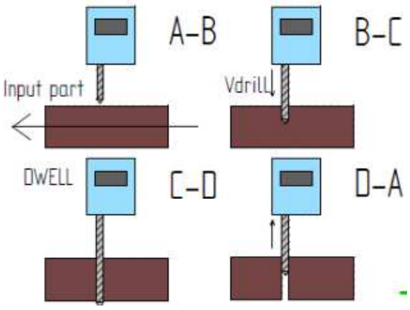
QUESTION 2

A customer needs a solution for a manufacturing application. This application consist in a driller machine that will be included in a production chain..We are hired to design a cam mechanism that fit the requirements given with good performance:



1 RISE. Drill in. h=50 mm β=60º. (Sketch: B-C)

2. DWELL. Finishing the drill. h=50 mm β=120º. (C-D)

3 FALL. Drill out. h=-50 mm β=45º. (D-A)

4 DWELL. New part positioning. h=0 mm β=135º. (A-B)

The mathematical functions for the displacement and velocity for the rise stage are given by the customer:

$$s = 10h\left(\frac{\theta}{\beta}\right)^3 - 15h\left(\frac{\theta}{\beta}\right)^4 + 6h\left(\frac{\theta}{\beta}\right)^5$$

$$v = \frac{1}{\beta}\left\{30h\left(\frac{\theta}{\beta}\right)^2 - 60h\left(\frac{\theta}{\beta}\right)^3 + 30h\left(\frac{\theta}{\beta}\right)^4\right\}$$

1) Define boundary conditions for all the stages.

2) Obtain a mathematical function symmetric to the rise equation useful for the fall stage.

3) Draw the displacement diagram and the velocity diagram using dimensionless units. (Express s and v as a function of the parameters given h and β)

4) The customer want to use a roller follower because it is easy to replace the roller with low cost of time.

Calculate the pressure angle for each position. Use a initial prime radius (r_p) of 110 mm and a eccentricity of ε= -2mm. Is the design acceptable? Why?

5) What measures can be taken to reduce the pressure angle?

6) Select an appropriated prime radius if ε=-5 mm?

7) Draw the cam surface profile with a prime radius of 100 mm, follower radius equal to 10 mm and a eccentricity of 5mm. Apply a reduction scale 4:1.

1) Define boundary conditions for all the stages.

RISE

$\beta = 60° = \frac{\pi}{3} rad$ $\quad \boldsymbol{\theta = 0 \rightarrow s = 0; \dot{s} = 0; \ddot{s} = 0 \quad \theta = \beta \rightarrow s = h; \dot{s} = 0; \ddot{s} = 0}$

1st DWELL

$\beta = 120° = \frac{2\pi}{3} rad$ $\quad \boldsymbol{\theta = 0 \rightarrow s = h; \dot{s} = \ddot{s} = 0 \quad \theta = \beta \rightarrow s = h; \dot{s} = \ddot{s} = 0}$

FALL

$\beta = 45° = \frac{\pi}{3} rad$ $\quad \boldsymbol{\theta = 0 \rightarrow s = h; \dot{s} = 0; \ddot{s} = 0 \quad \theta = \beta \rightarrow s = 0; \dot{s} = 0; \ddot{s} = 0}$

1st DWELL

$\beta = 135° = \frac{3\pi}{4} rad$ $\quad \boldsymbol{\theta = 0 \rightarrow s = 0; \dot{s} = \ddot{s} = 0 \quad \theta = \beta \rightarrow s = 0; \dot{s} = \ddot{s} = 0}$

2) Obtain a mathematical function symmetric to the rise equation useful for the fall stage.

