

Arithmetic Combinational Circuits

© Luis Entrena, Celia López,
Mario García, Enrique San Millán
Universidad Carlos III de Madrid

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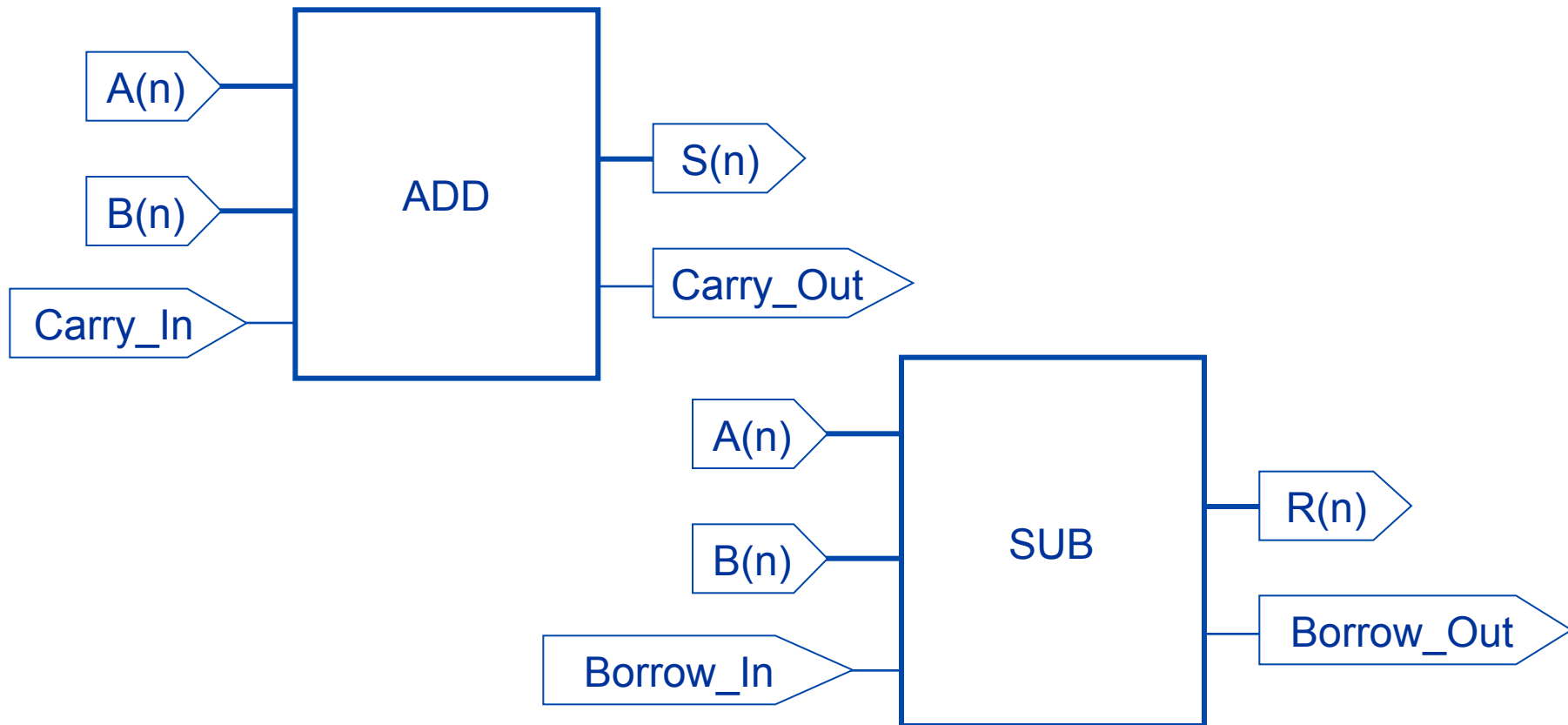
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Adders and Subtractors circuits

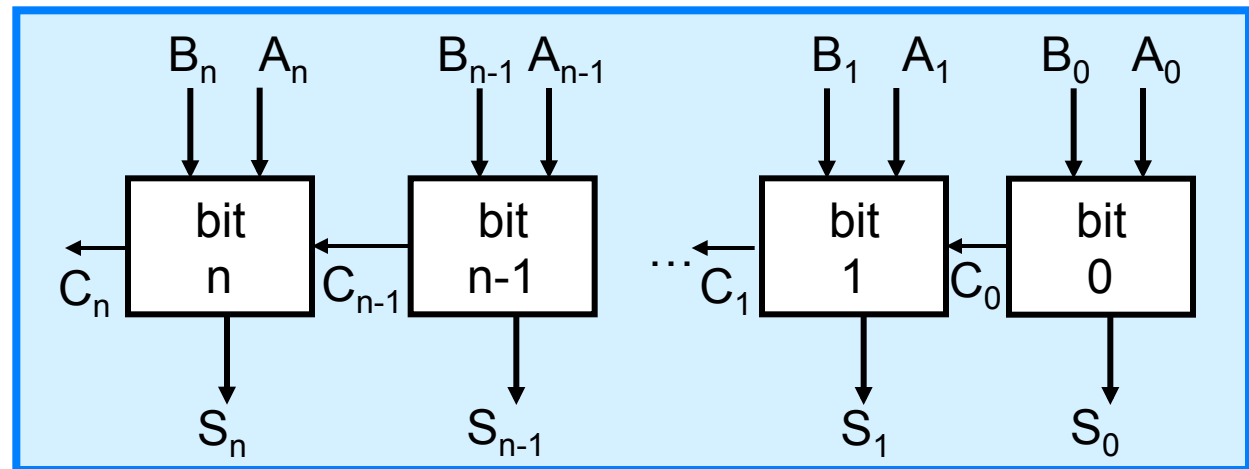


Adder with serial propagation of Carry.

Decimal and binary Addition

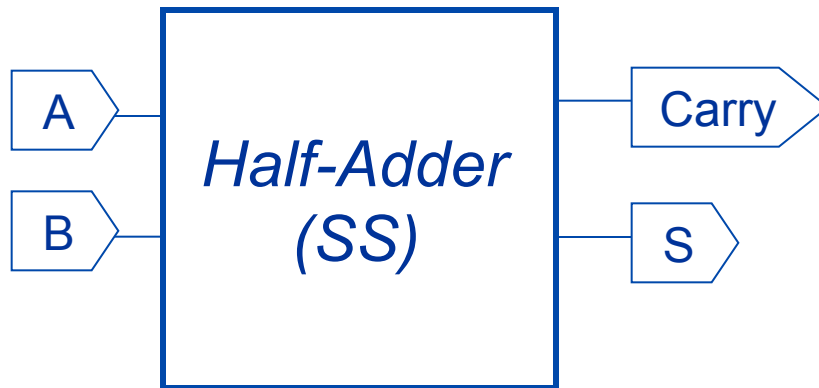
① ①		
86d	→	1010110b
25d	→	0011001b
111d	→	1101111b

- Operands: **n bits**
- Result: **n+1 bits**

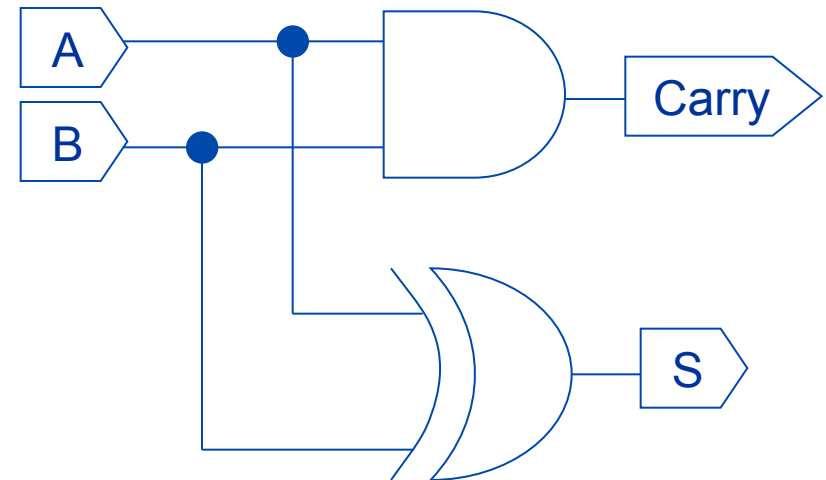


Adder with serial propagation of carry.

