MACHINE THEORY
Bachelor in Mechanical Engineering

INTRODUCTION TO GEAR DESIGN

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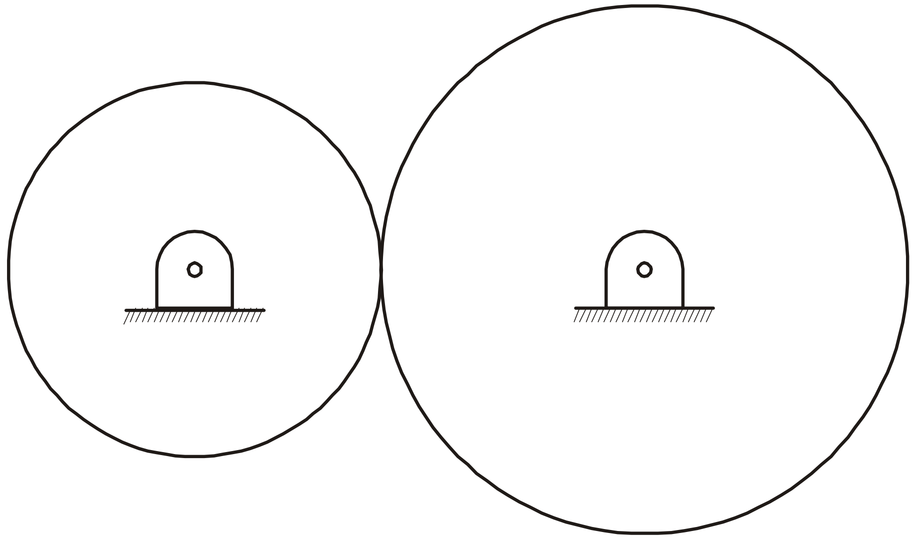
Objectives

- Understand why teeth profiles are needed.
- Understand the involute curve, its properties and how it is created.
- Learn the basic terminology of gears.
- Understand the equivalent kinematic diagram of gears.
- Identify different types of gears.

Why do we need gears?

Rolling circles, what if friction is not enough?



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Possible solutions

But, what's wrong with these designs?

They cannot transmit constant speed

Conjugated action

- When two profiles are designed to produce a constant relation of angular velocity.

How to find the profile?

- Unfortunately it is not a trivial task.
- But, fortunately only two profiles are widely used:
  - The involute
  - And the cycloid

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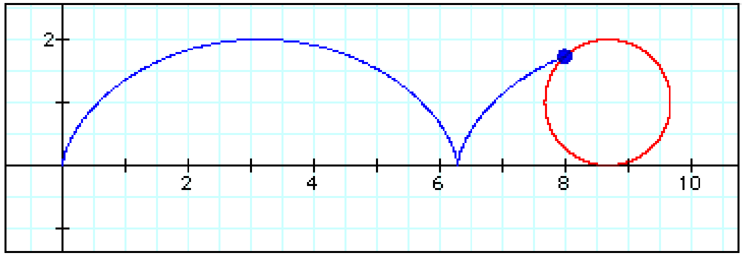
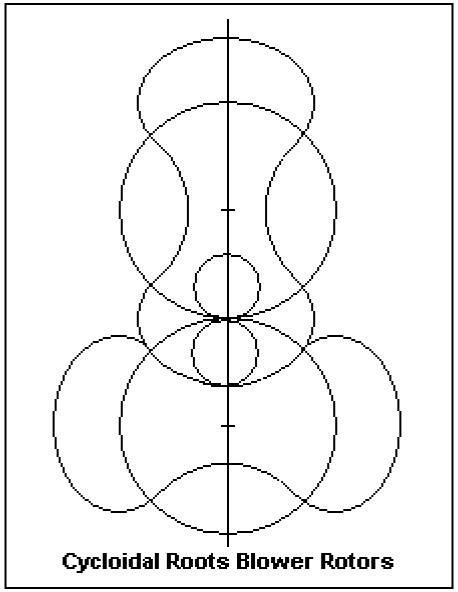
The cycloid





Check the second video of the references.

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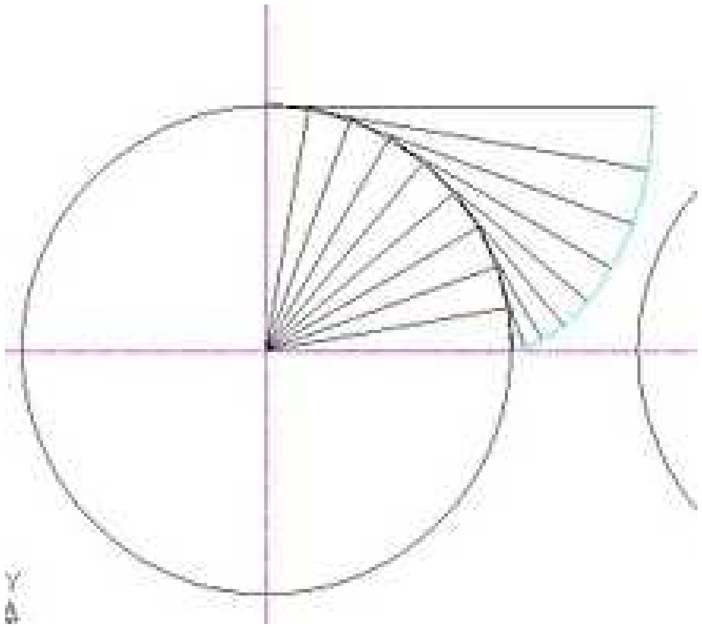
The involute