As we saw in Cam Design Lecture I, C constants are opposite to the Rise stage. We also have to recalculate C_0 for new boundary conditions (θ=0 s=h).

$$\theta = 0 \rightarrow s = h \qquad \boldsymbol{C_0 = h}$$

$$\begin{Bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} h \\ 0 \\ 0 \\ -10 \\ 15 \\ -6 \end{Bmatrix}$$

Then, the equations for the fall are:

$$\boldsymbol{s = h - 10h\left(\frac{\theta}{\beta}\right)^3 + 15h\left(\frac{\theta}{\beta}\right)^4 - 6h\left(\frac{\theta}{\beta}\right)^5}$$

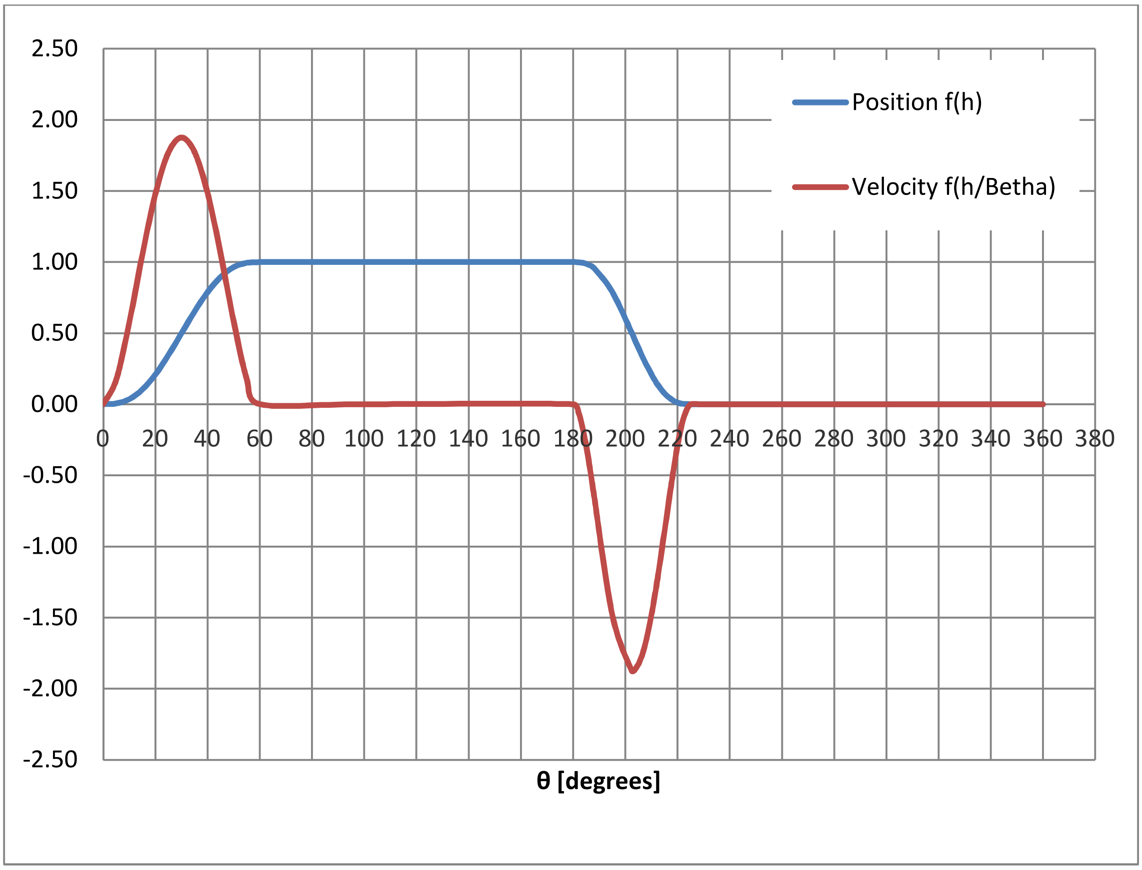
$$\boldsymbol{v = \frac{1}{\beta}\left\{-30h\left(\frac{\theta}{\beta}\right)^2 + 60h\left(\frac{\theta}{\beta}\right)^3 - 30h\left(\frac{\theta}{\beta}\right)^4\right\}}$$

3) Draw the displacement diagram and the velocity diagram using dimensionless units. (Express s and v as a function of the parameters given h and β)

We have to obtain some points in order to draw the diagrams.