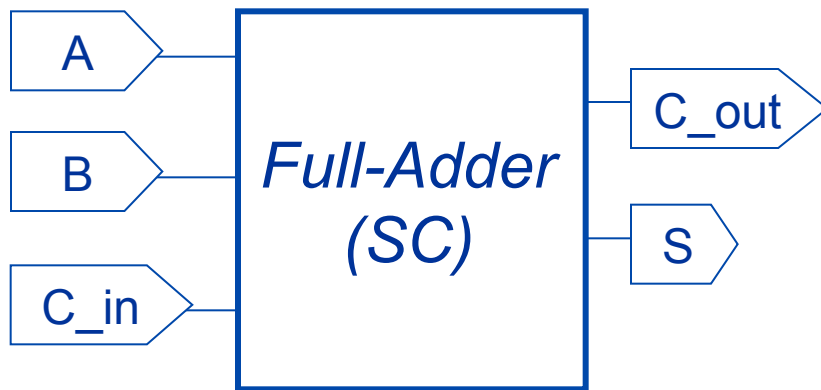
Half Adder



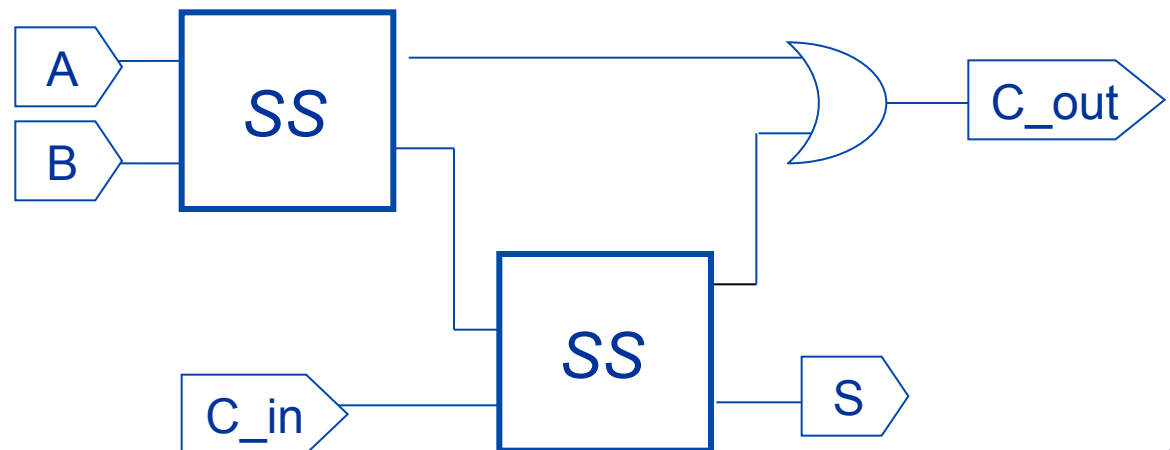
A	B	S	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



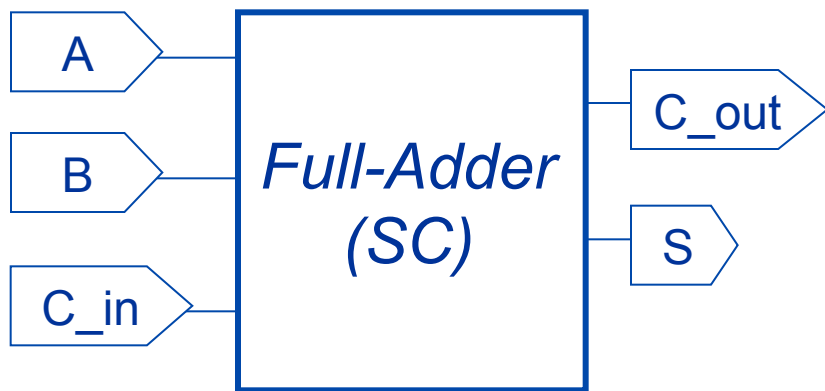
Adder with serial propagation of carry. Full Adder



A	B	C_in	S	C_out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Adder with serial propagation of carry. Full Adder



