functionality of the circuit on the right.

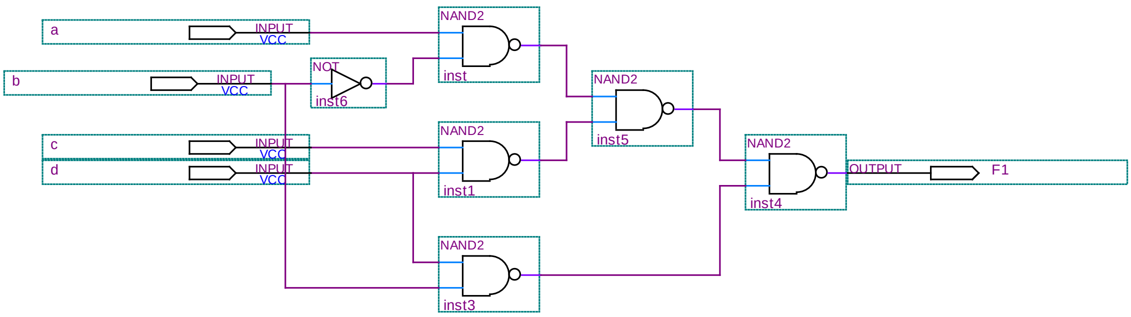
7. Given the following boolean function:

$$f(d,c,b,a) = \sum_4 (0,2,4,7,10,11,15) + \Delta_4 (5,6,13)$$

a) Find a simplified expression for f as a product of sums.

b) Find a simplified expression for f as a sum of products.
c) Find an expression using only NAND operations (it is not required to show the graphical representation of the circuit).
d) Implement f with a decoder with active-high outputs and active-high enable. Point out first which is the most suitable size for the decoder.
e) Implement f with a MUX2 (multiplexer with 4 data inputs) and additional logic if needed (only one MUX2).

8. Consider two functions F1 and F2. F1 is shown in the figure:



and F2 is expressed as:

$$\mathbf{F2(A,B,C,D) = \Sigma_4(1,5,7,9,11,12)+\Delta_4(13,14,15)}$$

a) Obtain the simplified logic function of F1 (schematic shown in the figure), expressed as a sum of products.
b) Obtain the simplified logic function of F2 expressed as a product of sums.
c) Implement F1 using a 4:16 decoder with active-high outputs and additional logic.
d) Implement F2 using an 8:1 multiplexer and the minimum possible number of logic gates.

9.

a) Convert 1501_{10} to natural binary and octal, and encode it using Gray's code.
b) Encode $E3_{16}$ using BCD code.
c) Express $+33_{10}$ in natural binary, and $+33_{10}$ and -33_{10} in both 1's complement and 2's complement systems. Choose the minimum possible number of bits for each representation.
d) Perform the following operations as sums of 9 bits, with the operands expressed in 2's complement: 33+239, -33+239. Point out when there is carry and/or overflow and justify it. (239_{10}= 11101111_2).

10. Hand in your solutions of this problem in this same sheet.

a) Represent 16510 y 9710 in natural binary, octal, hexadecimal, natural BCD and Gray code.

	165_{10}	97_{10}
Natural binary		
Octal		
Hexadecimal		
Natural BCD		
Gray code		

b) Represent 671_8 in natural BCD:

c) Represent $+165_{10}$, -165_{10}, $+97_{10}$, -97_{10} using 2s-complement representation. Use the minimum possible number of bits for all of them.

	Representation in 2C with minimum number of bits
$+165_{10}$	
-165_{10}	
$+97_{10}$	
-97_{10}	

d) Perform the following operations in 2s-complement. Use the minimum number of bits so that there is no overflow in any operation.

165-165	165+97	97-165

11. Let A = 10011101_2 and B = 01111011_2:
    a) Represent A in Octal, Hexadecimal and natural-BCD code
    b) Suppose that A and B are representing unsigned numbers:
        1. Determine the decimal values of A and B
        2. Perform the operation A+B in binary.
        3. Indicate if there is overflow in the operation. Justify your answer.
    c) Suppose now that A and B are represented in 2's complement
        4. Determine the decimal values of A and B
        5. Perform the operation A+B in binary.
        6. Indicate if there is overflow in the operation. Justify your answer.

12.
    a) Represent number 459_{10} in Octal, Hexadecimal and natural BCD code.

    b) Represent numbers A=+43 and B=-36 using 8-bit 2s-complemement.

    c) Perform operation A+B in 8-bit 2s-complement. Point out if there is overflow in this operation. Justify your answer.

13.
    a) Convert 1501_{10} to binary, octal and hexadecimal.
    b) Encode $E3_{16}$ using BCD code.
    c) Encode $+33_{10}$ in natural binary, and $+33_{10}$ and -33_{10} in both 1's complement and 2's complement systems. Choose the minimum possible number of bits for each representation.
    d) Perform the following operations as sums of 9 bits, with the operands expressed in 2's complement: 33+239, -33+239. Point out when there is carry and/or overflow and justify your answer. (239_{10}= 11101111_2).

14.
    a) Convert into decimal, octal, hexadecimal and BCD the binary number 101110_2.
    b) Convert into binary the decimal number $25{,}4375_{10}$.
    c) Using representations of 7-bit 2s complement, compute the operations A+B y A-B being A= -45_{10} and B= -17_{10}. Point out if there is overflow in any of the operations.