θ [degrees]	θ/β [dimensionless]	s f(h) [-]	v f(h/ β) [-]
0	0	0	0
20	1/3	≈0.2h	≈1.5 h/ β
40	2/3	≈0.8h	≈1.5 h/ β
60	1	h	0
180	0	h	0
195	1/3	≈0.8h	≈-1.5 h/ β
210	2/3	≈0.2h	≈-1.5 h/ β
225	1	0	0
360	0	0	0

Note: Because of the appropriated **θ/β** step selection to evaluate s f(h) and v f(h/β) (equal to 1/3), the values of displacement and velocity for the rise and the fall obtained are "symmetric".



As it can be seen in figure above, the maximum and minimum velocity are obtained around 30º(β_{RISE}/2) and 202.5º (β_{FALL}/2). It would be very convenient to obtain a velocity and displacement in this points.

θ [degrees]	θ/β [dimensionless]	s f(h) [-]	v f(h/ β) [-]
0	0	0	0
20	1/3	≈0.2h	≈1.5 h/ β
30	1/2	0.5h	≈1.88 h/ β
40	2/3	≈0.8h	≈1.5 h/ β
60	1	h	0
180	0	h	0
195	1/3	≈0.8h	≈-1.5 h/ β
202.5	1/2	0.5h	≈-1.88 h/ β
210	2/3	≈0.2h	≈-1.5 h/ β
225	1	0	0
360	0	0	0

4) The customer want to use a roller follower because it is easy to replace the roller with low cost of time.

Calculate the pressure angle for each position. Use a initial prime radius (R_p) of 110 mm and a eccentricity of ε= -2mm. Is the design acceptable? Why?

$$\phi = \tan^{-1}\left\{\frac{v - \varepsilon}{s + \sqrt{R_p^2 - \varepsilon^2}}\right\}$$

Where:

v is velocity expressed in length/rad.

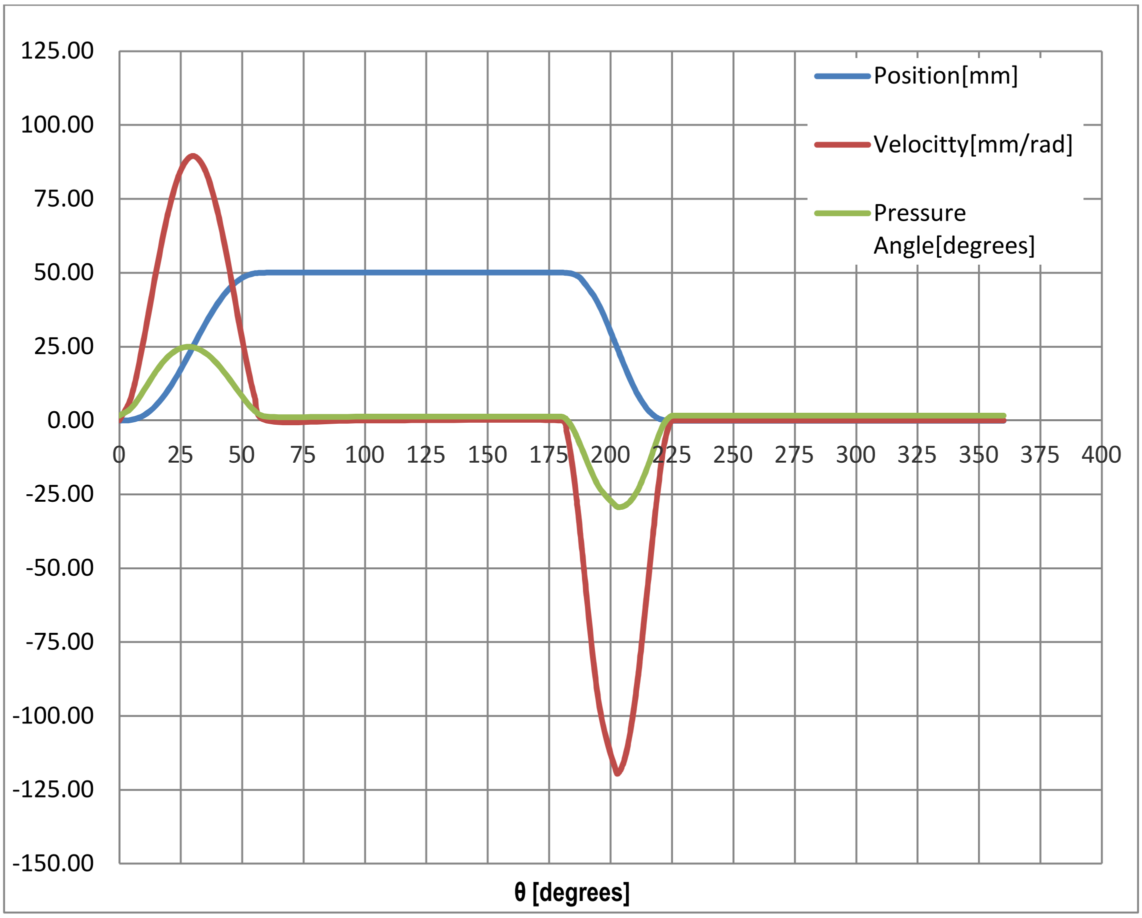
s is displacement expressed in the same length units used in velocity.

