Bachelor in Statistics and Business
Universidad Carlos III de Madrid

Mathematical Methods II
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Name ____________________ Student number ____________________

FINAL EXAM

January 21, 2012

Instructions

- You have 3 hours to answer the final exam.
- Marks per question are given in bold. **Total marks: 10 points**
- The mark of this exam corresponds to the 60% of this course.
- Write your name in all the sheets.
- It is not allowed to use lecture notes, scientific calculators or cellphones during the exam.

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1. Consider the following system of linear equations depending on the parameter h:

$$\begin{cases} 4x & & +z & = & 2 \\ -x & +y & +2z & = & 0 \\ & -4y & -9z & = & h \end{cases}$$

   (a) Describe the solution set in terms of the parameter h. **[1]**

   (b) For that or those values of h that make the system consistent:

      i. Write the solution set in parametric vector form. **[0.5]**

      ii. Is that solution set a subspace of $\mathbb{R}^3$? **[0.5]**

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2. Let T be a transformation between vector spaces defined by

$$\begin{aligned} T\colon \mathbb{R}^2 &\longrightarrow \mathbb{R}^3 \\ (x, y) &\longmapsto (2x+3y, -y, -x+2y) \end{aligned}$$

   (a) Check that the transformation T is linear. **[0.75]**

   (b) Find the matrix corresponding with the linear transformation T. **[0.5]**

   (c) Find a basis for the column space of the matrix associated with T. **[0.5]**

(d) Let $\mathcal{C} = \{c_1, c_2, c_3\} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$ be a basis for $\mathbb{R}^3$. The matrix in (b) is given in standard basis both in the domaing as in the image of T. Find the matrix associated with the linear transformation T such that the image is written in the basis $\mathcal{C}$ for $\mathbb{R}^3$. (*Hint: Find a suitable change-of-basis matrix in* $\mathbb{R}^3$.) **[1.25]**

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3. Let $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$.

(a) Find the eigenvalues of A and their algebraic multiplicities. **[0.75]**

(b) Orthogonally diagonalize the matrix A. Give the matrices P and D. **[1.5]**

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4. Answer the following questions and **always** justify your answers.

(a) If the dimension of the nullspace of a 4×2 matrix A is 1, what is the rank of A? Why? **[0.5]**

(b) Let $A = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$. Find the matrix norm $\|A\|_2$ and describe all the matrices in a singular value decomposition of A. **[0.75]**

(c) Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ and $\lambda = 2$ be an eigenvalue of A. Find an **ORTHOGONAL basis** for the eigenspace of eigenvalue 2. **[1]**

(d) Consider the following network. Find the system of linear equations that describe the general flow pattern. **Do not solve the system.** **[0.5]**