

# **MACHINE THEORY**

## **Bachelor in Mechanical Engineering**

### **CAMS DESIGN II**

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# 1. Introduction

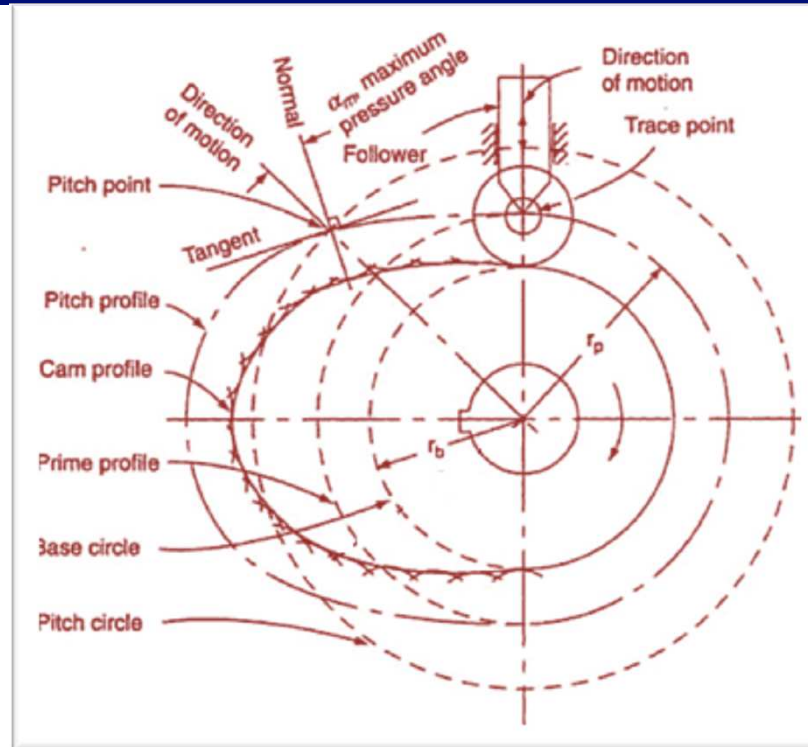
**There are different ways to obtain the cam profile:**

- ◉ **Graphical cam profile synthesis: We will focus on this one.**
- ◉ **Analytical cam profile synthesis.**

**There are two major factors which affect cam size:**

- 1. The pressure angle  $\Phi$ .**
- 2. The radius of curvature.**

## 2. Nomenclature and definitions



**Cam profile:** the actual surface contour of the cam.

**Base circle ( $r_b$ ):** the smallest circle that can be drawn tangent to the cam surface concentric with the cam shaft. All radial cams will have base circle.

**Tracer point:** point at the center of the follower that generates the pitch curve.



## 2. Nomenclature and definitions

**Pressure angle  $\Phi$**  : the angle between the normal to the pitch curve and the instantaneous direction of motion of the follower.

**Pitch point**: a point on the pitch curve where the pressure angle is maximum.

**Pitch curve**: the path generated by the trace point as the follower is rotated about a stationery cam (kinematic inversion). There is no pitch curve for flat-faced followers.

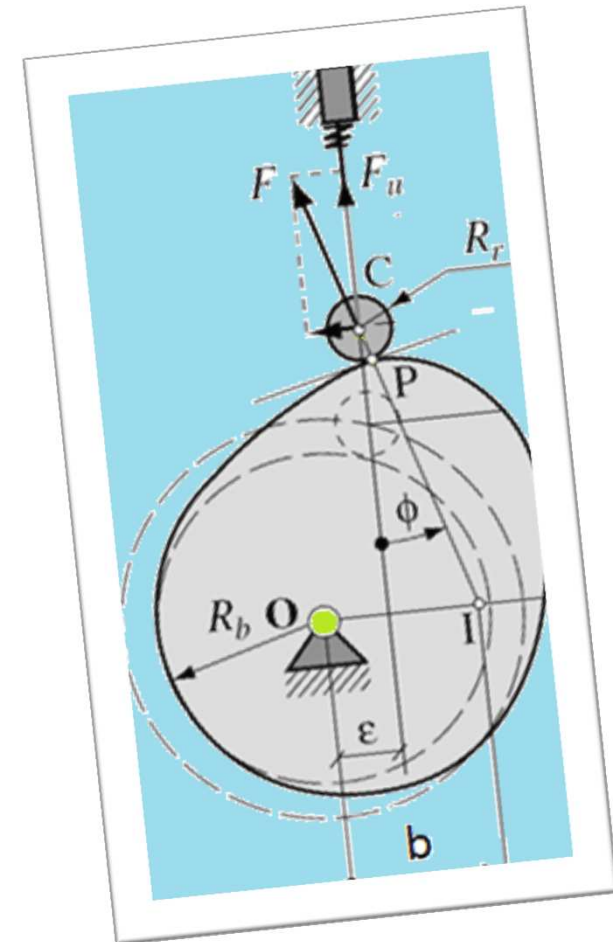
**Prime circle ( $r_p$ )**: the smallest circle drawn on the pitch curve from the cam center. Only for roller or curved followers (mushroom) .

## 3. Eccentricity

### 1. Eccentricity $\epsilon$

If the axis of motion of the follower does not intersect the ground linkage in the cam, there is an eccentricity  $\epsilon$  which is defined as:

***“Perpendicular distance between the follower’s axis of motion and the center of the cam”***

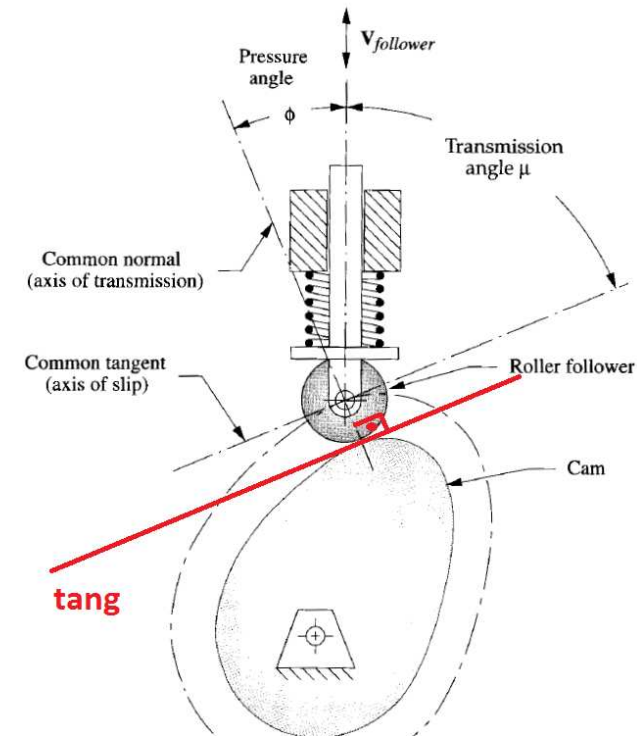


## 4. Pressure angle

### 1. The pressure angle $\Phi$

**Is the angle between the direction of motion of the follower and the direction of transmission (only valid for single D.O.F).**

The direction of transmission is normal to the cam contour.