ε is the eccentricity.

R_p is the prime radius.

θ [degrees]	θ/β [dimensionless]	s [mm]	v [mm/rad]	ϕ [degrees]
0	0	0	0	1.04
20	1/3	10.5	70.74	31.12
30	1/2	25	89.52	34.14
40	2/3	39.5	70.74	25.95
60	1	50	0	0
180	0	50	0	0
195	1/3	39.5	-94.31	-31.69
202.5	1/2	25	-119.37	-41.01
210	2/3	10.5	-94.31	-37.46
225	1	0	0	1.04
360	0	0	0	1.04

Representing these values:



Looking to figure above, it can be derived that the pressure angle function is quite similar to the velocity function. However, the maximum and minimum pressure angle are not exactly located at the same position (θ) than the maximum and minimum velocity. Despite this difference, we can say for this case that the values of the maximum and minimum pressure angle are obtained for θ=30 and 202.5º.

Then the maximum absolute pressure angle for this configuration is:

ϕ=41º > 30º. Design is not acceptable.

5) What measures can be taken to reduce the pressure angle?

1. Increase the prime radius (R_p)
2. Add some eccentricity (ε):
    - ε<0 will decrease ϕ for the fall stage but will increase it for the rise.
    - ε>0 will increase ϕ for the rise stage but will increase it for the fall.

See some examples

Value of prime radius ε=0	Max	Min		
70	43.3	-52.3		
90	37.9	-46.7		
110	33.55	-41.9		
140	28.5	-36.1		
170	24.6	-31.6	0.60	A reduction of 40% is possible
200				
Value of eccentricity Rp=170	Max	Min		
0	24.6	-31.6		
-1	24.9	-31.4		
-2	25.14	-31.2		
-3	25.4	-31		
-4	25.7	-30.7		
-5	25.9	-30.5	0.96	A reduction of 4% is possible

6) Select an appropriated prime radius if ε= -5 mm?

$$R_p = \sqrt{\left(\frac{v-\epsilon}{\tan\phi} - s\right)^2 + \epsilon^2}$$

In a first approximation, the minimum and maximum pressure angle are obtained for θ= 202.5 and 30º (angles in eq. above must be expressed in radians).

θ [degrees]	θ/β [dimensionless]	s [mm]	v [mm/rad]	Acceptable ϕ
30	1/2	25	89.52	30
202.5	1/2	25	-119.37	-30

Rp> 138.8mm

Rp> 173.2 mm

Rp=139 mm for the maximum velocity condition

Rp=174 mm for the minimum velocity condition

Therefore, the prime radius selected shall be equal or higher than 174 mm.

6) If the required Prime radius is too large, what are the options to reduce it but the pressure angle remain constant?

1. Increase the roller radius (Rf): $\boldsymbol{R_p = R_b + R_f}$
2. Re-design the problematic steps (rise and fall) for example by increasing **β** and therefore reducing the maximum and minimum velocity.