A	B	C _{in}	S	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

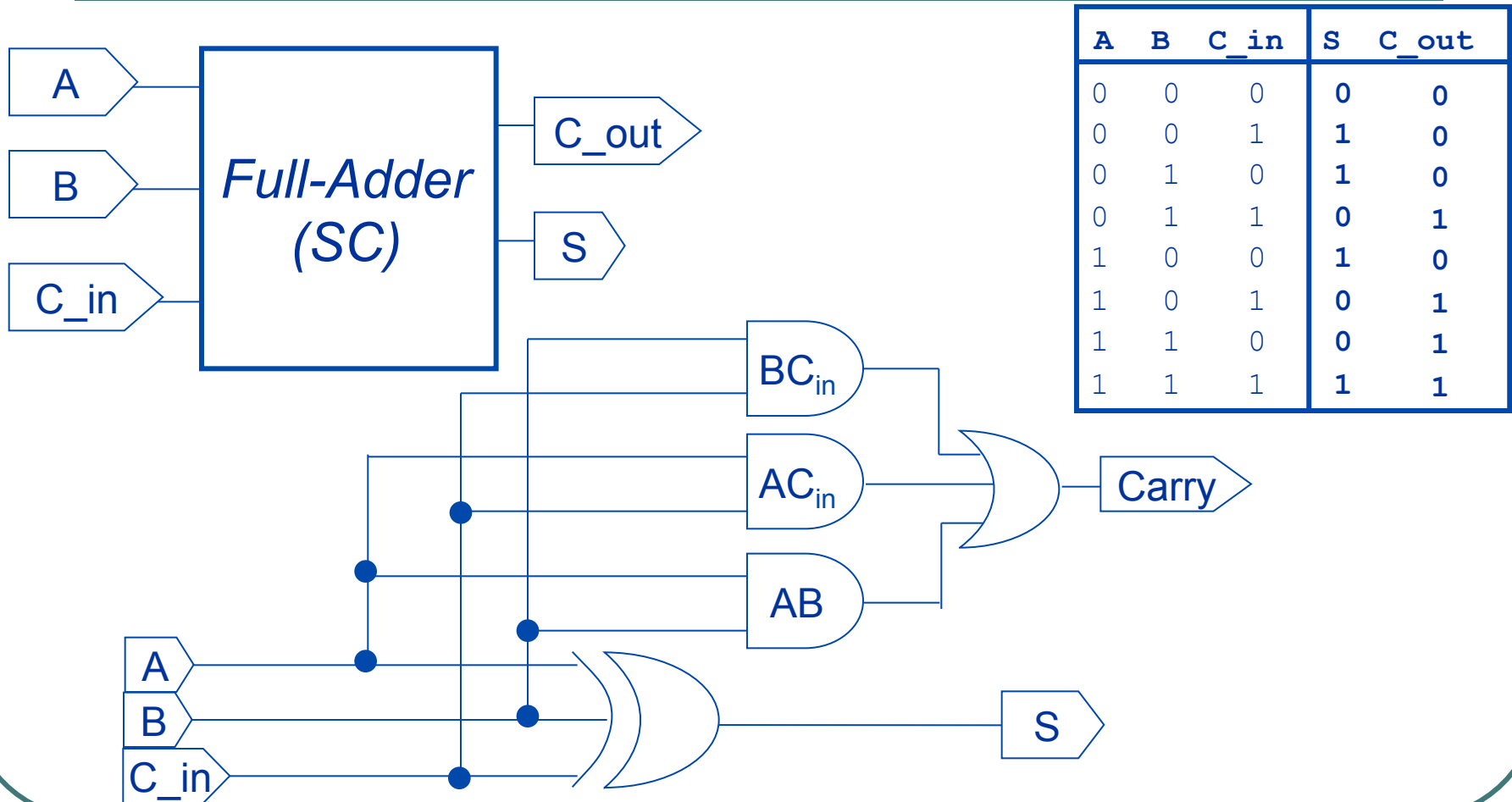
S

0	1	0	1
1	0	1	0

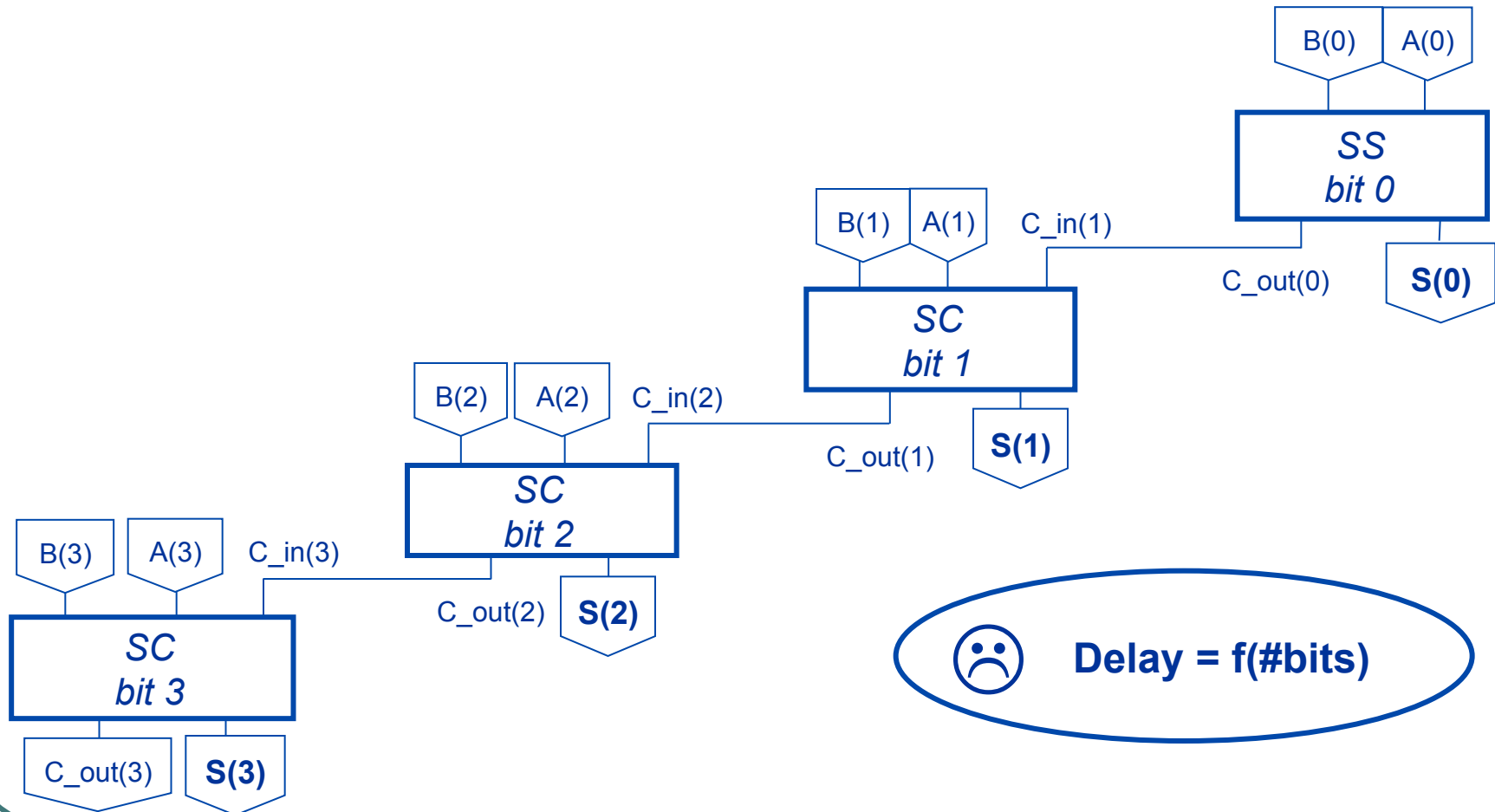
C_{out}

0	0	1	0
0	1	1	1

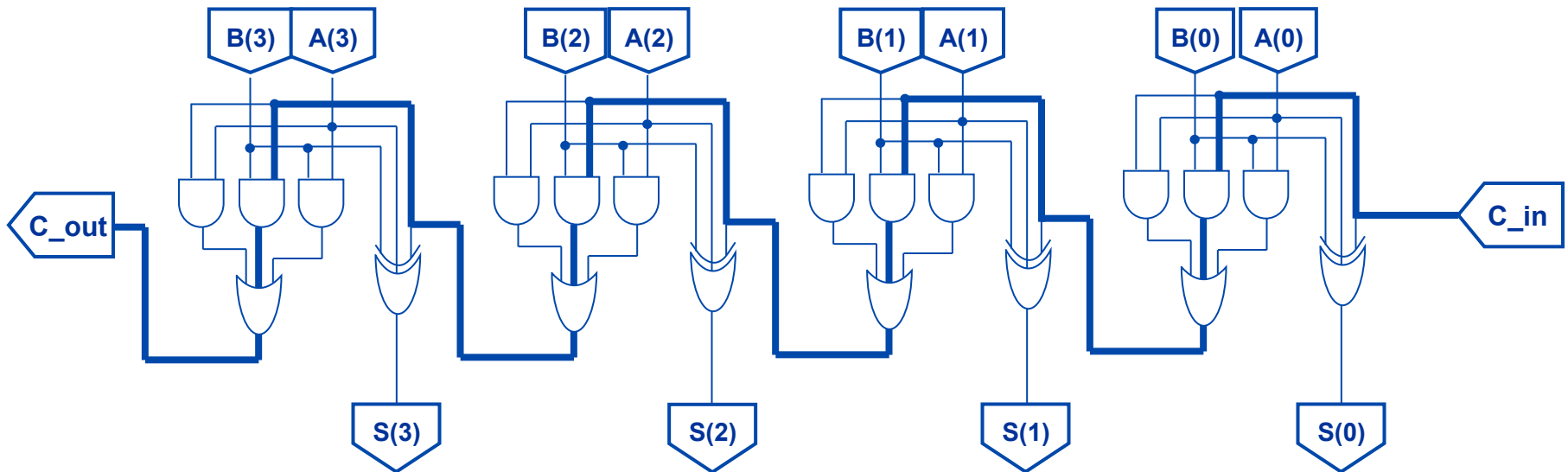
Adder with serial propagation of carry. Full Adder