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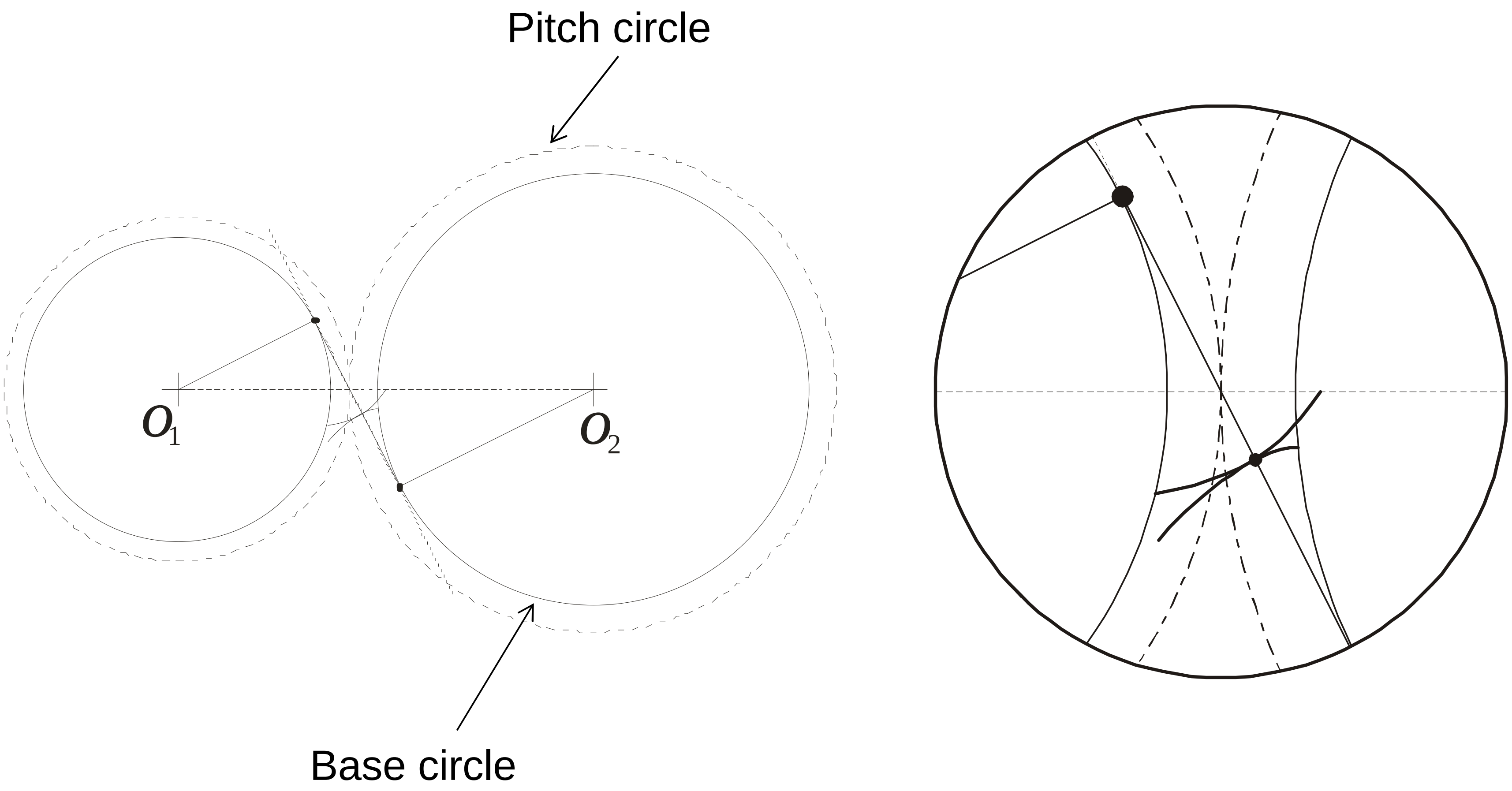
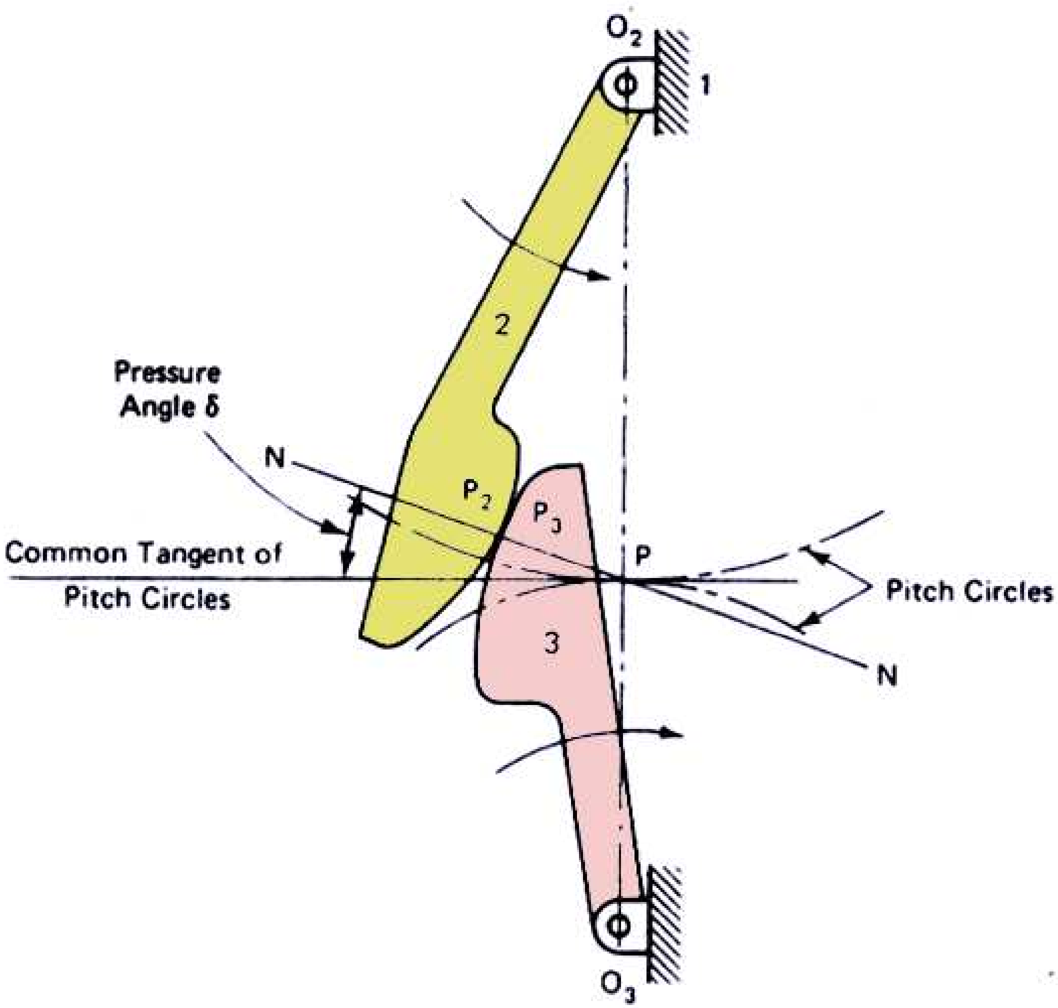
Kinematic equivalence

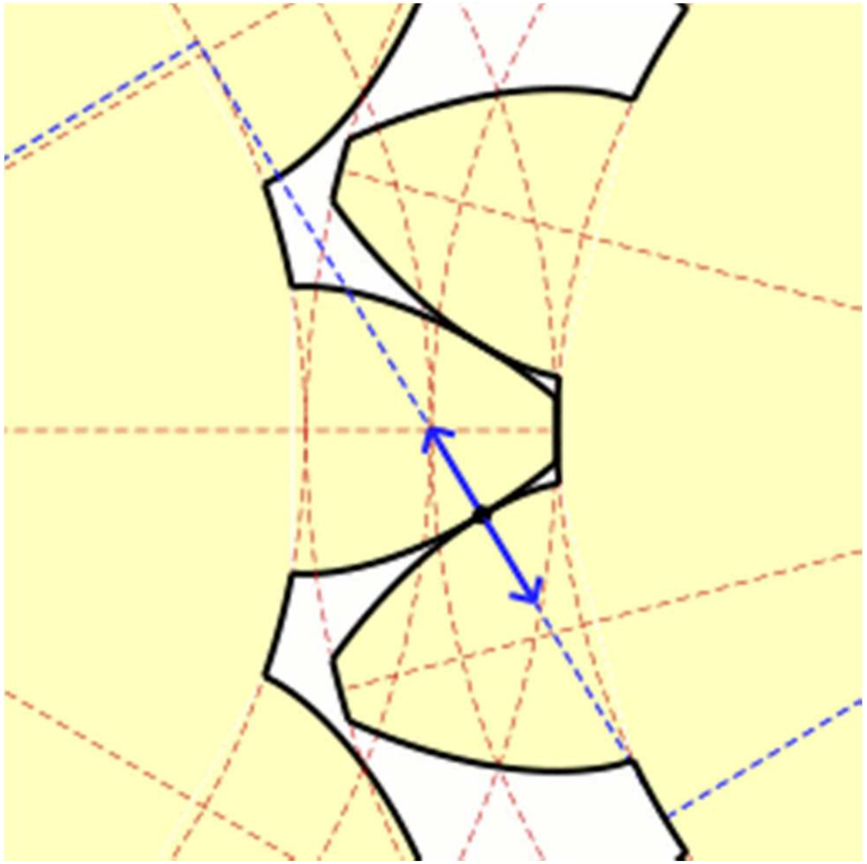


What is the pressure angle?



