Ignacio Valiente, José Luis Pérez Díaz, David Mauricio Alba Lucero y Efrén Diez Jiménez

PRACTICE SESSION: CAM DESIGN.

QUESTION 1

Design a polynomial equation to fit requirements below:

$$\theta = 0 \rightarrow s = 0; \dot{s} = 0; \ddot{s} = 0;\ \dddot{s}\ \textit{no conditions given}$$

$$\theta = \beta \rightarrow s = h; \dot{s} = v; \ddot{s} = 0;\ \dddot{s}\ \textit{no conditions given}$$

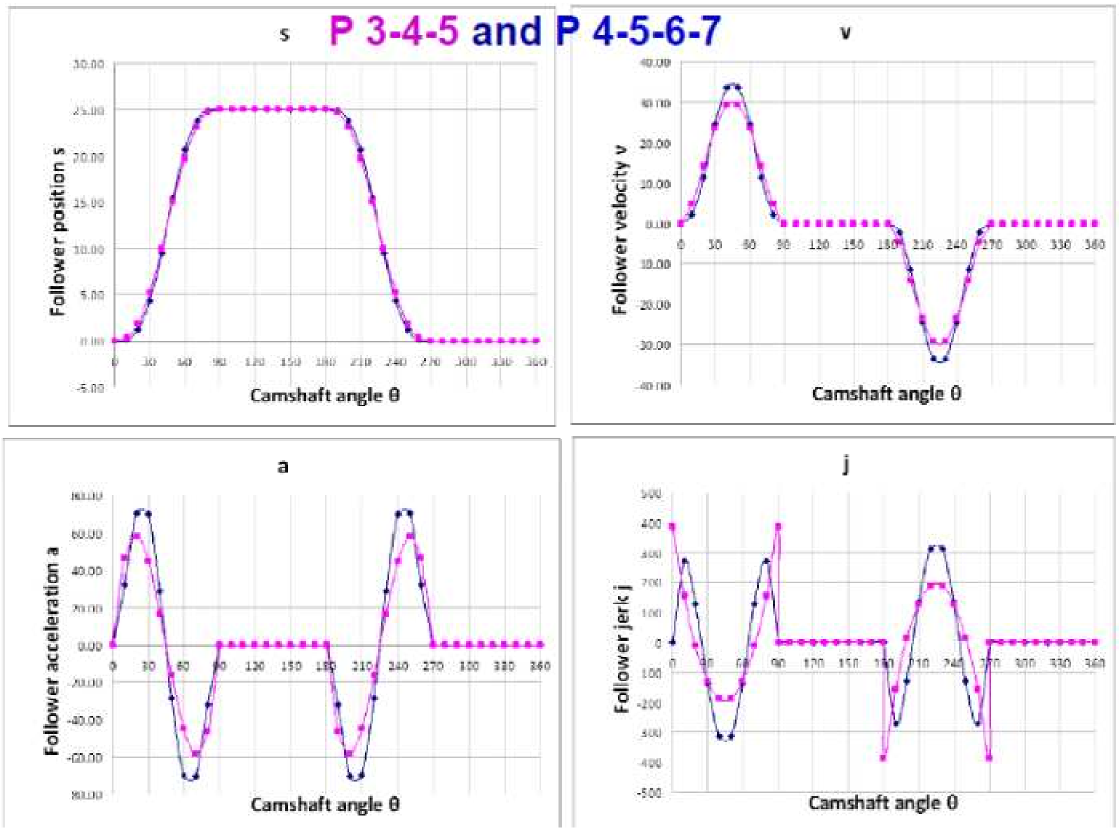
1) What kind of polynomial function should be used? Why?

2) Define the appropriated equations for the rise given.

---

1) What kind of polynomial function should be used? Why?

A polynomial 3-4-5 equation would be enough because there are no jerk boundary conditions. However, using a polynomial 4-5-6-7 function, will give us a continuous jerk function but higher speed and acceleration.



2) Define the appropriated equations for the rise given.

General equations are:

$$\boldsymbol{s = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5}$$

$$\boldsymbol{v = \frac{1}{\beta}\left\{C_1 + 2C_2\left(\frac{\theta}{\beta}\right) + 3C_3\left(\frac{\theta}{\beta}\right)^2 + 4C_4\left(\frac{\theta}{\beta}\right)^3 + 5C_5\left(\frac{\theta}{\beta}\right)^4\right\}}$$

$$\boldsymbol{a = \frac{1}{\beta^2}\left\{2C_2 + 6C_3\left(\frac{\theta}{\beta}\right) + 12C_4\left(\frac{\theta}{\beta}\right)^2 + 20C_5\left(\frac{\theta}{\beta}\right)^3\right\}}$$

Now, we apply the boundary conditions:

$\theta = 0 \rightarrow s = 0$ $\quad \boldsymbol{C_0 = 0}$

$\theta = 0 \rightarrow \dot{s} = 0$ $\quad \dot{s} = \frac{C_1}{\beta} = 0 \quad \boldsymbol{C_1 = 0}$

$\theta = 0 \rightarrow \ddot{s} = 0$ $\quad \ddot{s} = \frac{C_2}{\beta^2} = 0 \quad \boldsymbol{C_2 = 0}$

At the end of the rise the boundary conditions are:

$\theta = \beta \rightarrow s = h$ $\quad \boldsymbol{C_3 + C_4 + C_5 = h}$

$\theta = \beta \rightarrow \dot{s} = v$ $\quad \boldsymbol{3C_3 + 4C_4 + 5C_5 = \beta v}$

$\theta = \beta \rightarrow \ddot{s} = 0$ $\quad \boldsymbol{6C_3 + 12C_4 + 20C_5 = 0}$

Using the Jacobi method

$\mathbf{B = A \cdot x}$

$$\mathrm{x} = \begin{Bmatrix} C_3 \\ C_4 \\ C_5 \end{Bmatrix} \quad \mathrm{A} = \begin{Bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{Bmatrix} \quad B = \begin{Bmatrix} h \\ \beta v \\ 0 \end{Bmatrix}$$

Solving the system we obtain values of the constants:

$$\begin{Bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 10h - 4\beta v \\ 7\beta v - 15h \\ 6h - 3\beta v \end{Bmatrix}$$

Finally, the concrete equations for displacement, velocity and acceleration are obtained introducing the constant values in the general formulation:

$$s = (10h - 4\beta v)\left(\frac{\theta}{\beta}\right)^3 + (7\beta v - 15h)C_4\left(\frac{\theta}{\beta}\right)^4 + (6h - 3\beta v)\left(\frac{\theta}{\beta}\right)^5$$

$$v = \frac{1}{\beta}\left\{3 \cdot (10h - 4\beta v)\left(\frac{\theta}{\beta}\right)^2 + 4 \cdot (7\beta v - 15h)C_4\left(\frac{\theta}{\beta}\right)^3 + 5 \cdot (6h - 3\beta v)\left(\frac{\theta}{\beta}\right)^4\right\}$$

$$a = \frac{1}{\beta^2}\left\{6 \cdot (10h - 4\beta v)\left(\frac{\theta}{\beta}\right) + 12 \cdot (7\beta v - 15h)C_4\left(\frac{\theta}{\beta}\right)^{23} + 20 \cdot (6h - 3\beta v)\left(\frac{\theta}{\beta}\right)^3\right\}$$