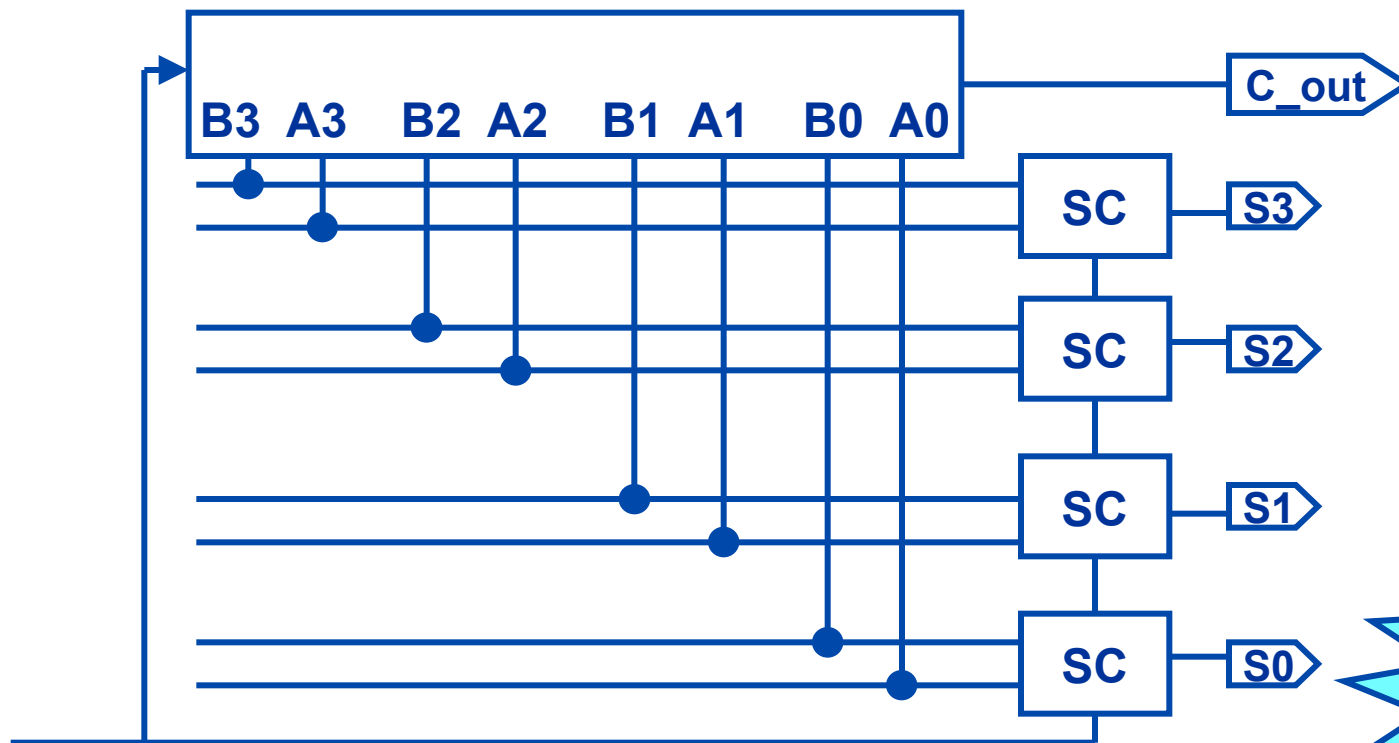
Adder with serial propagation of carry. N-bits adder



Adder with serial propagation of carry. N-bits adder

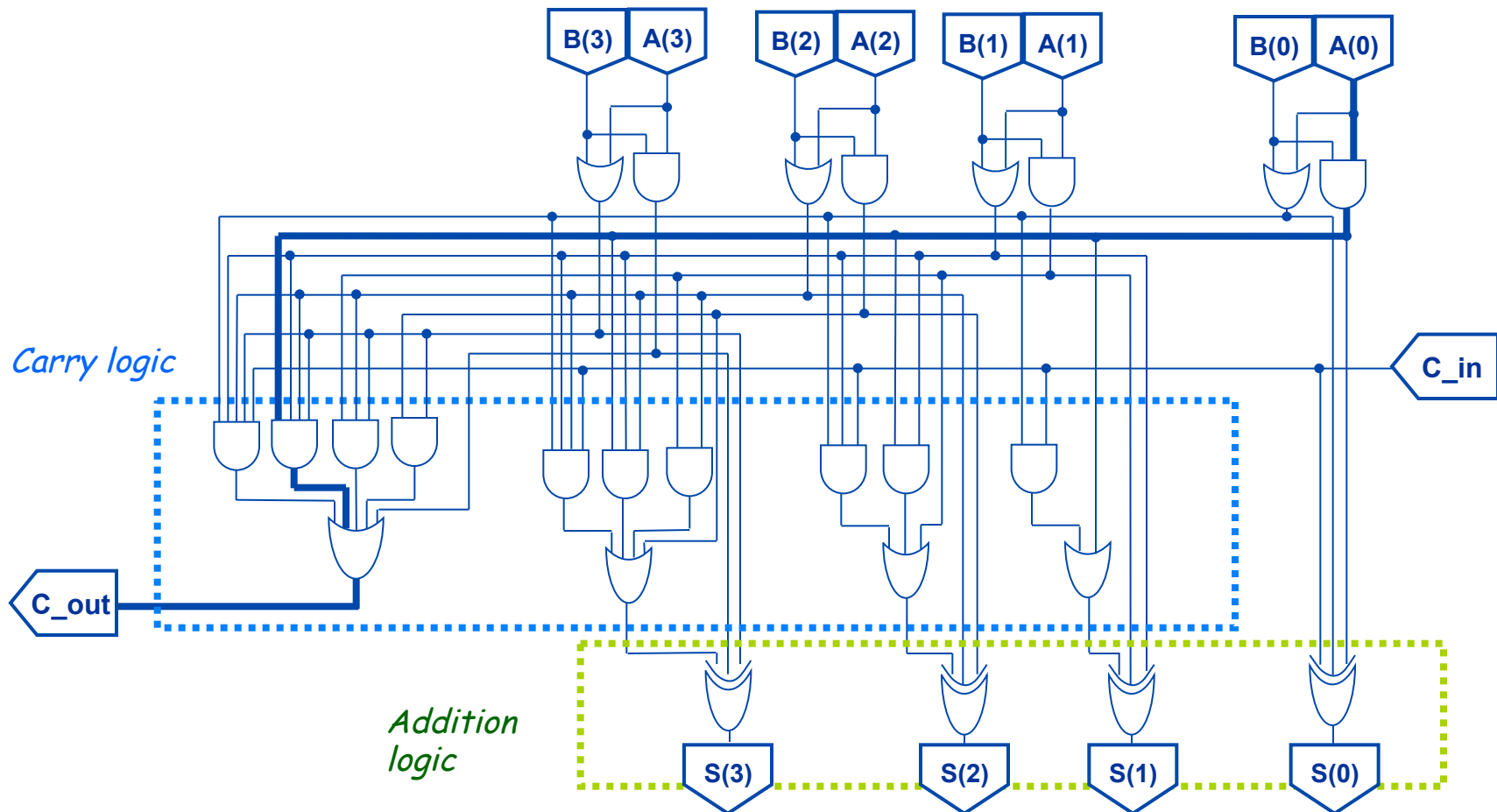


Adder with carry look ahead.