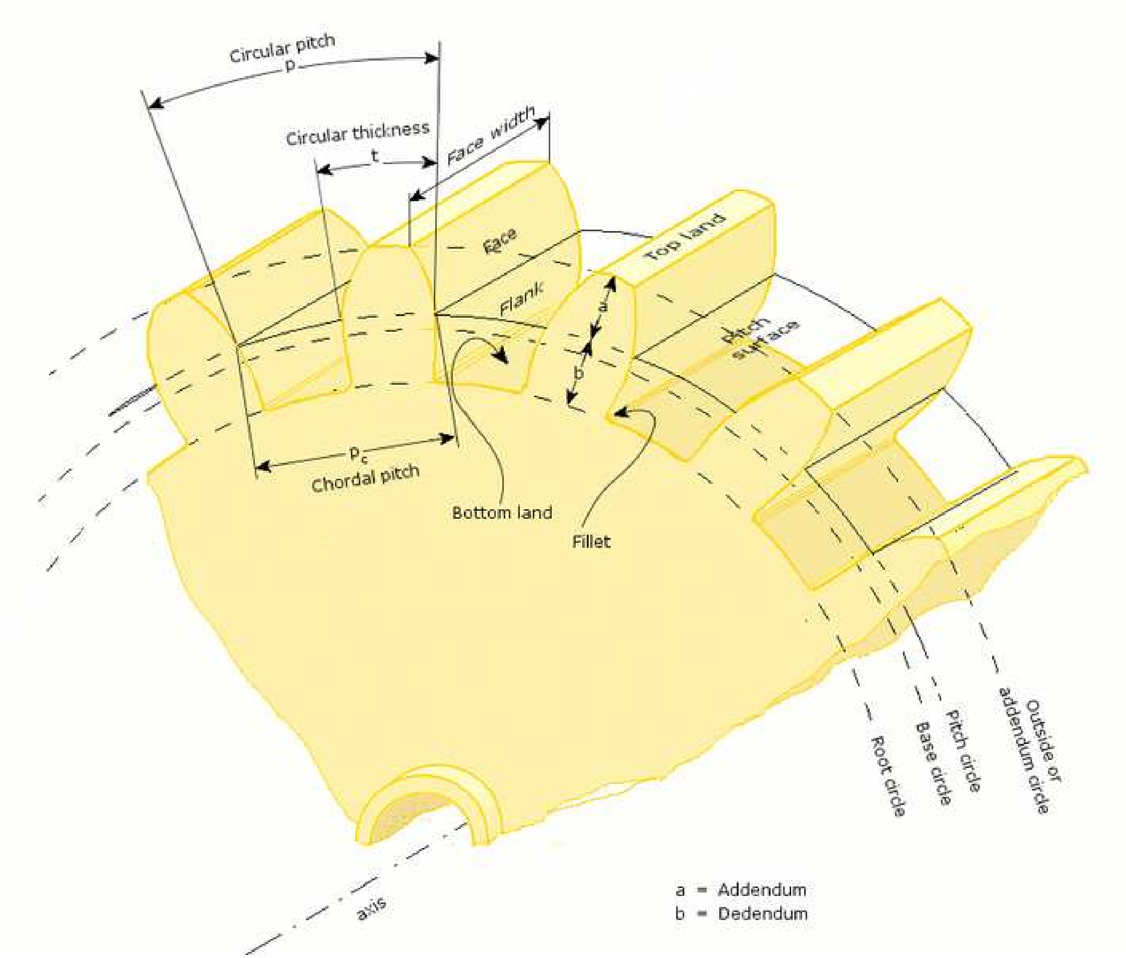
It is an standard value:
20º, 25º and in the past 14.5º

The pressure angle

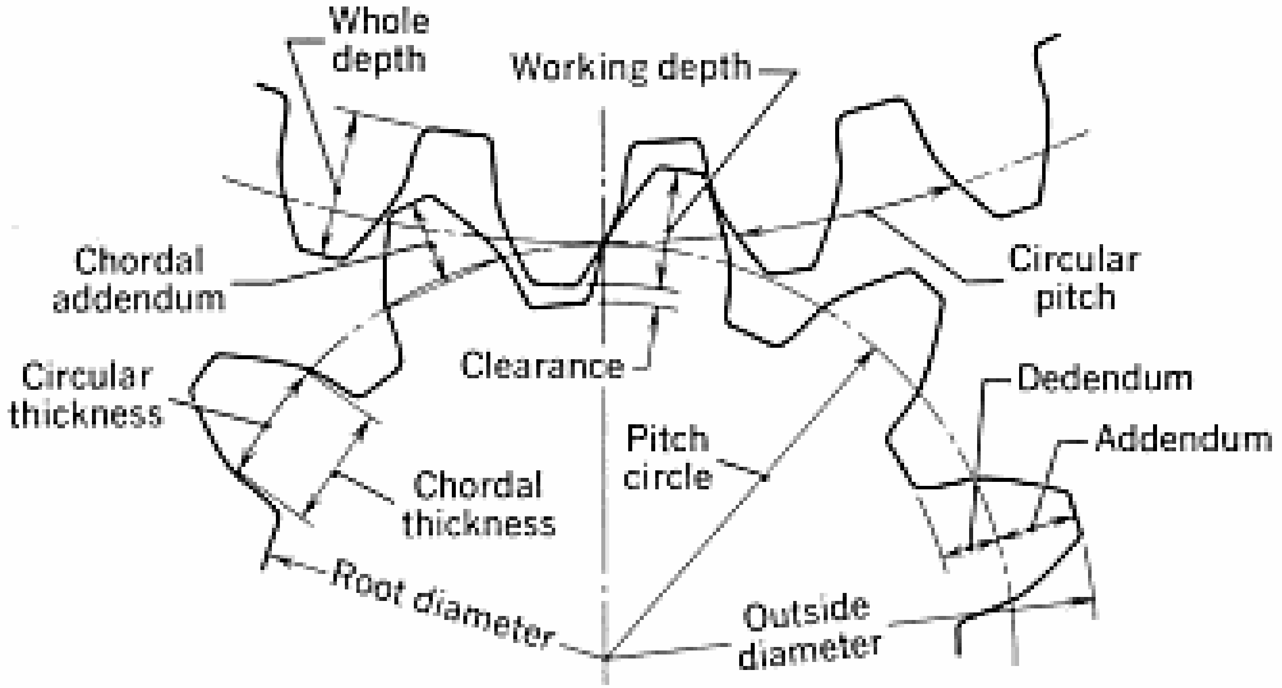


Gear tooth nonmeclature

- Pitch circles
- Circular pitch
- Addendum
- Clearance
- Working depth
- Whole depth
- Tooth
- Thickness
- Backlash!!



Backlash



Spur gear terms.

Basic relationships

Diametral Pitch

$$P = \frac{N}{D}$$

N= Number of teeth
D= Pitch circle diameter

Circula Pitch

$$CP = \frac{\pi D}{N}$$

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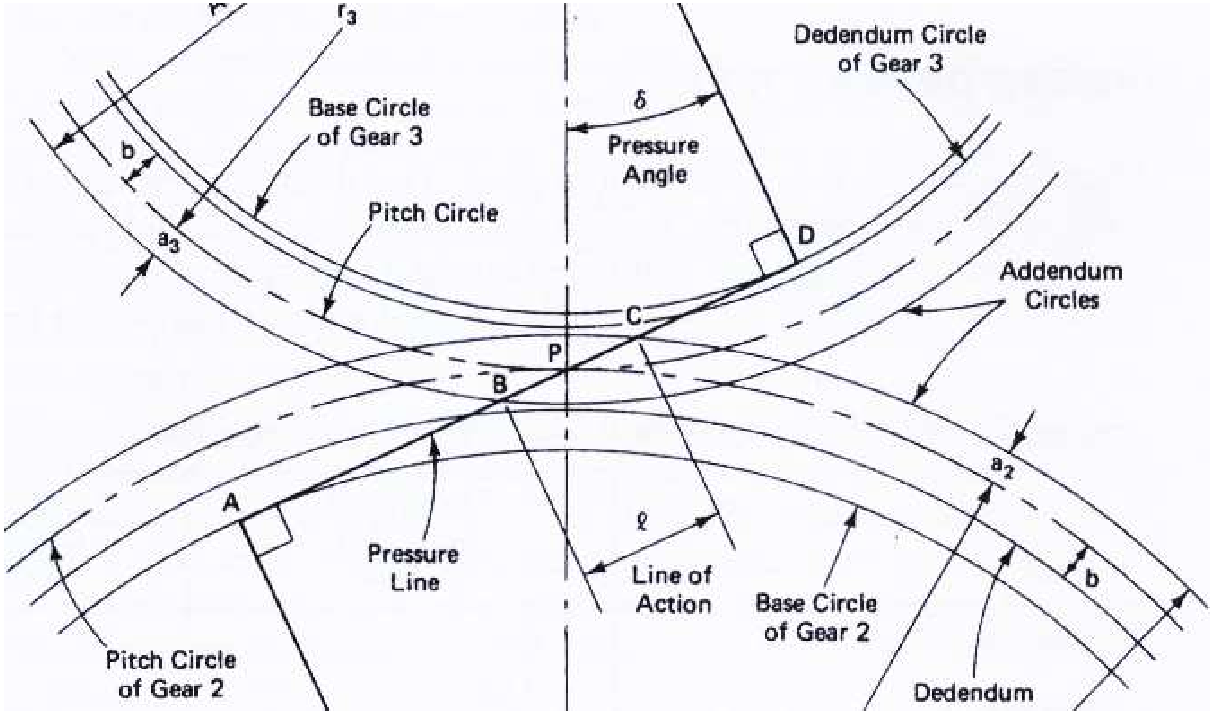
Basic relationships