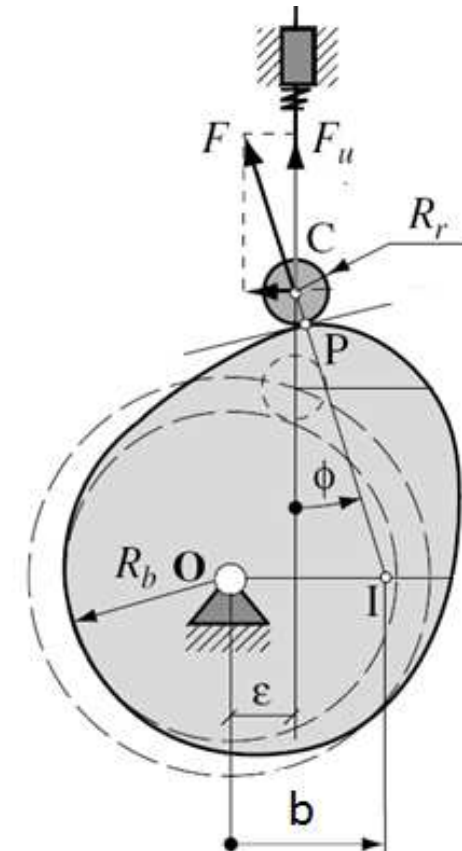


The lower the value of the pressure angle, the best the transmitted force will be transformed into motion of the follower. If the pressure angle is too high, the follower sliding or friction will be increased.

## 4. Pressure angle

### 1. The pressure angle $\Phi$

- Pressure angle  $\Phi$  determines the relation between the cam-follower contact force  $F$  and the useful force  $F_u$  (component of  $F$  in the direction of motion).
- In practice, the pressure angle should not exceed  **$30^\circ$**  for translating followers, oscillating followers can have an angle up to  **$35^\circ$** . If these values are exceeded, the cam may tend to jam a translating follower in its guides.

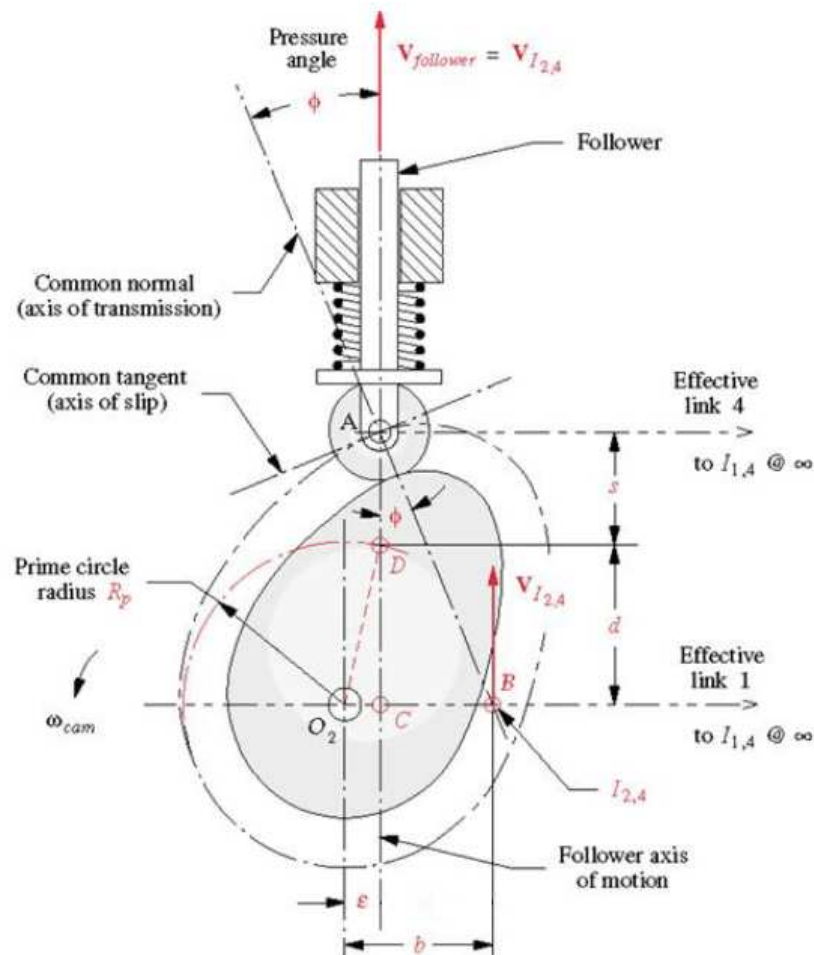




## 4. Pressure angle

The pressure angle  $\phi$  can be expressed:

$$\Phi = \tan^{-1} \left\{ \frac{b - \varepsilon}{s + d} \right\}$$

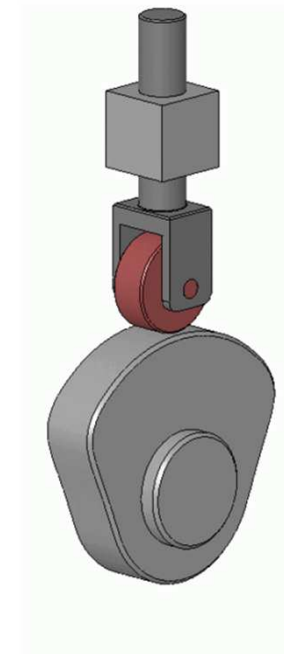
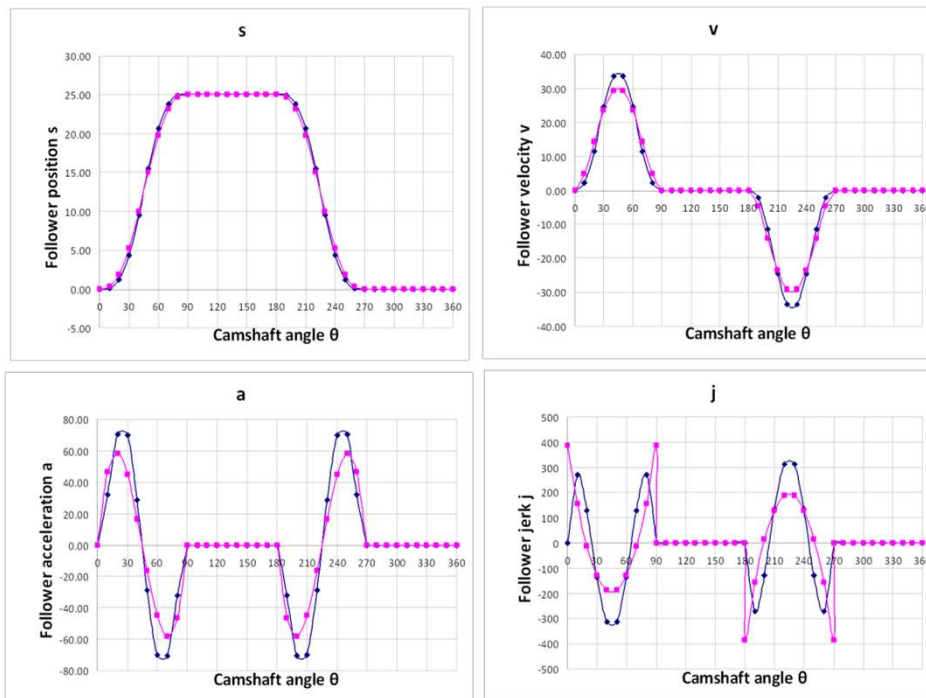


From geometry:  $d = \sqrt{R_p^2 - \varepsilon^2}$

$$\Phi = \tan^{-1} \left\{ \frac{b - \varepsilon}{s + \sqrt{R_p^2 - \varepsilon^2}} \right\}$$

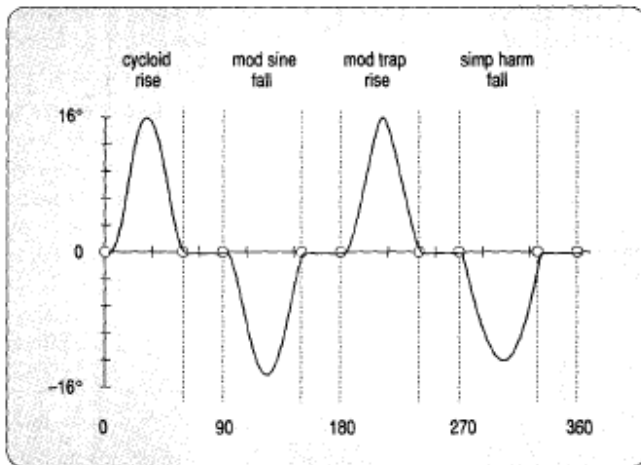
# 1. Introduction

Once the svaj diagrams have been defined to met the specifications of our application, the next step is to shape the cam. We need to know some definitions about the cam profile.



