
Name

Student number

Quiz 1 (Unit 1 and 2)

October 3, 2011

Instructions

- You have 90 minutes to answer the quiz.
- Marks per question are given in bold. **Total marks: 10 points**
- The mark of this quiz is part of the 40% corresponding with the continuous evaluation of this course.
- Write your name in all the sheets.
- It is not allowed to use lecture notes, scientific calculators or cellphones during the exam.

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1. Discuss the following systems of linear equations. Whenever is possible solve them by means of the **reduced echelon form**. (Do not forget to specify the row operations, to identify basic and free variables.)

$$(a) \begin{cases} 6x_1 + 4x_2 + 16x_3 = 0 \\ 2x_1 + 4x_2 + 8x_3 = 0 \\ -2x_1 - 2x_2 - 6x_3 = 0 \end{cases} \quad [1]$$

$$(b) \begin{cases} x_1 + x_3 = 3 \\ 2x_1 + x_2 - x_3 + x_4 = 5 \\ 2x_1 - x_3 + x_4 = -1 \\ -x_2 - x_3 = -1 \end{cases} \quad [1]$$

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2. Let $\begin{pmatrix} 1 & 3 & k \\ 4 & h & 8 \end{pmatrix}$ be the augmented matrix of a system. Determine h and k such that the solution set of this system [2]

- (a) is empty;
- (b) has a unique solution;
- (c) contains infinitely many solutions.

3. Compute the following determinants developing by a row or a column:

(a) $\begin{vmatrix} 1 & 2 & -1 \\ 4 & 0 & 3 \\ 6 & 4 & 1 \end{vmatrix}$. [0.5]

(b) $\begin{vmatrix} 9 & 1 & 9 & 9 & 9 \\ 9 & 0 & 9 & 9 & 2 \\ 4 & 0 & 0 & 5 & 0 \\ 9 & 0 & 3 & 9 & 0 \\ 6 & 0 & 0 & 7 & 0 \end{vmatrix}$. [0.5]

4. Determine if the following matrices are invertible or not. If they are invertible, compute the inverse of the matrix.

(a) $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 5 \end{pmatrix}$ (only use row operations to solve this exercise). [1]

(b) $B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ (do not use row operations to solve this exercise). [1]

5. Determine all the matrices that commute, with respect to the product, with $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. [1.5]

6. Answer the following questions and justify your answers.

(a) Describe the solution set of a system with 2 equations and 3 unknowns whose coefficient matrix has two pivots. [0.25]

(b) Give an example of an inconsistent overdetermined system of linear equations. [0.25]

- (c) Show that the partitioned matrix $A = \left(\begin{array}{c|c} I & 0 \\ \hline 0 & -I \end{array} \right)$ satisfies that $A^2 = I$. This kind of matrices are called involutive. **[0.25]**
- (d) If the determinant of $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is equal to 2, find the determinant of $\begin{pmatrix} a & 5b \\ c & 5d \end{pmatrix}$. **[0.25]**
- (e) Let C and D be square $n \times n$ matrices such that $\det C = 3$ and $\det D = 2$. Compute the determinant of $C^T D^2 C^{-1}$. **[0.25]**
- (f) Let U be a square $n \times n$ matrix such that $U^T U = I_n$. Show that $\det U = \pm 1$. **[0.25]**