Note: Using SC levas software, you can play with all this parameters and see how they affect to the pressure angle and the curvature radius.

7) Draw the cam surface profile with a prime radius of 100 mm, follower radius equal to 10 mm and a eccentricity of 5mm. Apply an appropriated reduction scale.

The step for the drawing are:

1. Divide in sector the full circle according to your specifications.

2. Draw the prime circle and choose a initial position.

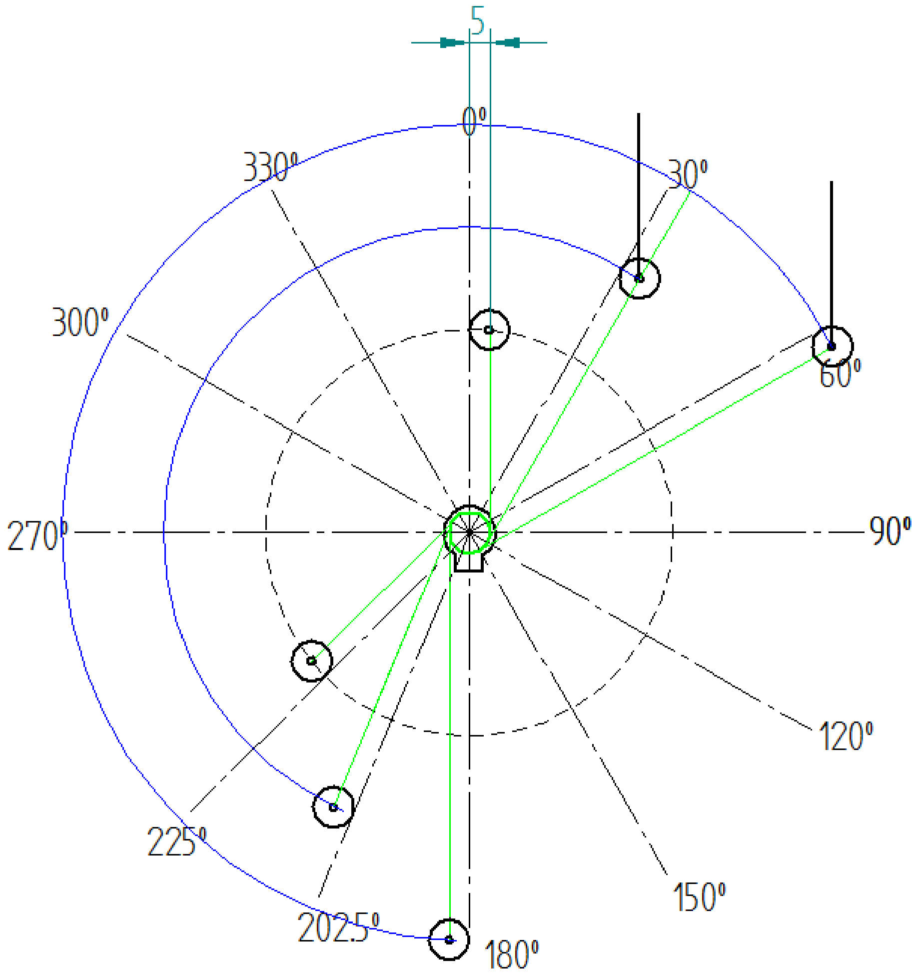
3. Add eccentricity (+5mm) to the initial position Draw a full circle centered in the cam shaft with radius equal to the eccentricity given (+5mm).

4. Draw an arc (blue arc) centered at the cam shaft with radius equal to the prime circle radius plus the elevation (s) for each position.

5. Rotate both, the follower and the eccentricity guide (green line) to each position.

6. The intersection of the blue arc and the follower guide (green lines) is the location of the new center of the roller follower for each position.

7. Obtain the pitch curve and the real cam surface profile connecting these points. (This is up to you)


5
0º
30º
60º
90º
120º
150º
180º
202.5º
225º
270º
300º
330º