Carry
look-ahead

Adder with carry look ahead.



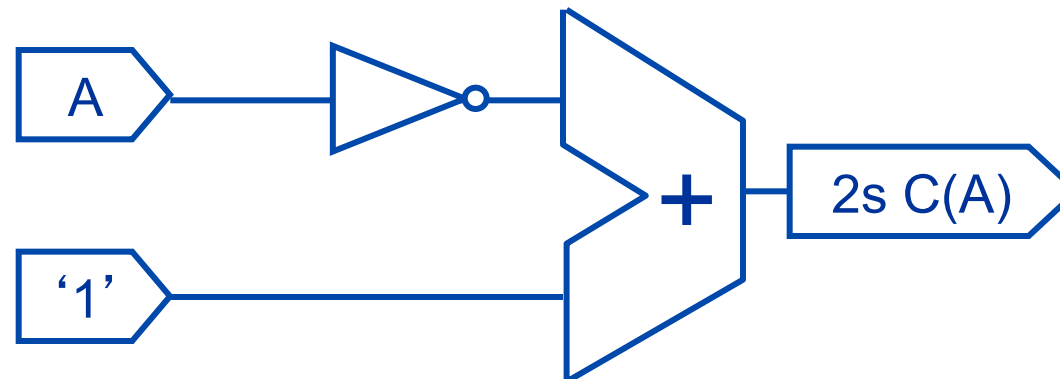
Adder/Subtractor with 2s complement.

2-s complement

- Positive numbers



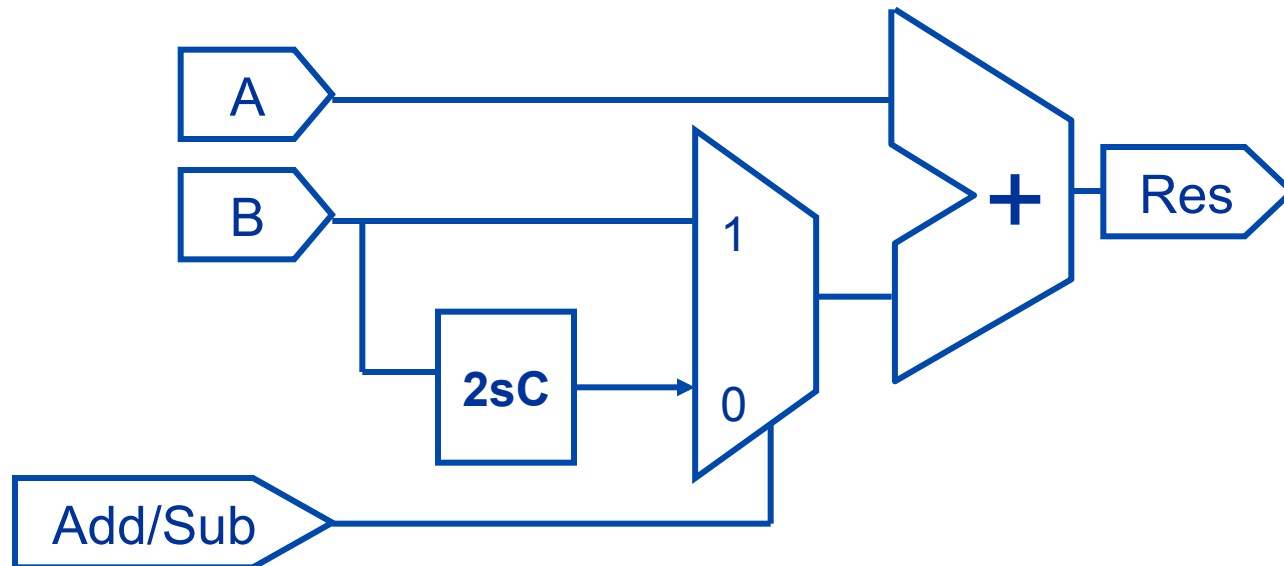
- Negative numbers



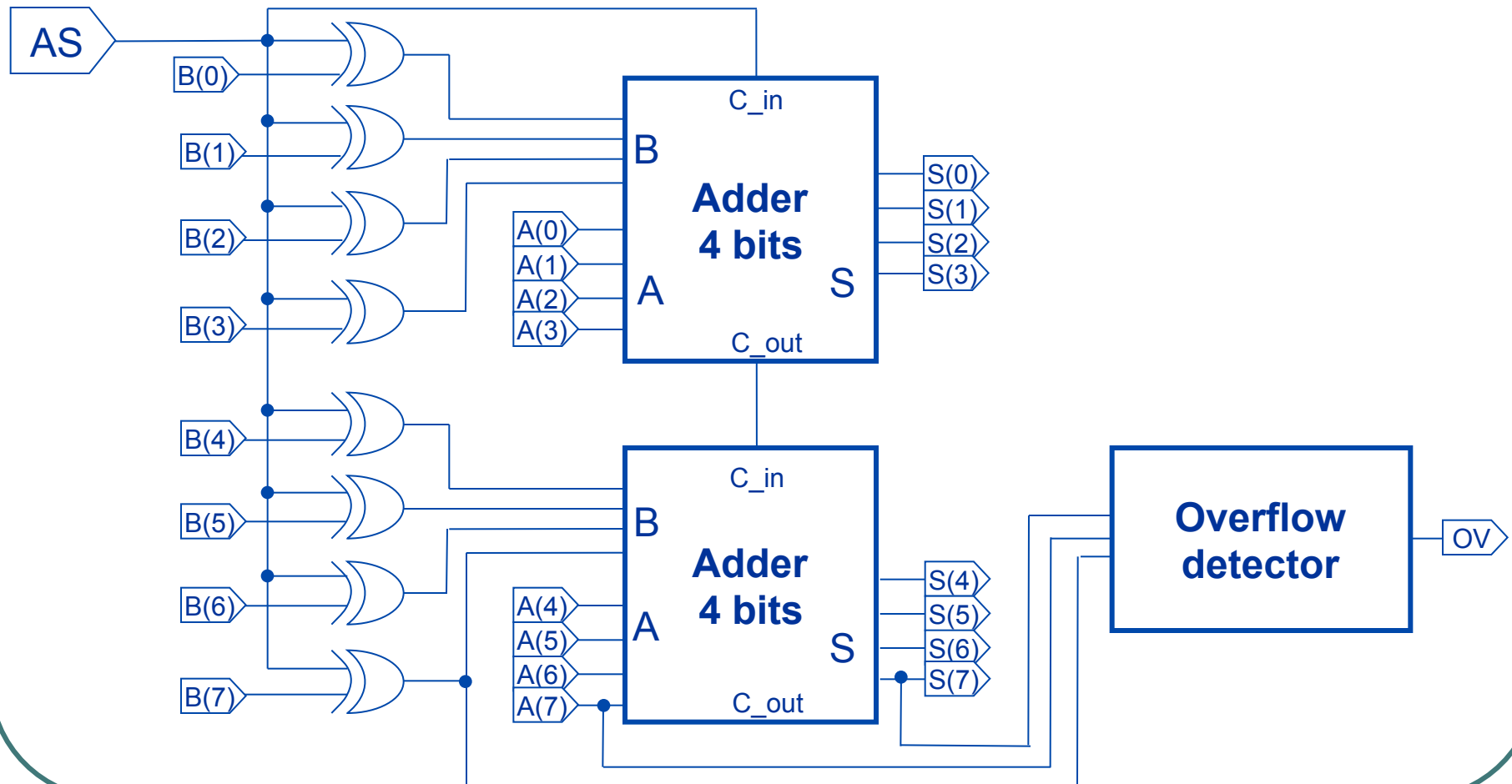
Adder/Subtractor with 2s complement.