## 5. Prime Radius selection

Selecting a **base radius for flat followers** and **prime radius for roller or curved followers** has a strong influence on the pressure angle. The way to proceed is to choose a initial value of  $R_p$  and  $\varepsilon$  and then calculate  $\Phi$  (using informatics tool). Adjust  $R_p$  until an acceptable arrangement is found.

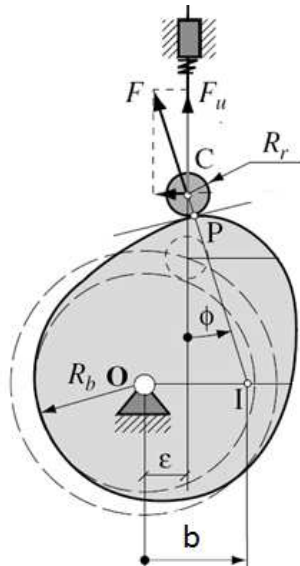


Lower values of  $R_p$  will decrease the cost of manufacturing the cam, but will increase pressure angle for the same roller diameter.

Footnote: Pressure angle function will be similar in shape to velocity diagram in your application.

## 6. Using eccentricity

If a suitably small cam cannot be obtained with acceptable pressure angle  $\Phi$ , then the eccentricity can be modified in order to reduce the pressure angle. Take into account that  $\epsilon$  can be positive or negative.



$$\Phi = \tan^{-1} \left\{ \frac{b - \epsilon}{s + \sqrt{R_p^2 - \epsilon^2}} \right\}$$

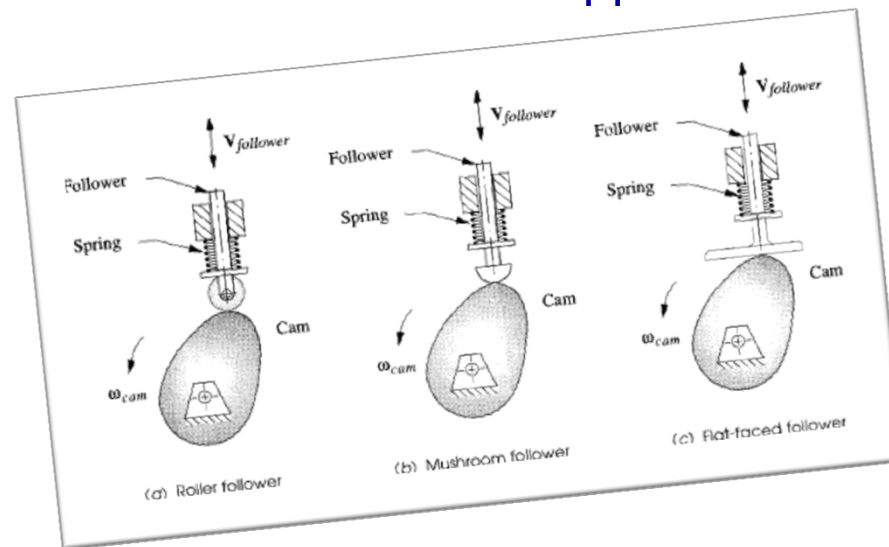
$$\phi = \tan^{-1} \left\{ \frac{v - \epsilon}{s + \sqrt{R_p^2 - \epsilon^2}} \right\}$$

Velocity in  
[length/rad]

If adjustments in  $R_p$  and  $\epsilon$  do not yield acceptable, the design must be revised and modified.

## 7. Graphical Cam Profile Synthesis

When the base/prime radius and the eccentricity are selected, the next step is to draw the cam profile. Obviously the method will depend on the type of follower we have selected for our application.

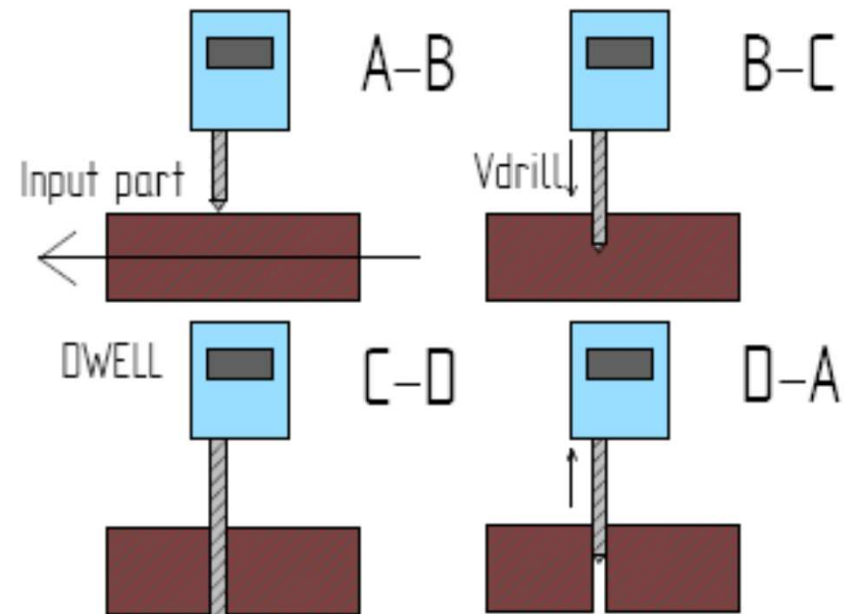
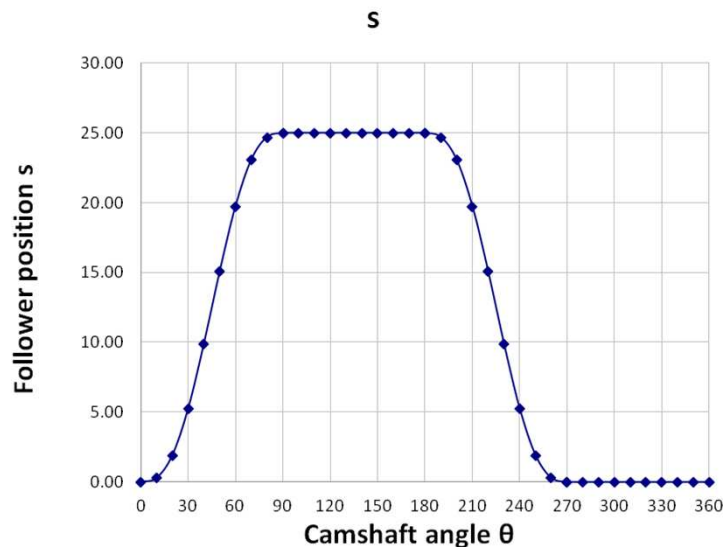


Nevertheless there are some steps which are applicable for all of them.