Base pitch:

$$BP = CP\frac{D_b}{D}$$

D = Pitch circle diameter

D_b = Base circle diameter



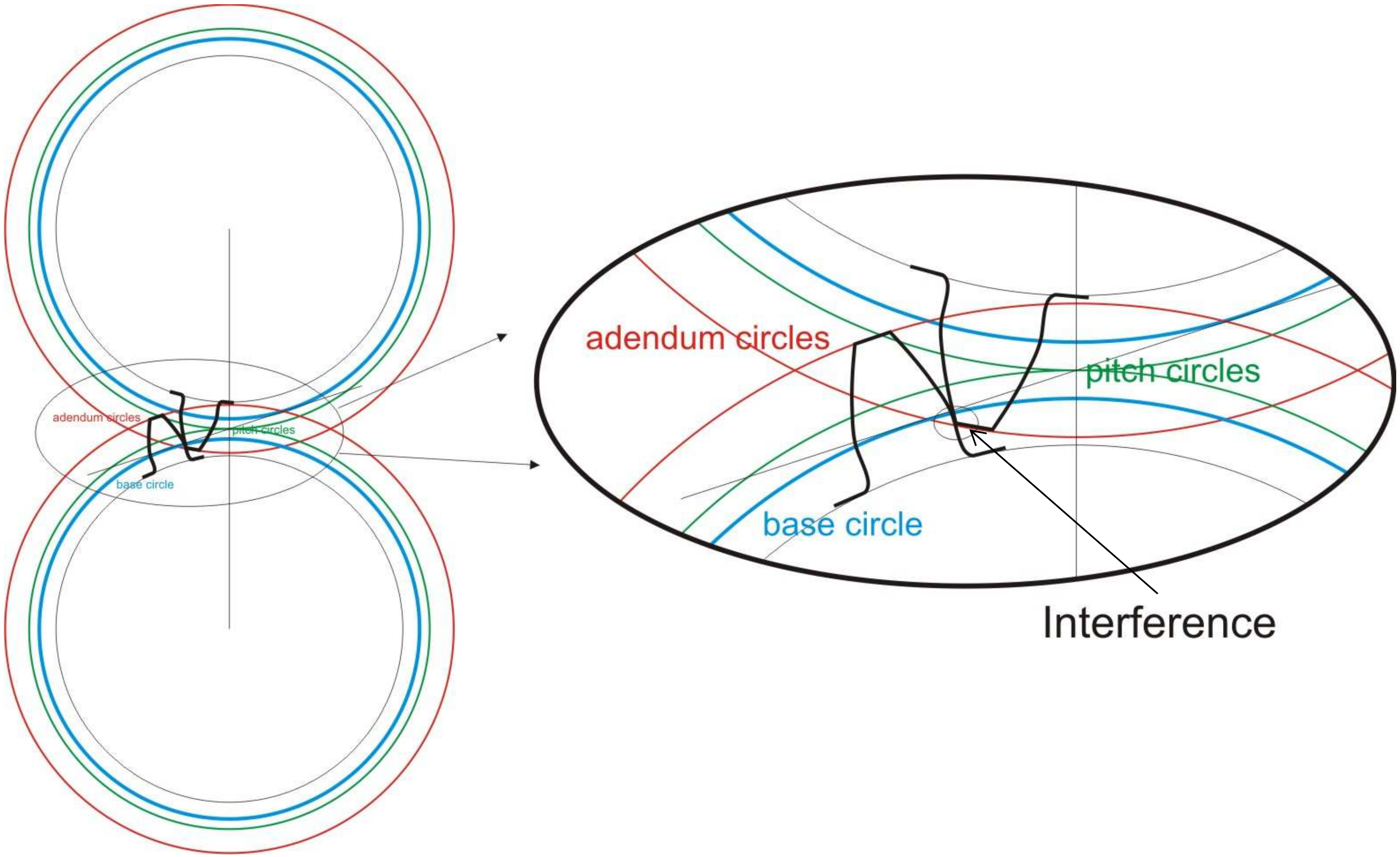
Line of action:

$$l = \left|\overline{AC} - \overline{AP}\right| + \left|\overline{DB} - \overline{DP}\right|$$

$$l = \left|\sqrt{(r_2 + a_2)^2 - (r_2\cos\delta)^2}\right| - r_2\sin\delta + \left|\sqrt{(r_3 + a_3)^2 - (r_3\cos\delta)^2}\right|$$

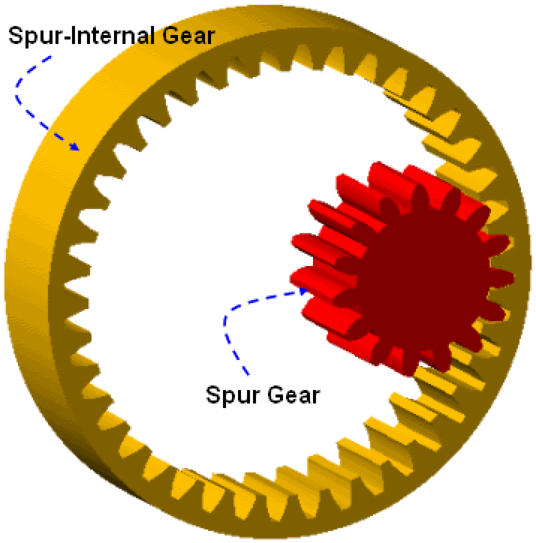
Contact ratio: $$CR = \frac{l}{BP}$$

Interference



Spur gears

- The smallest gear it is usually called pinion



Internal ring gear

Spur gears

Bevel gears

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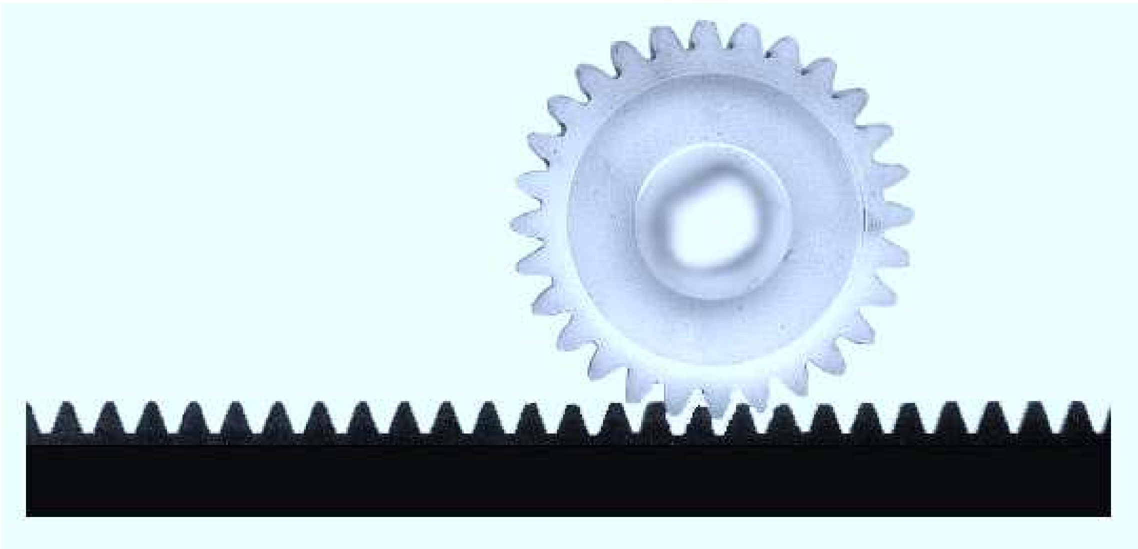
Spur gears