$$A - B = A + (-B)$$



Adder/Subtractor with 2s complement.



Adder/Subtractor with 2s complement.

Exercise



Multiplier.

Decimal and binary multiplier



86d	→	1010110b
15d	→	0001111b
1290d	→	10100001010b

Decimal

86d	
15d	
30	5x6
40	5x8 shifted left 1 position.
6	6x1 shifted left 1 position.
8	8x1 shifted left 2 positions.
1290	

- Operands: **n bits**
- Result: **2*n bits**

Multiplier.

Binary

$$A * B = A * (b_{n-1} * 2^{n-1} + b_{n-2} * 2^{n-2} + \dots + b_1 * 2^1 + b_0 * 2^0)$$

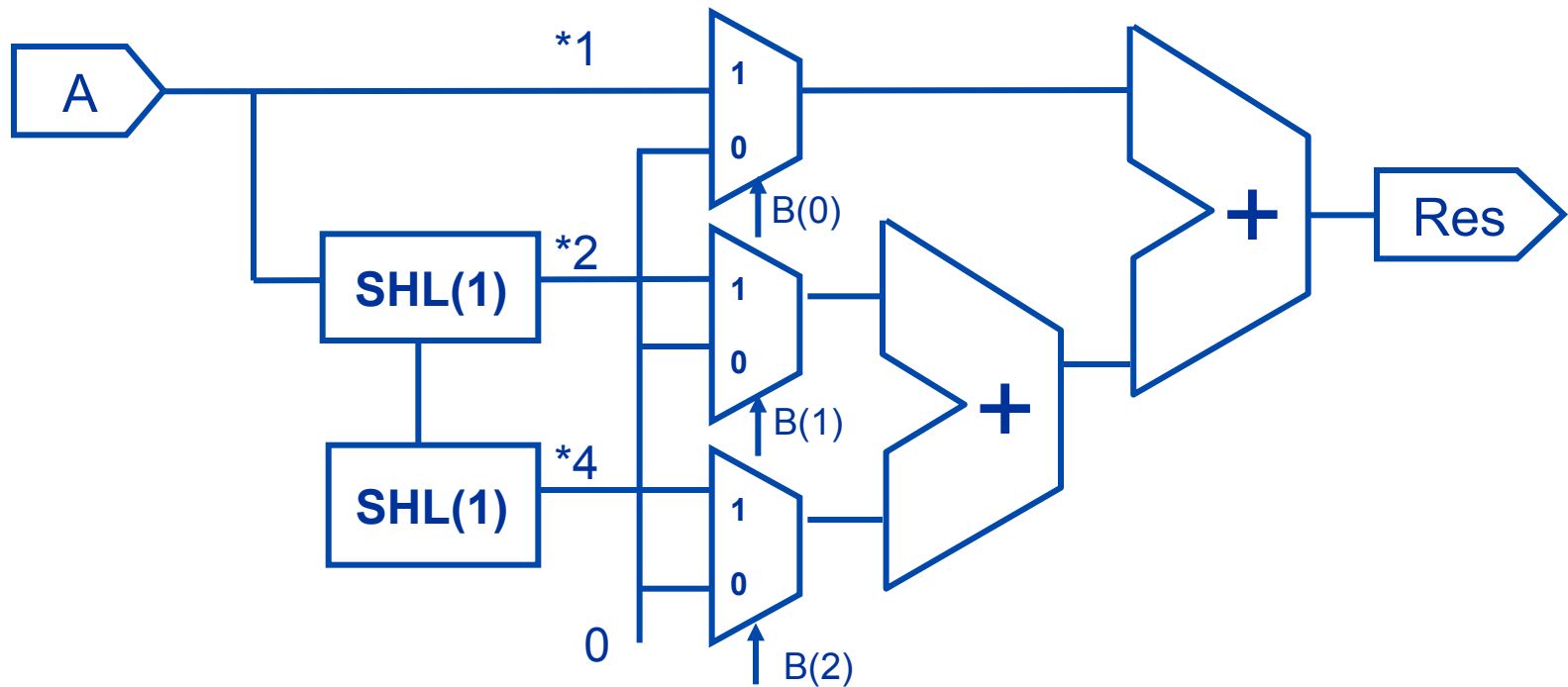
!! '1s' o '0s' !!

The binary multiplication of two numbers A (m bits) y B (n bits) consists in an adding of as many elements as bits in B (n). Every element i is number A shifted left i-times when the corresponding weight of B is '1'.
In other case the element is '0'.

Multiplier.



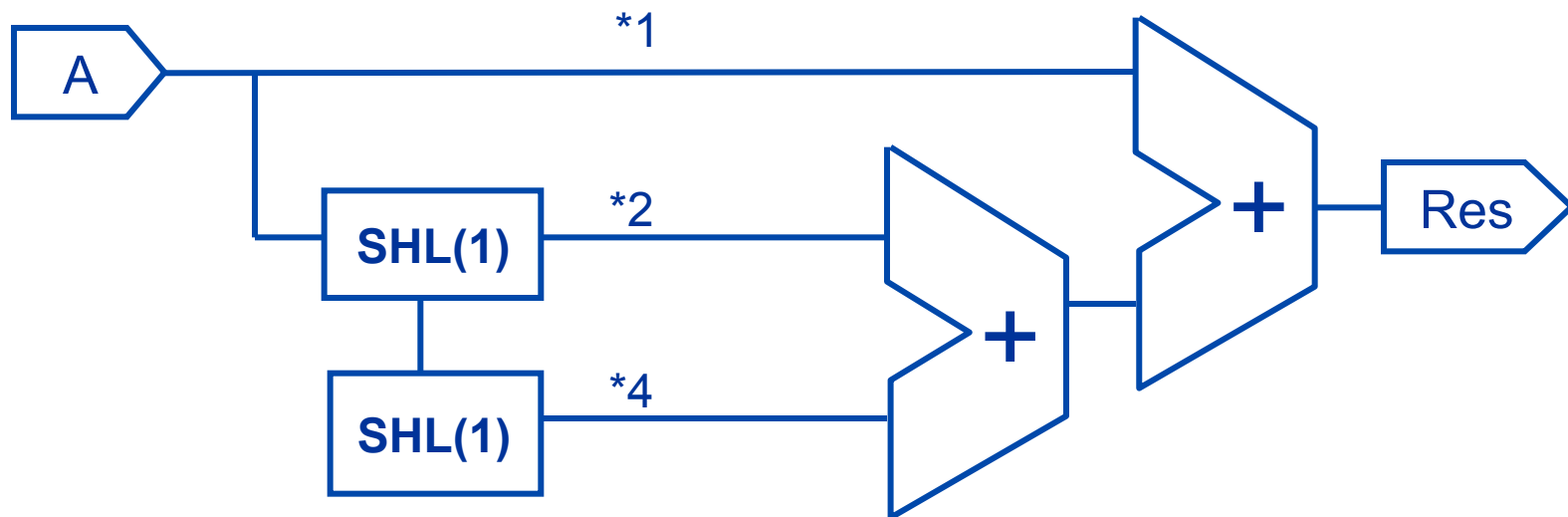
$$A * B = A * (4*B(2)+2*B(1)+1*B(0))$$



Multiplier.