# 7. Graphical Cam Profile Synthesis

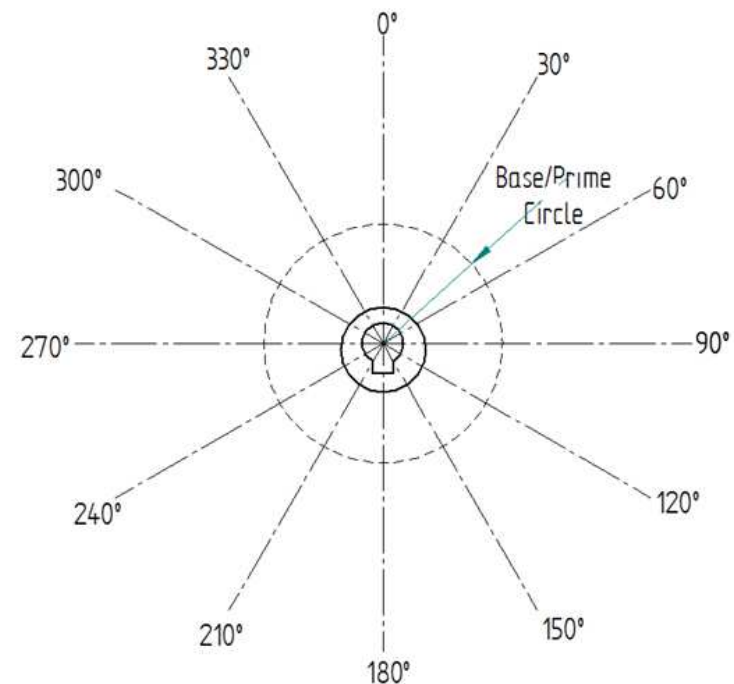
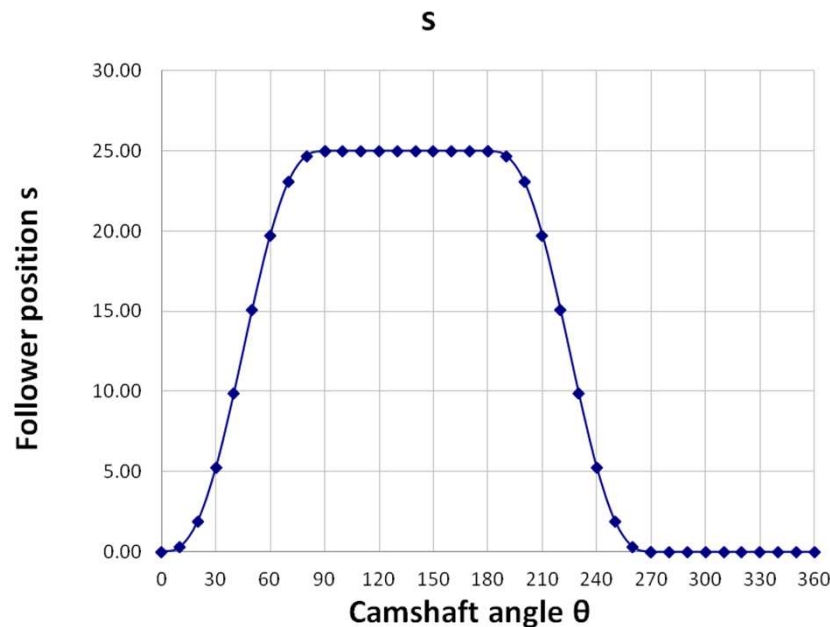
## Let me remind you the example of the previous class:

- E.g.:
- A cam mechanism will be designed for a driller machine.
- Operation conditions:
  - One part each 20s. Initial position: **A**
  - DRILL IN: 25 mm in 5 s. **B-C**
  - DWELL: 5 s finishing the drill. **C-D**
  - DRILL OUT: 25 mm in 5s. **D-A**
  - DWELL: Wait for another part. **A-B** 5s



# 7. Graphical Cam Profile Synthesis

1. Divide the displacement diagram in **constant camshaft angle sectors** and draw the **base/prime circle** you have previously selected. The higher number of sectors, the more accurate the profile of the cam will be. E.g.  $\theta=30^\circ$

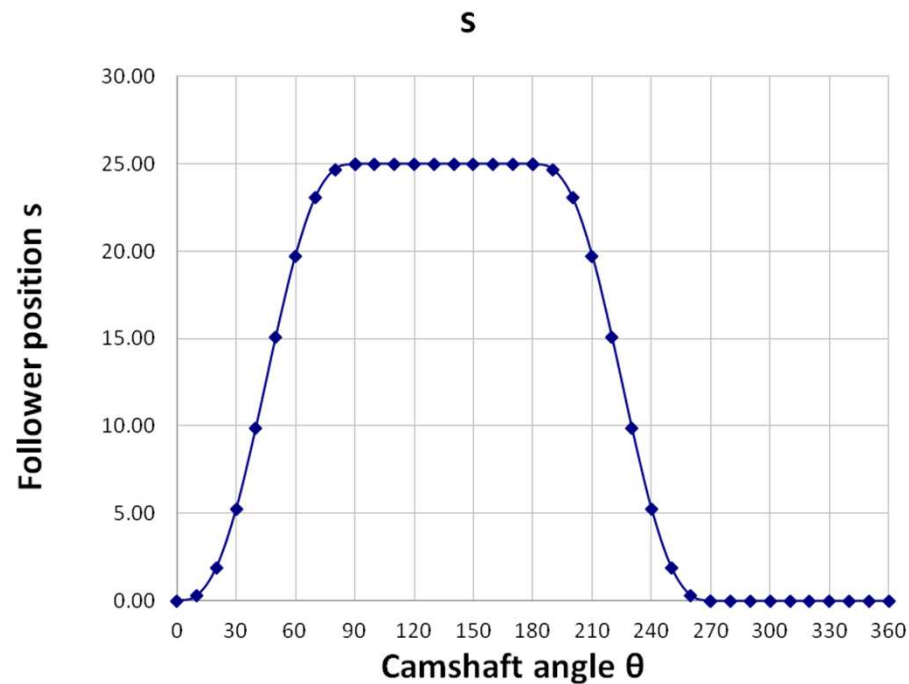






# 7. Graphical Cam Profile Synthesis

## 2. Obtain displacement points for the sectors:

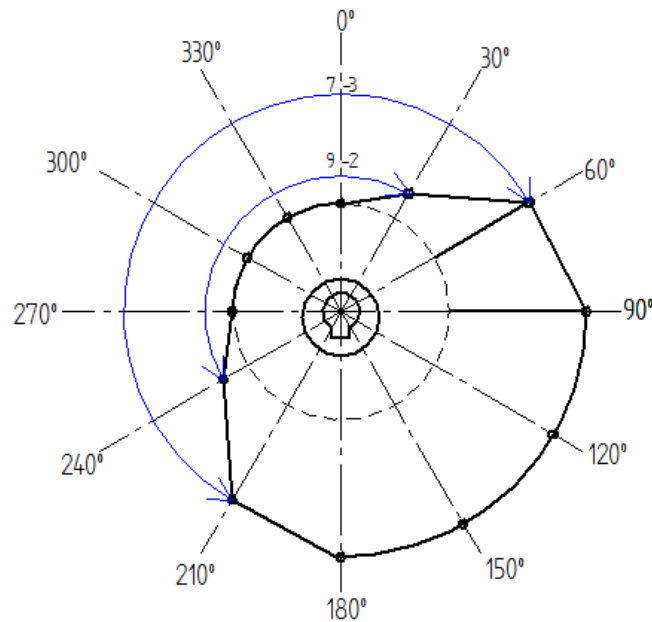


Position	$\theta$ [degrees]	S [mm]
0	0	0
1	30	5
2	60	20
3	90	25
4	120	25
5	150	25
6	180	25
7	210	20
8	240	5
9	270	0
10	300	0
11	330	0
12	360	0