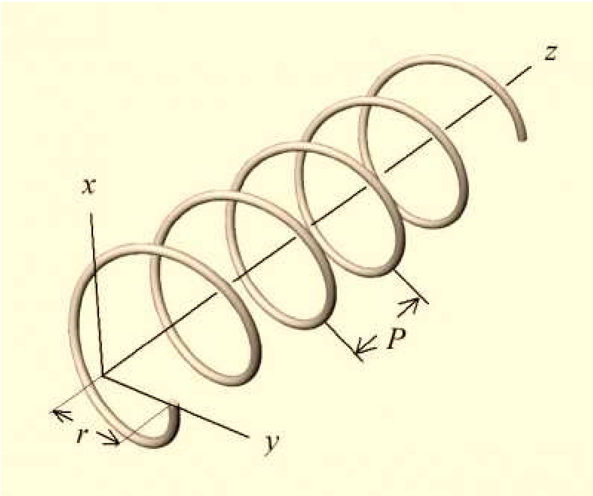
- What if the diameter of the base circle approaches to infinite?



We get a rack and pinion pair. It does convert rotational motion into linear motion.

Helical gears

- They provide a smoother movement than spur gears.

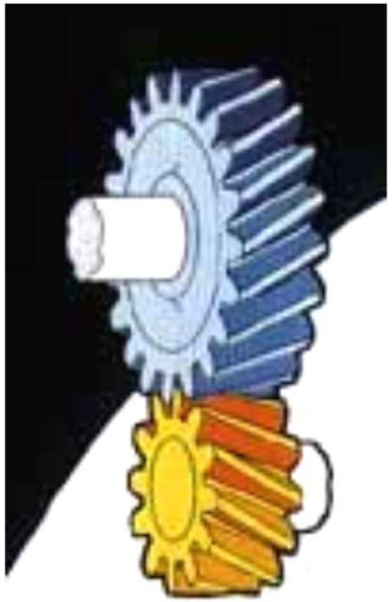


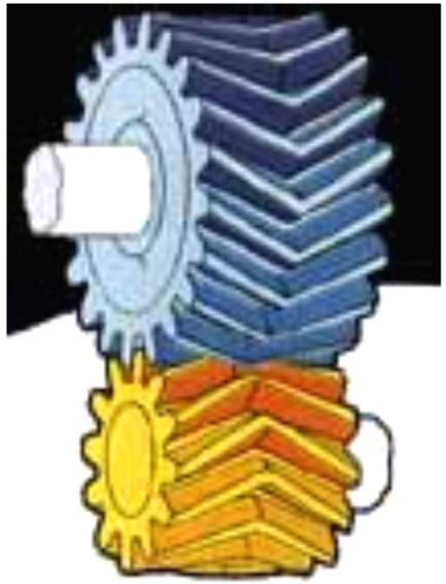
$$x = r\cos(t), y = r\sin(t), z = pt$$

r is the diameter of the
helix and p is the pitch

Helical gears

- Their disadvantage (besides that they are more complex and therefore more expensive) is that they produce forces along the axis. Although there is a solution: Herringbone gears.

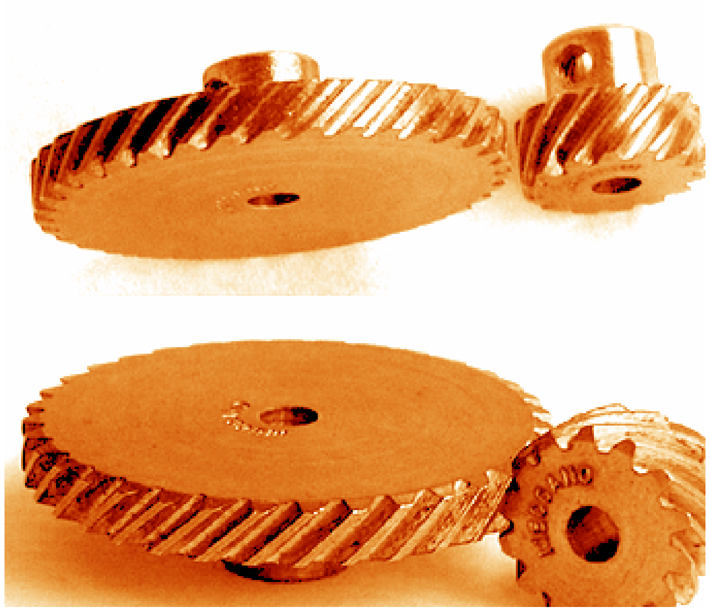


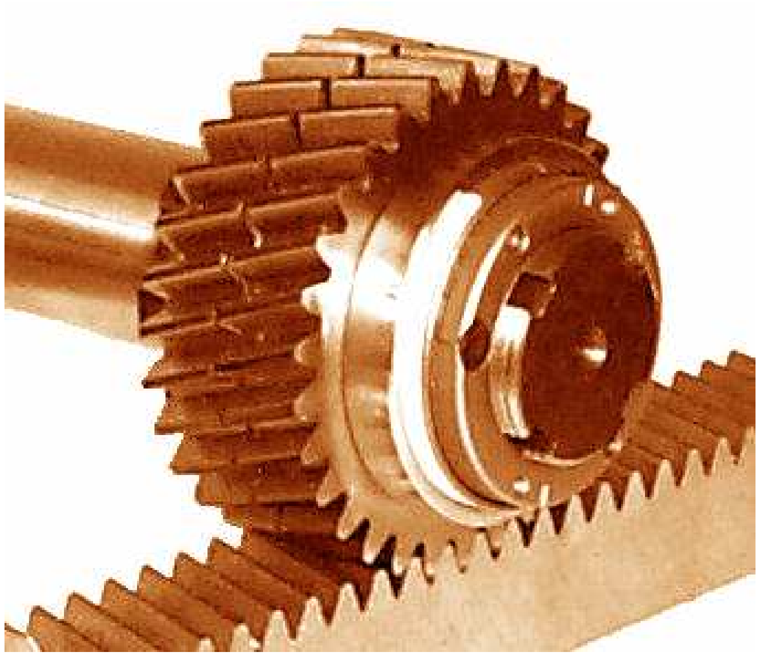


Herringbone gear

Helical gears

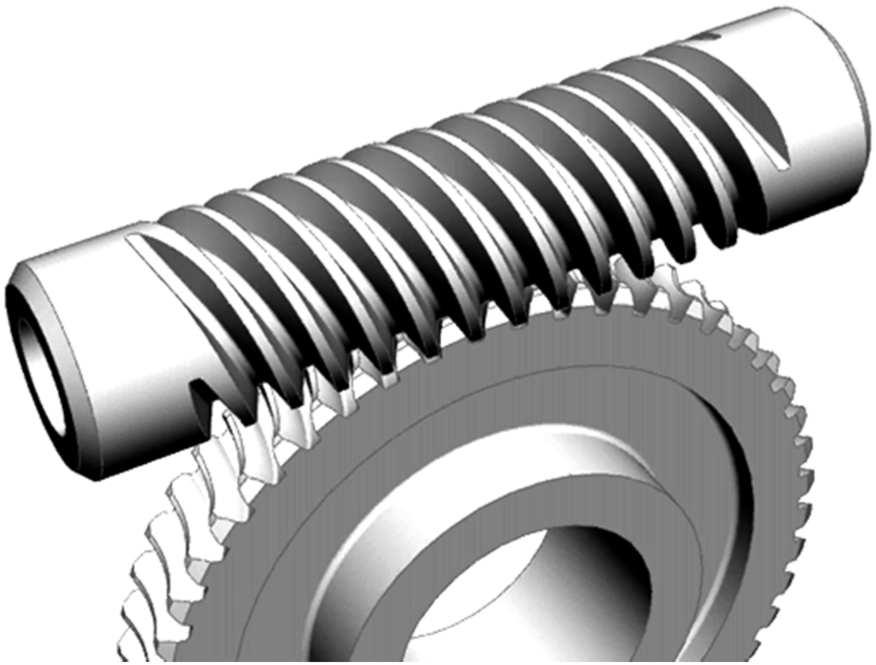
- They can also transmit movement between axis that do not intersect. And there also is the rack and pinion version.