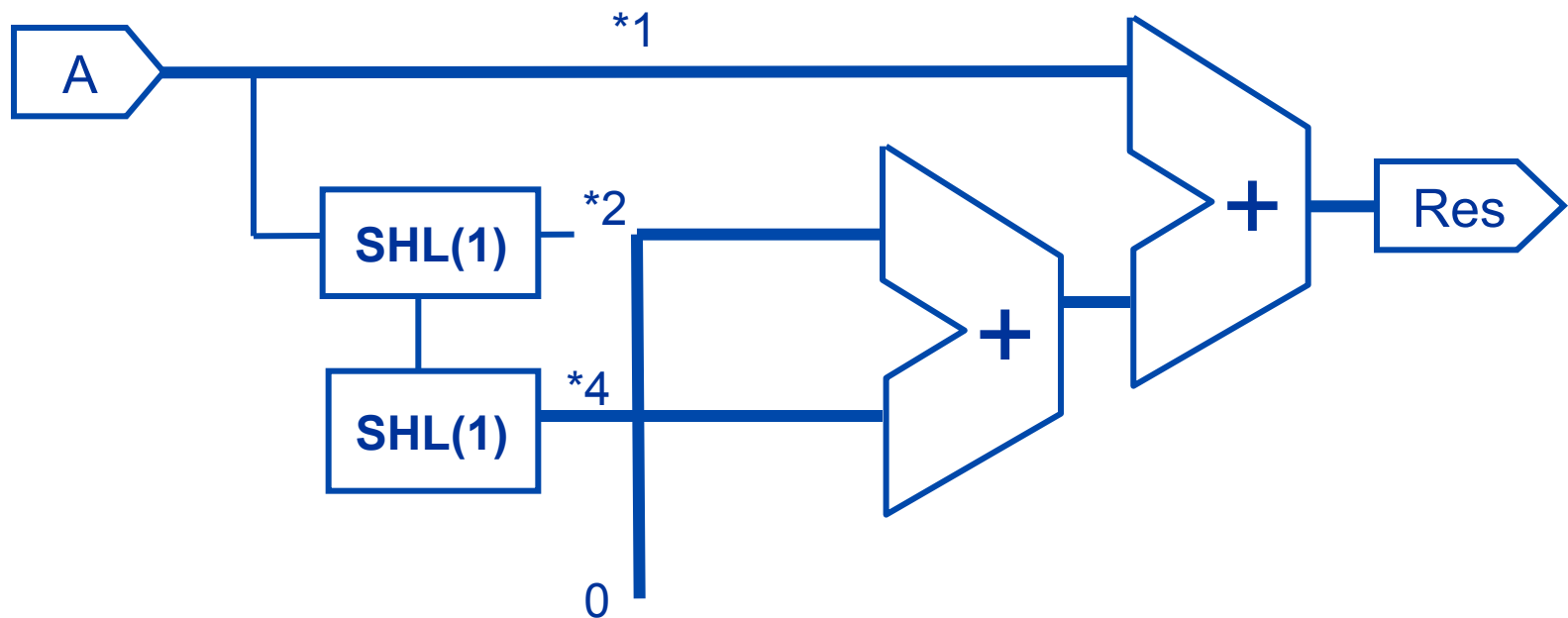
$$A * 7 = A * (4+2+1)$$



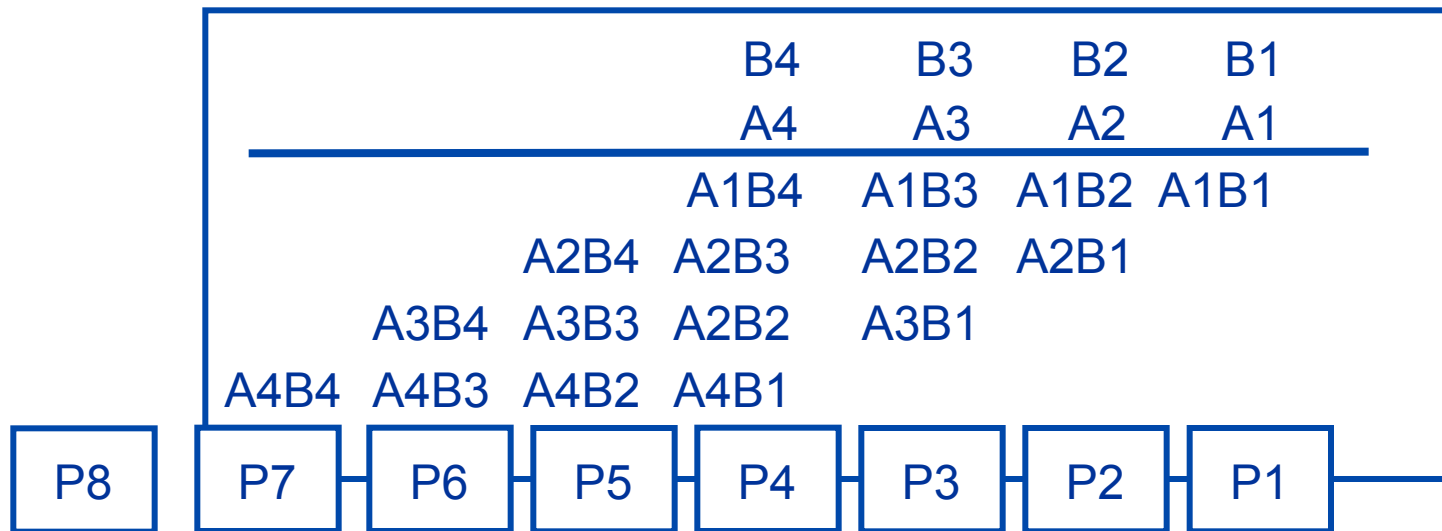
Multiplier.