# 7. Graphical Cam Profile Synthesis

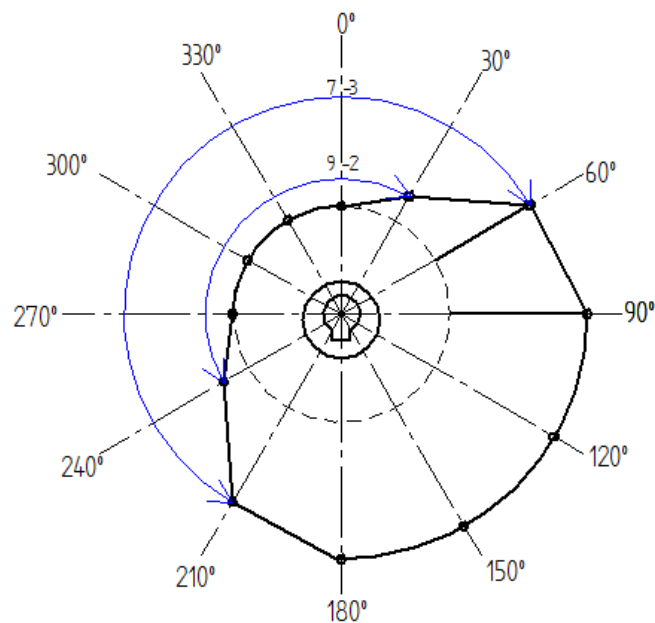
3. Locate this points and connect them. You also can increase resolution in rise and fall pieces rather than in dwells:



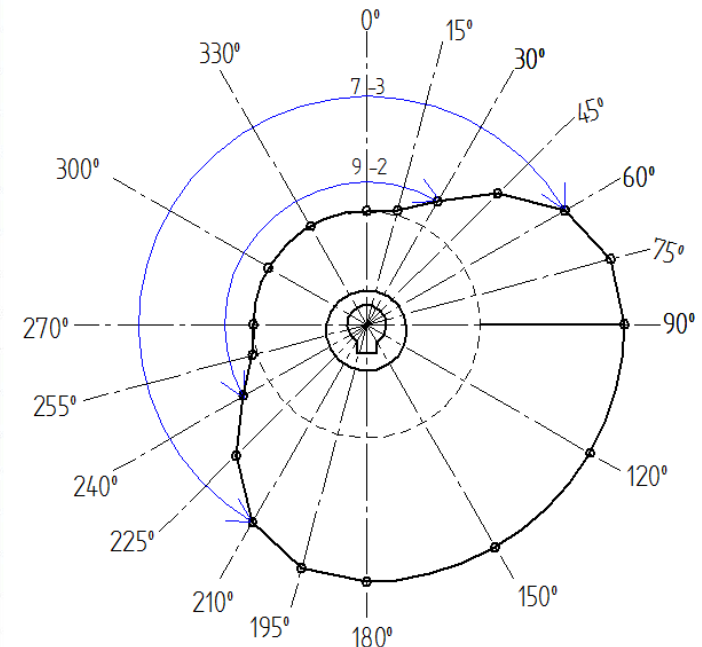
Position	$\theta$ [degrees]	S [mm]
0	0	0
1	30	5
2	60	20
3	90	25
4	120	25
5	150	25
6	180	25
7	210	20
8	240	5
9	270	0
10	300	0
11	330	0
12	360	0

# 7. Graphical Cam Profile Synthesis

3. You also can increase resolution in rise and fall pieces rather than in dwells:

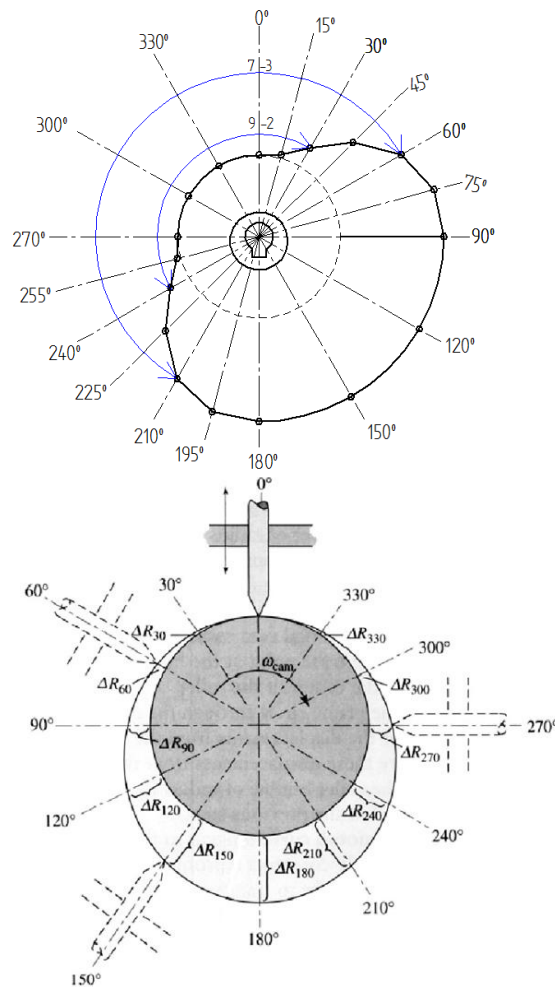


Number	$\theta$ [degrees]	S
0	0	0
1	15	0.88734568
2	30	5.24691358
3	45	12.5
4	60	19.7530864
5	75	24.1126543
6	90	25
7	120	25
8	150	25
9	180	25
10	195	25
11	210	24.1126543
12	225	19.7530864
13	240	12.5
14	255	5.24691358
15	270	0.88734568
16	300	0
17	330	0
18	360	0



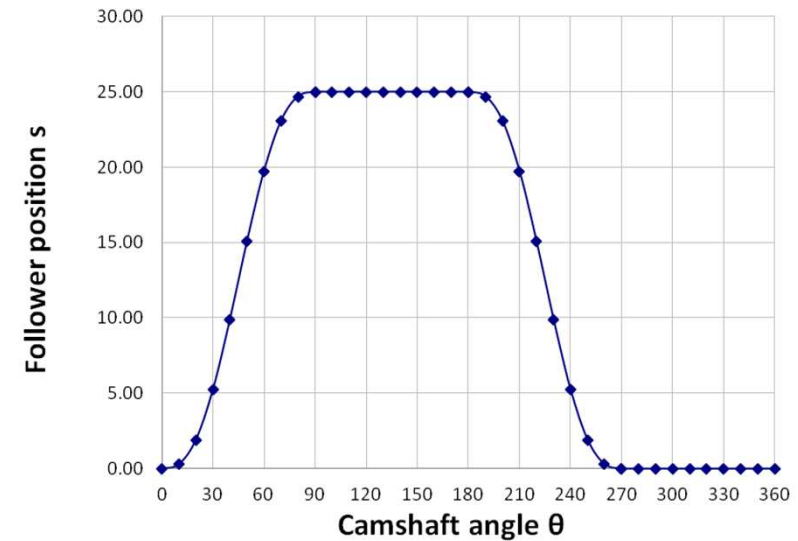
# 7. 1 Knife Edge Follower

This is the case of a knife edge follower.