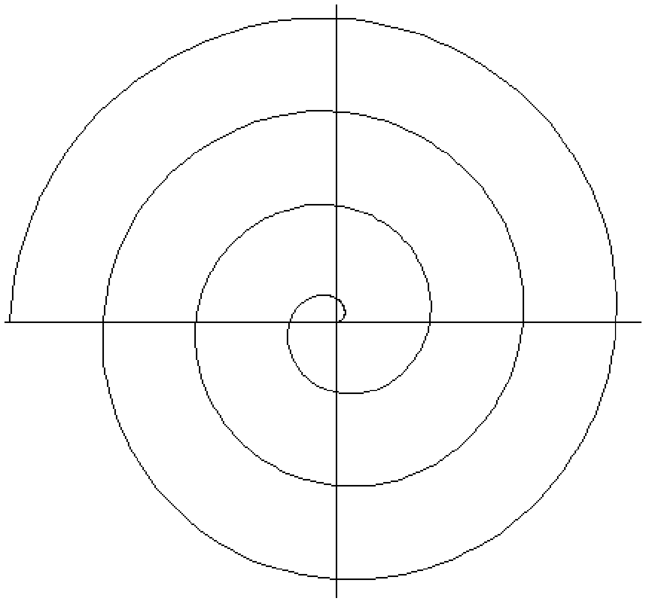
Worm gears

- They are a special case of helical gears, with the pion helix close o 90º, and the gear close to 0º



Spiral gears

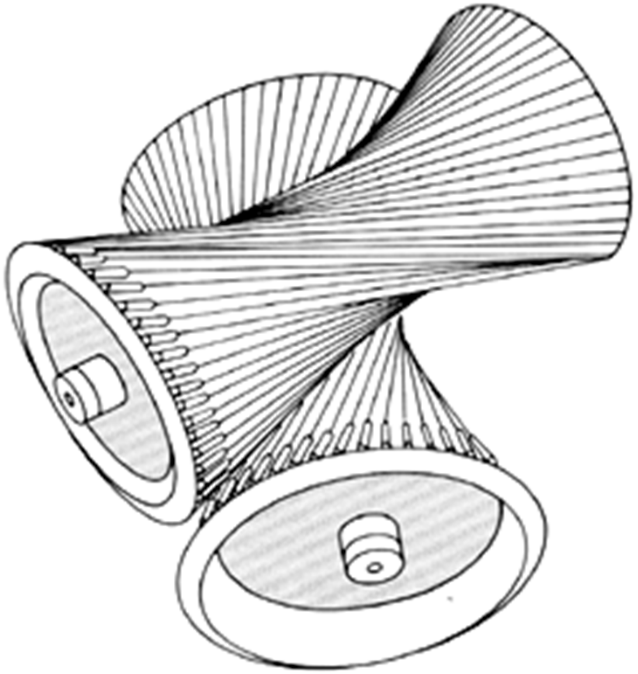
- An improvement of the bevel gears, they use the spiral curve to generate the tooth.



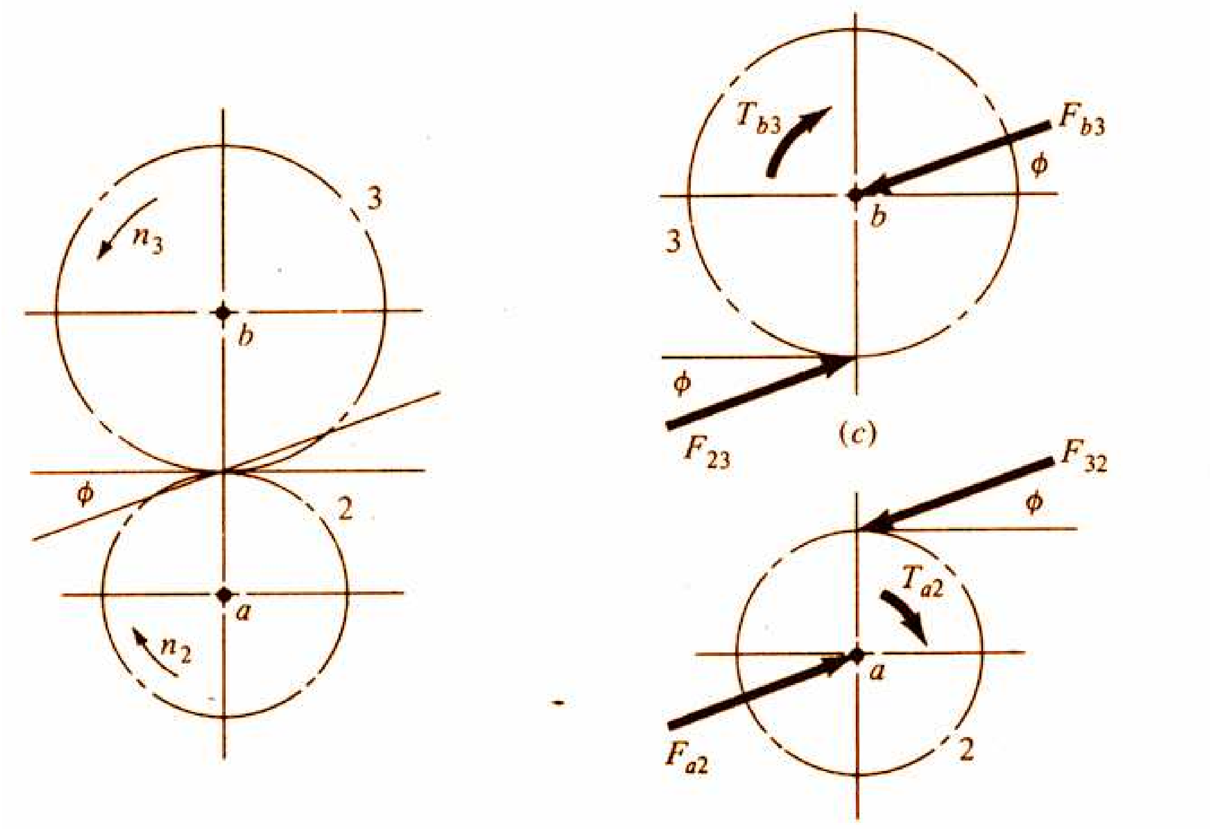
$$r = a + b\theta$$

Spiral Gears

- An special case are the hypoid gears, they allow higher degrees relations of transmission and smoother motion.



