Bachelor in Statistics and Business
Universidad Carlos III de Madrid

Mathematical Methods II
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Name ______________________ Student number ______________________

Quiz 2 (Units 3 and 4)

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Instructions

- You have 90 minutes to answer the quiz.
- Marks per question are given in bold. **Total marks: 10 points**
- The mark of this quiz is part of the 40% corresponding with the continuous evaluation of this course.
- Write your name in all the sheets.
- It is not allowed to use lecture notes, scientific calculators or cellphones during the exam.

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1. Let $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 3 \end{pmatrix}$, $v_3 = \begin{pmatrix} 2 \\ 2 \\ -1 \\ 5 \end{pmatrix}$, $v_4 = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}$ be vectors in $\mathbb{R}^4$.

   (a) Are the vectors v_1, v_2, v_3 and v_4 linearly independent? Why? **[1]**

   (b) Find a basis for the subspace spanned by v_1, v_2, v_3 and v_4. **[0.5]**

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2. Let $A = \begin{pmatrix} -2 & 4 & 6 \\ -1 & 2 & 3 \\ 2 & -4 & -6 \end{pmatrix}$ and $w = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

   (a) Find a basis for the subspaces $\operatorname{Col} A$ and $\operatorname{Nul} A$. **[1]**

   (b) Is w in $\operatorname{Nul} A$? Is w in $\operatorname{Col} A$? **[0.5]**

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3. Let $\mathcal{B}_1 = \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$ and $\mathcal{B}_2 = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ be bases for $\mathbb{R}^2$.

   (a) Find the change-of-basis matrix from $\mathcal{B}_1$ to the standard basis $\mathcal{B}_c$ for $\mathbb{R}^2$ and from $\mathcal{B}_2$ to the standard basis $\mathcal{B}_c$ for $\mathbb{R}^2$. **[0.5]**

   (b) Find the change-of-basis matrix from $\mathcal{B}_1$ to $\mathcal{B}_2$. **[1]**

   (c) Find the coordinate vectors $[x]_{\mathcal{B}_2}$ of $[x]_{\mathcal{B}_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ in the basis $\mathcal{B}_2$. **[0.5]**

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4. Let $A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$.

   (a) Find the characteristic polynomial of A and the eigenvalues of A. Give the algebraic multiplicity of each eigenvalue. **[0.75]**

   (b) Having in mind that the characteristic polynomial of A is $\lambda(\lambda-2)^3$, can the matrix A be diagonalized? Why? Give the geometric multiplicity of each eigenvalue. If A can be diagonalized, give the matrices P and D. **[1.75]**

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5. Answer the following questions and **always** reason your answers.

   (a) Is the set $W = \{(x, y, z) : \ 2x + y - z + 1 = 0\}$ a subspace of $\mathbb{R}^3$? Why? **[0.5]**

   (b) Find the matrix associated with the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x, -y + z)$. **[0.5]**

   (c) If the rank of a 3×7 matrix A is 2, what is the dimension of the null space of A? Why? **[0.5]**

   (d) Let A be a 5×5 matrix with two different eigenvalues. If one eigenspace has dimension 3 and the other one has dimension 2, is A diagonalizable? Why? **[0.5]**

   (e) Is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ an eigenvector of $\begin{pmatrix} 1 & -1 \\ 6 & -4 \end{pmatrix}$? If so, find the eigenvalue. **[0.5]**