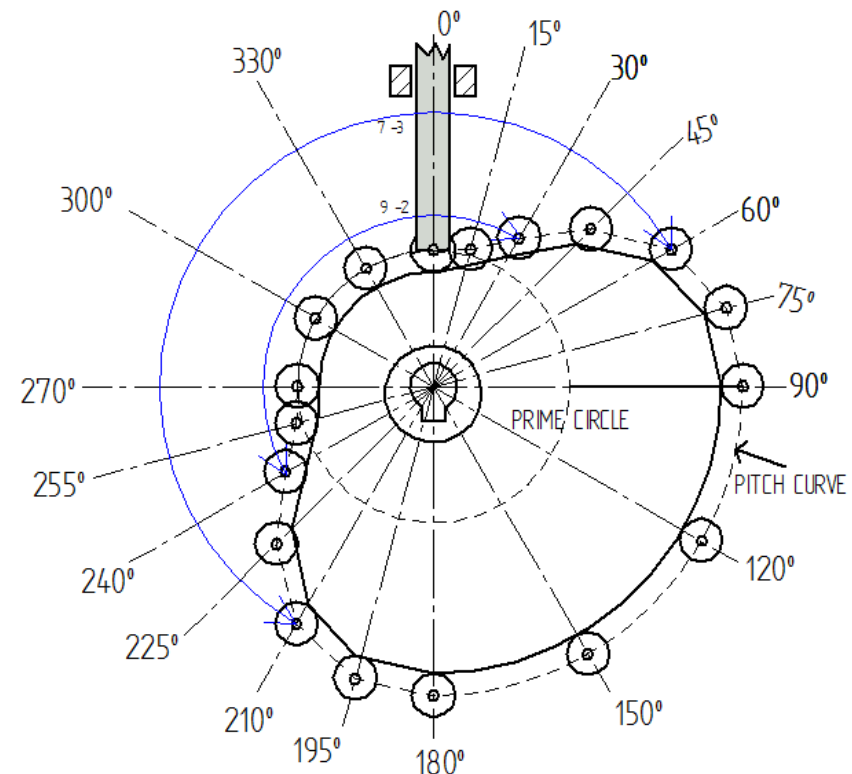
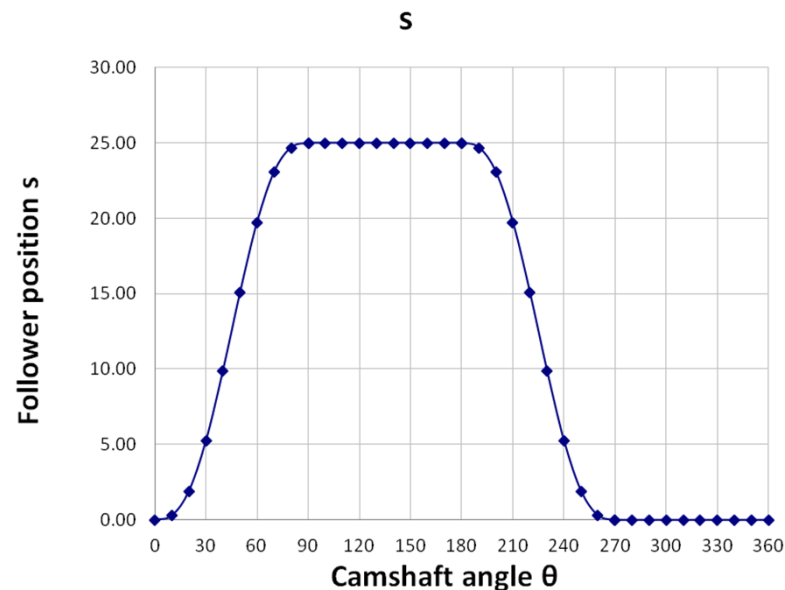
Position	$\theta$ [degrees]	$S$ [mm]
0	0	0
1	30	5
2	60	20
3	90	25
4	120	25
5	150	25
6	180	25
7	210	20
8	240	5
9	270	0
10	300	0
11	330	0
12	360	0

**S**



## 7. 2 Traslating Roller

You initially draw the prime circle. Procedure is the same, but you obtain the pitch profile instead of the cam surface profile (real profile). You should subtract the value of the follower roller radius to each point.

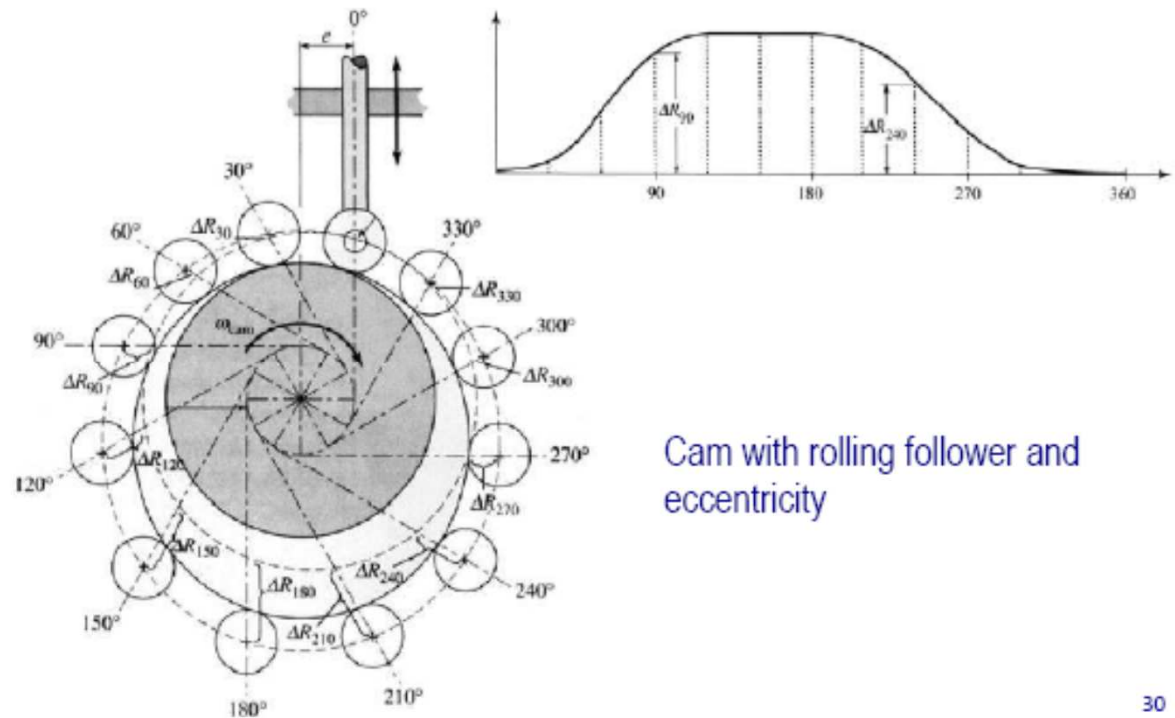


## 7. 3. Traslating Offset Roller

If any pressure angle is excessive, the eccentricity of the follower can be modified, and as we show previously, this will reduce the pitch angle. Procedure to obtain cam profile is very similar.

$$\Phi = \tan^{-1} \left\{ \frac{b - \varepsilon}{s + \sqrt{R_p^2 - \varepsilon^2}} \right\}$$