Force analysis of spur gears



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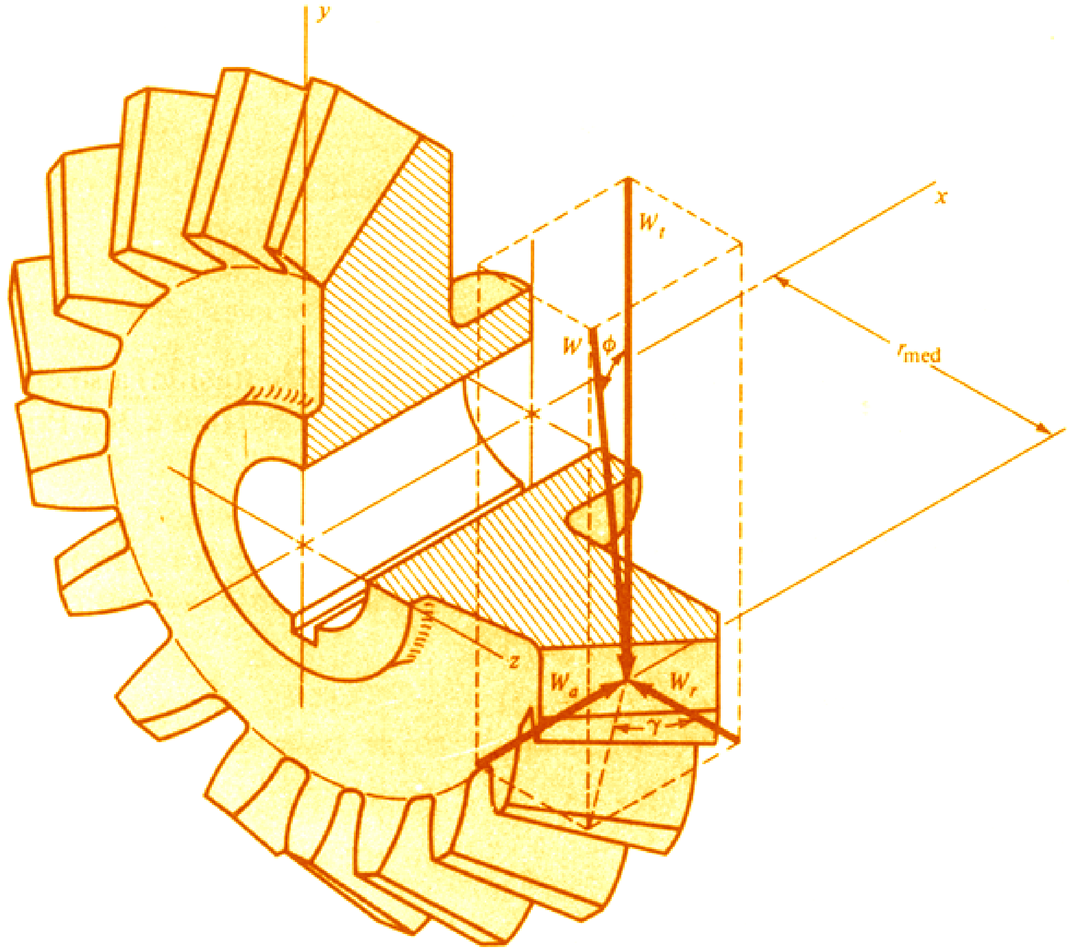
Force analysis of bevel gears

Assumption: All the forces are concentrated on the middle of the teeth



$$W_t = \frac{T}{r_{\text{med}}}$$

$$W_r = W_t \tan\phi \cos\gamma$$

$$W_a = W_t \tan\phi \sin\gamma$$

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Forces of helical gears

$$W_t = W \sin\phi_n$$

$$W_r = W \cos\phi_n \cos\psi$$

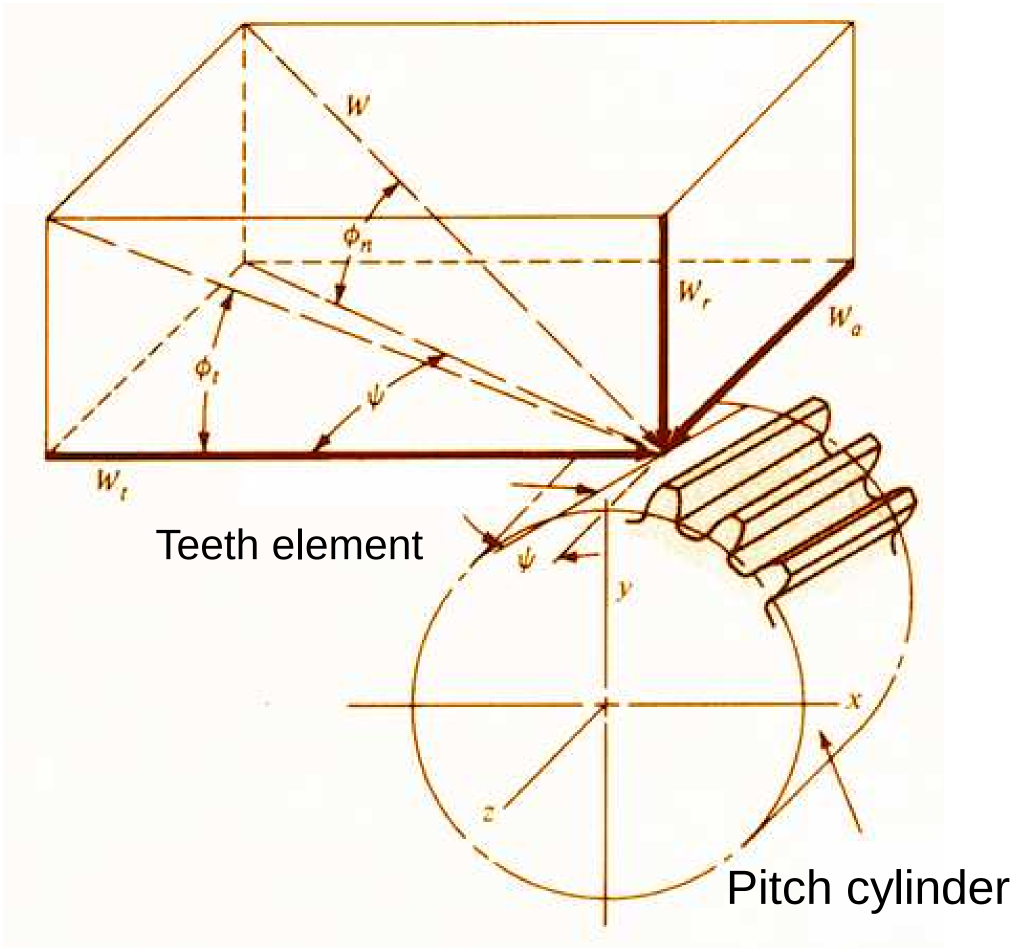
$$W_a = W \cos\phi_n \sin\psi$$

Usually W_t is given, therefore:

$$W = \frac{W_t}{\cos\phi_n \cos\psi}$$

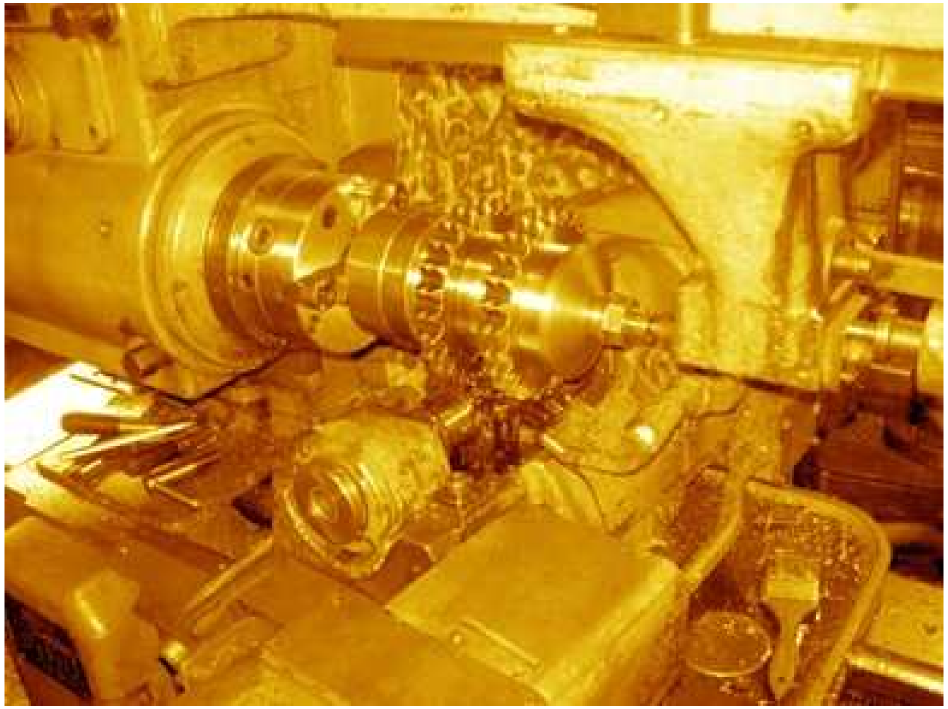
$$W_r = W_t \tan\phi_t$$

$$W_a = W_t \tan\psi$$



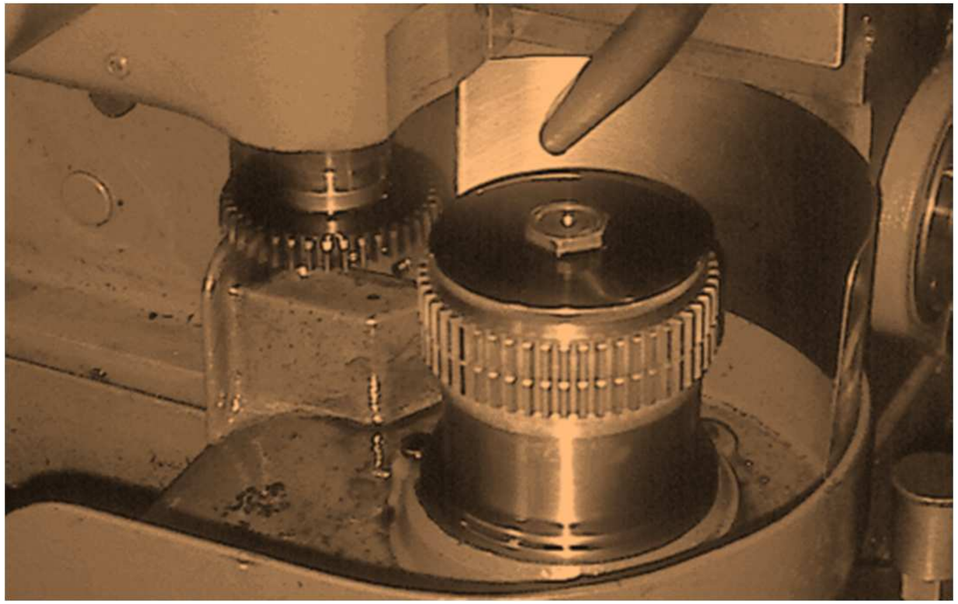
Gear manufacturing

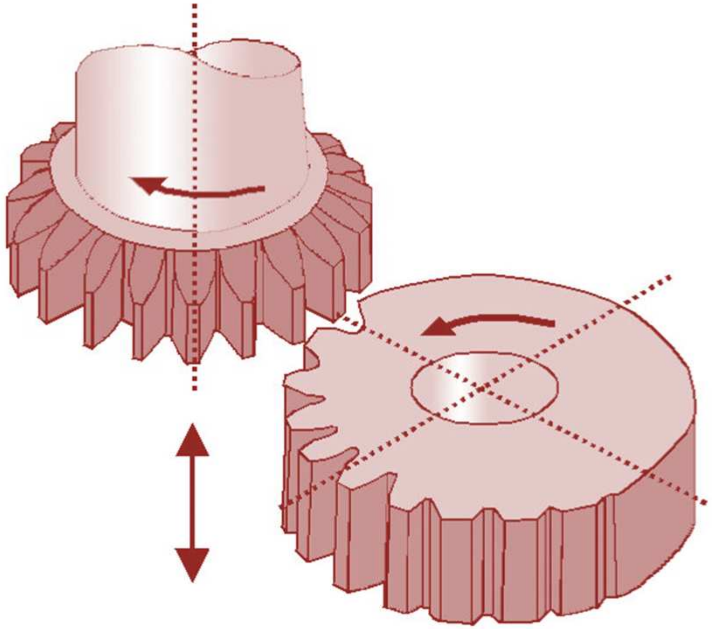
Universal milling



Gear manufacturing

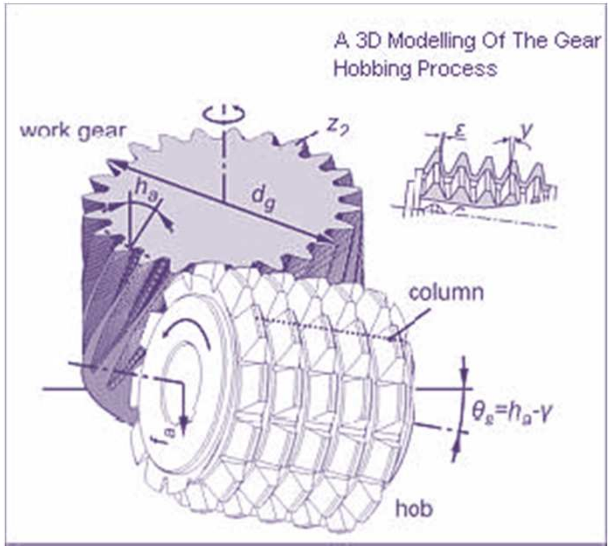
Gear shaping

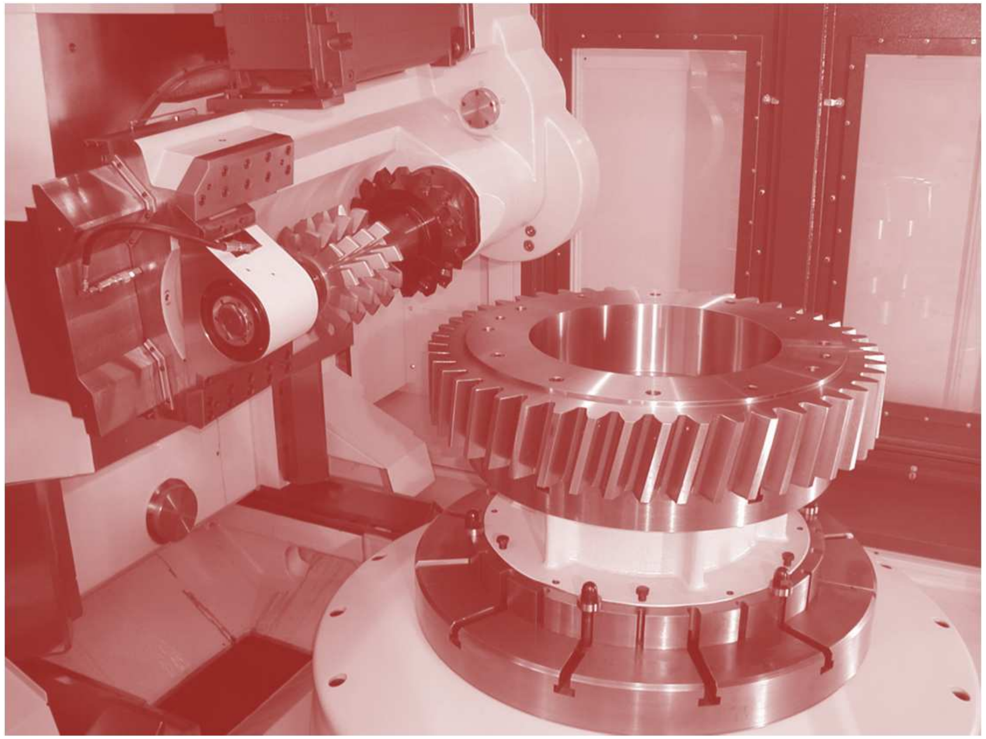




Gear manufacturing

- Hobbing, it cannot produce internall gears





Some videos

- http://www.youtube.com/watch?v=L1x1TRO36gU&feature=related
- http://www.youtube.com/watch?v=xF9CjluRFJ4ç
- http://www.youtube.com/watch?v=o6bPTTyHSBU&feature=related

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References

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- http://www.youtube.com/watch?v=Z1f29M4o3jI&feature=related