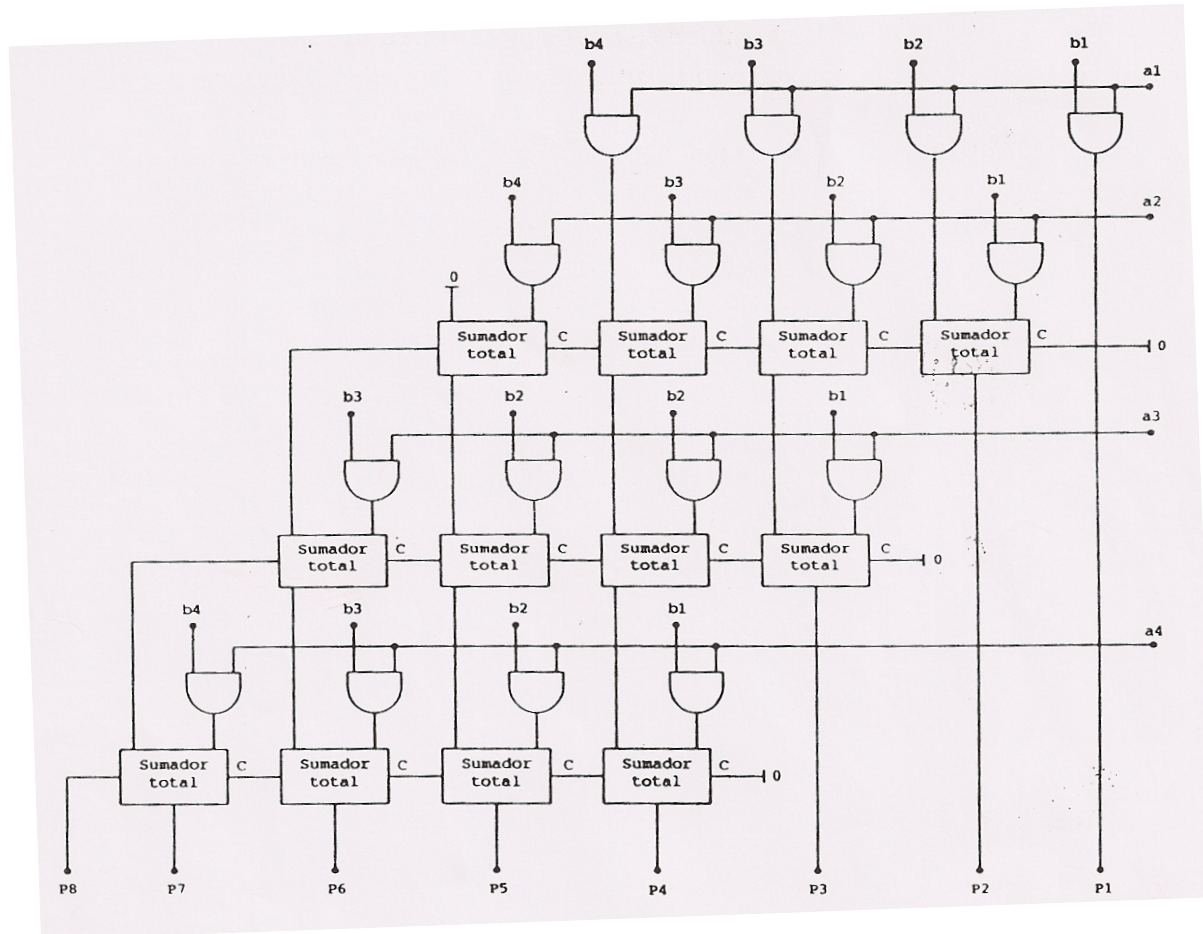
$$A * 5 = A * (4 + 0 + 1)$$



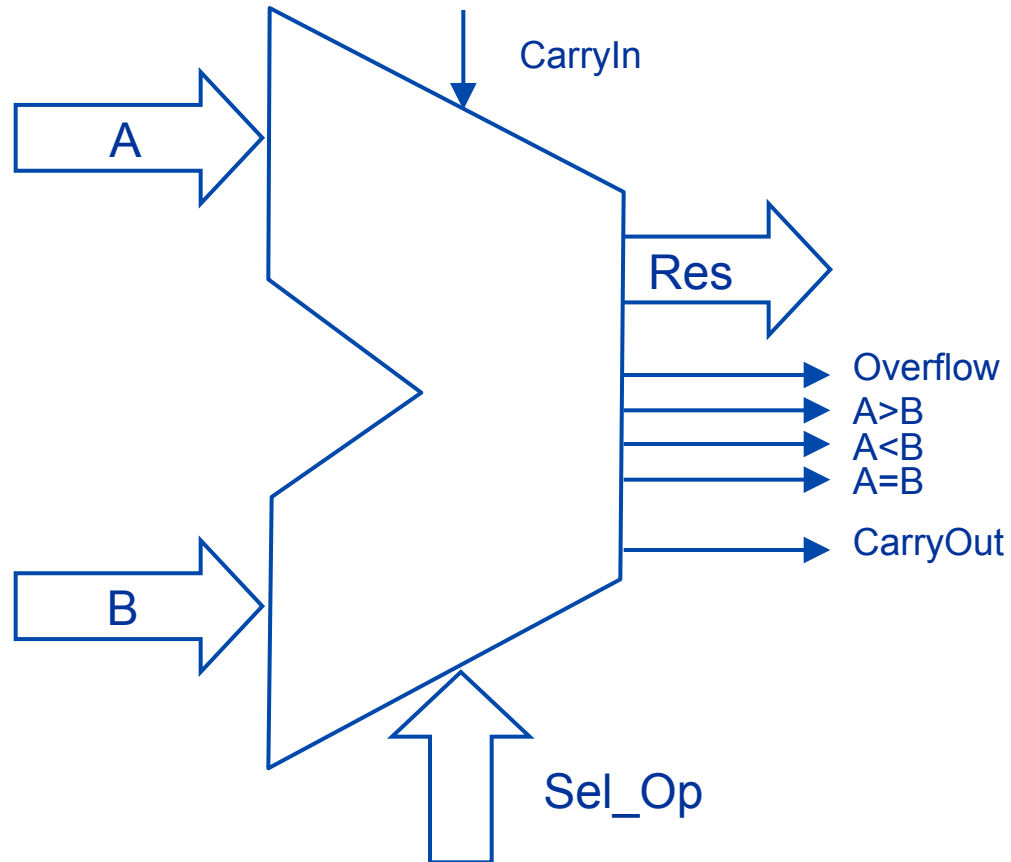
Multiplier.



Multiplier.



Arithmetic-Logic Unit



Arithmetic-Logic Unit.

Combinational

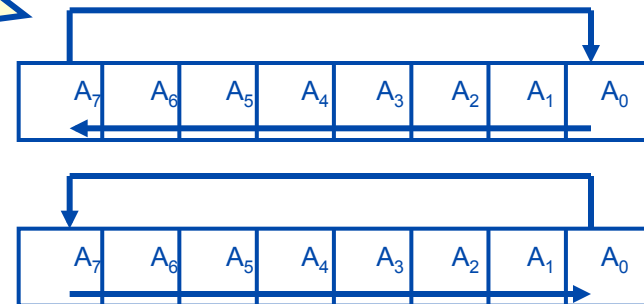
Block for execution of arithmetic-logic operations:

- Adding
 $A + B$
- Substraction
 $A - B$
- 2-s Complement
 $- B$
- Comparison
 $A > B$
 $A < B$
 $A = B$
- Shift Left
 $SHL(A) \leftarrow$
- Shift Right
 $SHR(A) \rightarrow$

SUBSTRACTION

Logic operations (bit to bit)

- AND
- OR
- XOR
- XNOR
- NOT



References

- “Circuitos y Sistemas Digitales”. J. E. García Sánchez, D. G. Tomás, M. Martínez Iniesta. Ed. Tebar-Flores
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- “Fundamentos de Sistemas Digitales”, T.L Floyd, Prentice-Hall