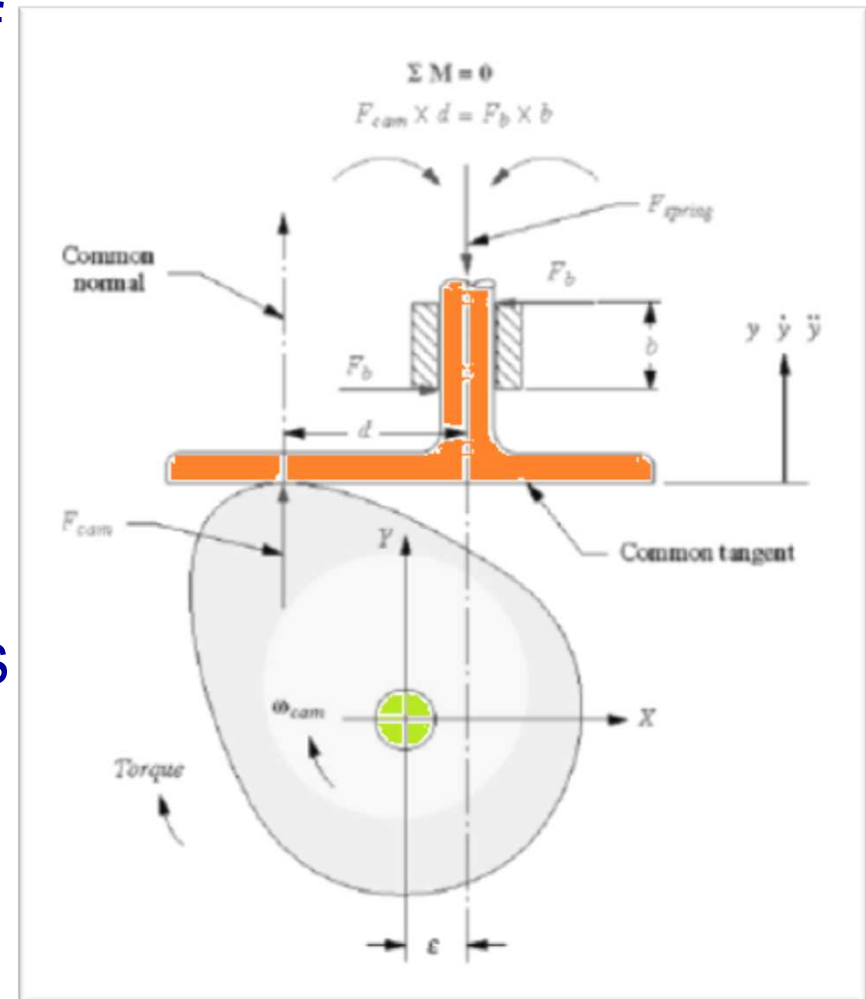
For a positive  $\omega$   
(counterclockwise), a  
positive value of  
eccentricity will decrease  
the pressure angle on the  
rise but will increase it on  
the fall.



Cam with rolling follower and eccentricity

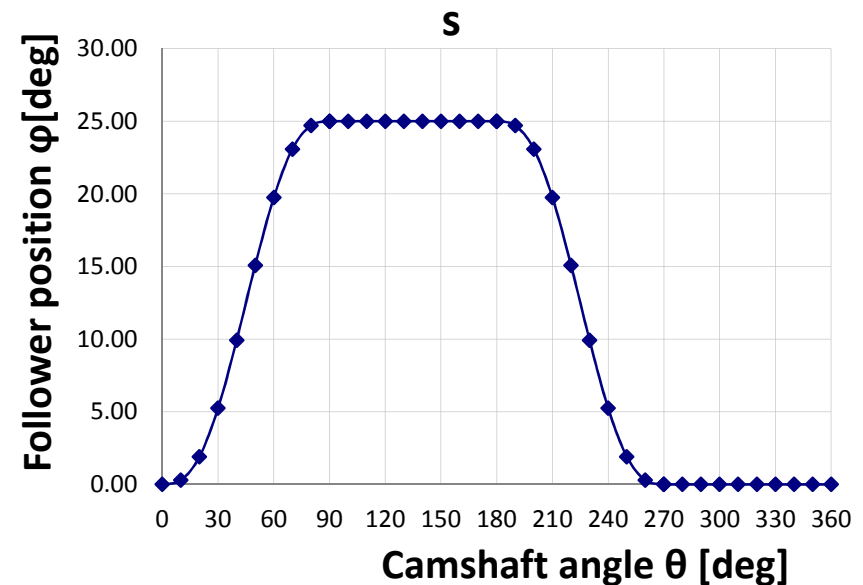
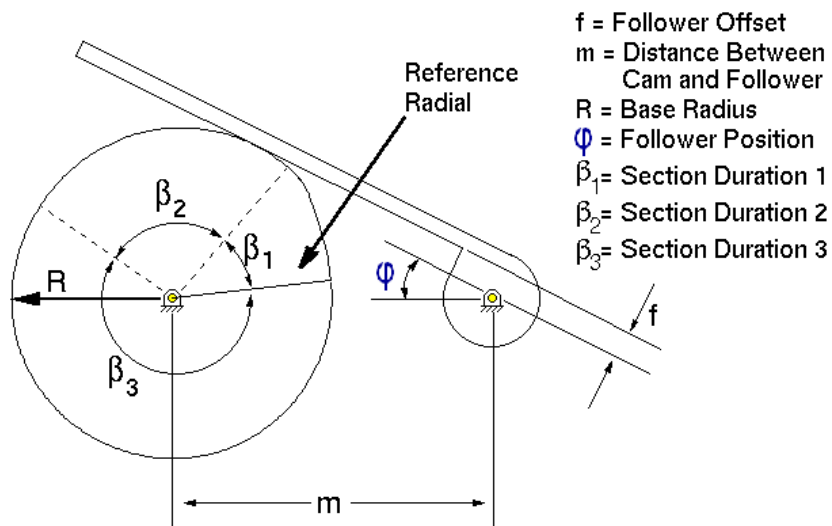
## 7.4 Overturning Moment in Translational Flat-Faced Followers

There is an overturning moment of the follower associated with the offset position in a radial cam with flat-faced follower. This moment tends to jam the follower in its guides. In this case we want to keep the radius of the cam as shorter as possible to minimize this effect.



## 7. 5 Oscillating Flat Faced

For oscillating followers in general, the displacement diagram will relate the cam shaft angle ( $\theta$ ) and the angle rotated by the follower arm ( $\phi$ ) what is our output variable.

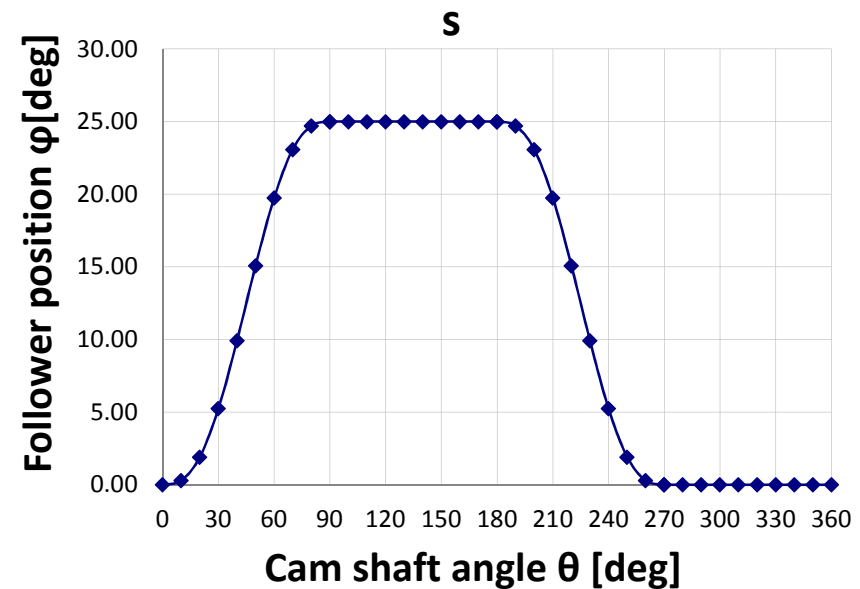




## 7. 5 Oscillating Flat Faced

1. You must obtain a number of points that you are going to use to build the cam profile.

Position	$\theta$ [degrees]	$\varphi$ (deg)
0	0	0
1	30	5
2	60	20
3	90	25
4	120	25
5	150	25
6	180	25
7	210	20
8	240	5
9	270	0
10	300	0
11	330	0
12	360	0

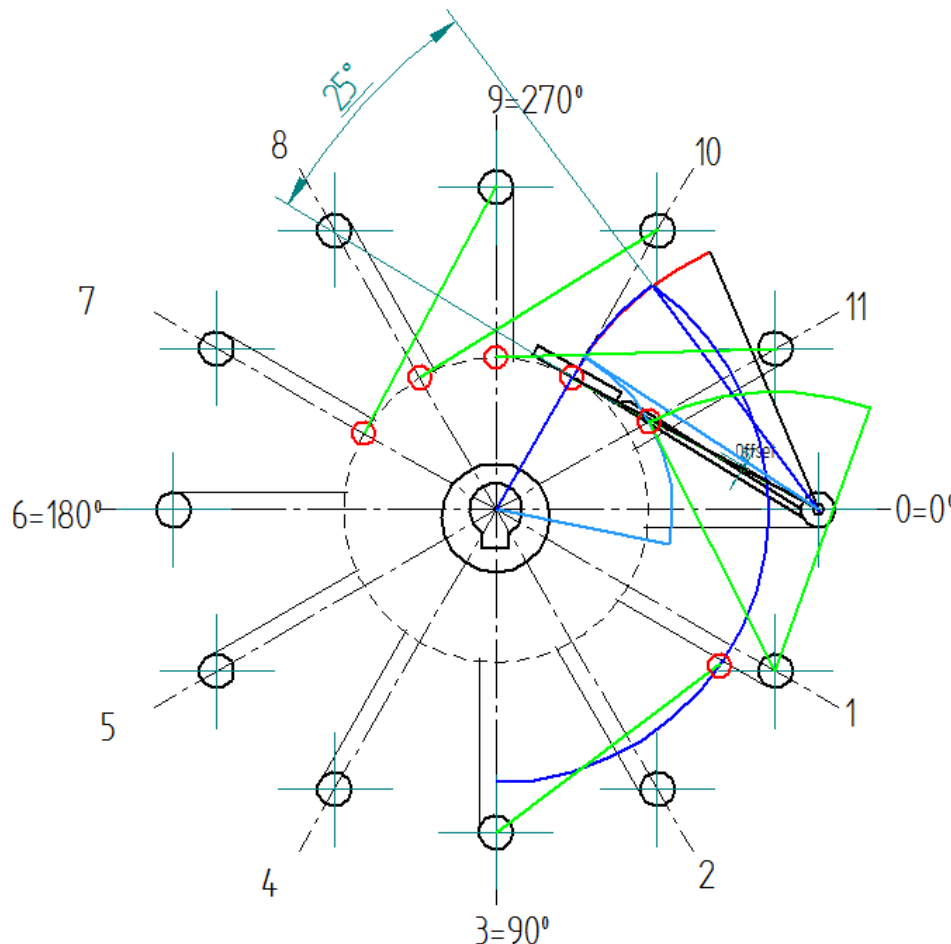




## 7.5 Oscillating Flat Faced

2. Locate the path points to build the cam profile.

Position	$\theta$ [degrees]	$\varphi$ (deg)
0	0	0
1	30	5
2	60	20
3	90	25
4	120	25
5	150	25
6	180	25
7	210	20
8	240	5
9	270	0
10	300	0
11	330	0
12	360	0



Intersection of two arcs:

1- From the center of the follower pivot to the initial contact point.

2- From initial position, an appropriated arc centered in the center of the camshaft.

3. Notice that in all the positions, the surface of the follower must be tangent to the cam profile.



## 7. 5 Oscillating Flat Faced

PLEASE COMPLETE THE EXAMPLE USING COMPASS AND  
RULER



## 8 Radius of curvature

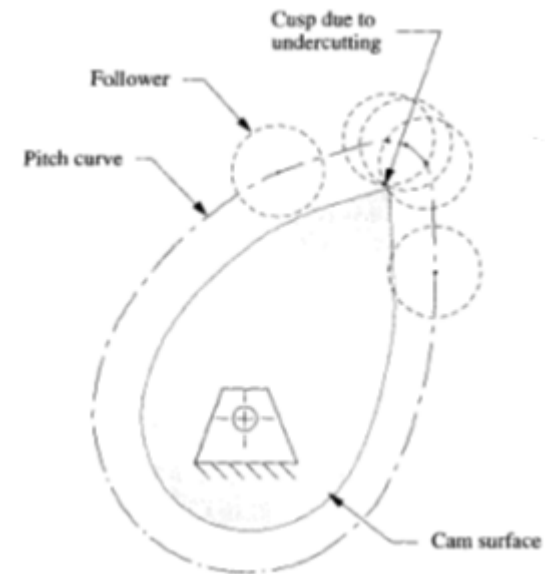
Is a mathematical property of a function. No matter how complicated the a curve's shape may be, nor how high the degree of the function, it will have always an instantaneous radius of curvature at every point of the curve.

When they are wrapped around their prime or base circle, they may be concave, convex or flat portions.

**Both, the pressure angle and the radius of curvature will dictate the minimum size of the cam and they must be checked.**

## 8 Radius of curvature

**Undercutting:** The roller follower radius  $R_f$  is larger than the smallest positive (convex) local radius. **No sharp corners for an acceptable cam design;**



The golden rule is to keep the absolute value of the minimum radius of curvature of the cam pitch curve at least 2 or 3 times large as the radius of the follower.



## 8. Radius of curvature

- Radius of curvature for roller follower:

$$\rho_{pitch} = \frac{\left[ (R_P + s)^2 + v^2 \right]^{3/2}}{(R_P + s)^2 + 2v^2 - a(R_P + s)}$$

Where:

**S, v and a** are displacement, velocity and acceleration you have obtained previously in your svaj diagram designed to fit the specification of your tasks.

**R<sub>p</sub>** is the radius of the prime circle.





## 9. Manufacturing considerations

- ◉ **Materials:** Hard materials as high carbon steels, cast iron. Sometimes made of brass, bronze and plastic cams (low load and low speed applications).
- ◉ **Production process:** rotating cutters. Numerical control machinery. For better finishing, the cam can be ground after milling away most of the unneeded material. Heat treatments are usually required to get sufficient hardness to prevent rapid wear.



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## 9. Manufacturing considerations

- ◉ **Geometric generation:** actual geometries are far from been perfect. Cycloidal function can be generated. Very few other curves can.





- ◉ **Erdman, A.G., Sandor, G.N. and Sridar Kota Mechanism Design. Prentice Hall, 2001. Fourth Edition.**
- ◉ **Robert L. Norton. Design of Machinery. Ed.Mc Graw Hill 1995. Second Edition**
- ◉ **Shigley, J.E. & Uicker, J.J. Teoría de máquinas y mecanismos. McGraw-Hill, 1998**