

## Lubrication and Sliding Bearings

**Literature:** Kinematic Chains and Machine Components Design (Part II: Machine Components; Chapter II.5 – Lubrication and Sliding Bearings, p. 607 – 638)

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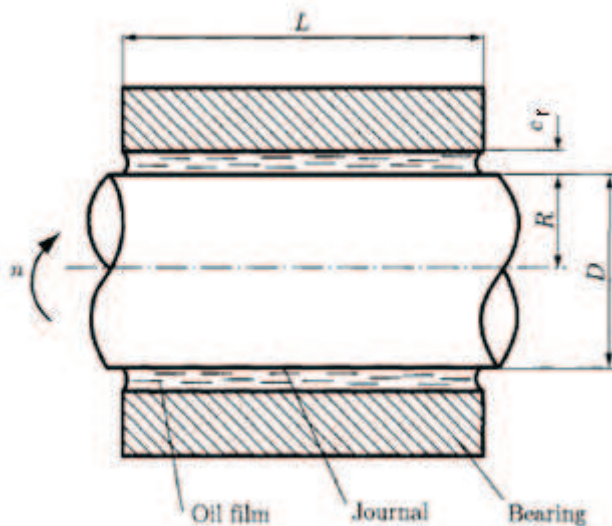
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### Dynamic Viscosity Units

	SI – Standard [Pa·s]	ASTM – Standard [cP]	British Standard [Reyn]
SI [Pa·s]	1	1000 cP	$\approx 1.4514 \cdot 10^{-4}$
ASTM [cP]	0.001	1	$\approx 1.4504 \cdot 10^{-7}$
British [Reyn]	$\approx 6890$	$\approx 6.8947 \cdot 10^6$	1

### Hydrodynamic bearing



### Nomenclature

- L – bearing length, [m]
- D – shaft diameter, [m]
- R – shaft radius, [m]
- $c_r$  – radial clearance, [m]
- $c_d$  – diametral clearance, [m]
- n – angular velocity of the shaft, [rev/s]
- V – surface velocity of the shaft, [m·s]



$\mu$  - dynamic viscosity of the oil, [Pa·s]  
 $P$  - load per unit surface area, [Pa]  
 $F_f$  - friction force, [N]  
 $T_f$  - frictional torque, [N·m]  
 $W$  - radial load, [N]  
 $S$  - Sommerfeld number  
 $H$  - power loss, [W]  
 $T$  - working temperature, [°C]  
 $h_0$  - minimum film thickness, [mm]  
 $p_{\max}$  - maximum film pressure, [Pa]  
 $\phi$  - position angle of minimum film thickness  
 $\theta_{p0}$  - the terminating position of the oil film  
 $\theta_{p\max}$  - the angular position of the point of maximum pressure  
 $Q$  - total flow  
 $Q_s$  - side leakage flow

### Expressions

Friction torque [N·m]:

$$T_f = FR = (\tau A) R = \left( \frac{2\pi R \mu n}{c} 2\pi RL \right) R = \frac{4\pi^2 \mu n LR^3}{c}$$

The radial load per unit of projected bearing area [Pa]:

$$P = \frac{W}{2RL}$$

Friction force [N]:

$$F_f = fW$$

$f$  - friction coefficient

$W$  - applied radial load

Friction torque [N·m]:

$$T_f = f W R = f (2RLP) R = 2R^2 f LP$$

Friction coefficient:

$$f = 2\pi^2 \left( \frac{\mu n}{P} \right) \left( \frac{R}{c} \right)$$

Sommerfeld number:

$$S = \frac{\mu n}{P} \left( \frac{R}{c} \right)^2$$

Power loss [W]:

$$H = 2\pi T_f n$$

### Problem 1

A shaft with a 120-mm diameter (Fig. II.5.17), is supported by a bearing of 100-mm length with a diametral clearance of 0.2 mm and is lubricated by oil having a viscosity of 60 mPa·s. The shaft rotates at 720 rpm. The radial load is 6000 N. Find the bearing coefficient of friction and the power loss.

**Solution** The pressure is calculated with the relation

$$P = \frac{W}{2RL} = \frac{6000}{2(0.06)(0.1)} = 500\,000 \text{ N/m}^2 = 500\,000 \text{ Pa},$$

where  $W = 6000 \text{ N}$ ,  $R = 0.06 \text{ m}$ , and  $L = 0.1 \text{ m}$ .

From Eq. (II.5.11), the coefficient of friction is

$$f = 2\pi^2 \left( \frac{\mu n}{P} \right) \left( \frac{R}{c} \right) = 2\pi^2 \left[ \frac{(0.06)(12)}{500\,000} \right] \left( \frac{60}{0.1} \right) = 0.017,$$

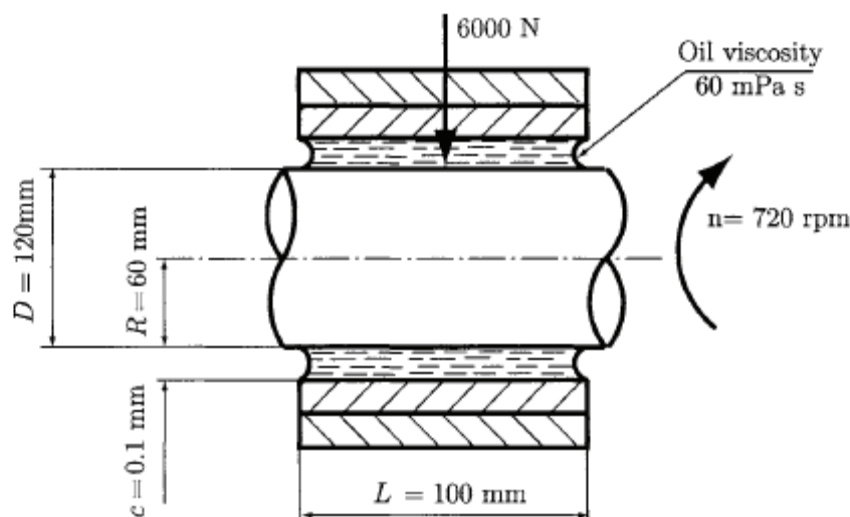
where  $\mu = 60 \text{ mPa} \cdot \text{s} = 0.06 \text{ Pa} \cdot \text{s}$ ,  $n = 720 \text{ rev/min} = 12 \text{ rev/s}$ ,  $R = 60 \text{ mm}$ , and  $c = 0.1 \text{ mm}$ .

The friction torque is calculated with

$$T_f = fWR = (0.017)(6000)(0.06) = 6.139 \text{ N} \cdot \text{m}$$

The power loss is

$$H = 2\pi T_f n = 2\pi (6.139)(12) = 462.921 \text{ N} \cdot \text{m/s} = 462.921 \text{ W}.$$





### Additional problem

The working temperature in this case was 60 °C and SAE 50 grade oil was used. What happens to the oil working parameters if the working temperature decreases to 50°C or rises to 100 °C?

#### **Solution:**

1. If  $T = 50\text{ °C} \rightarrow \mu = 100\text{ mPa}\cdot\text{s} = 0.1\text{ Pa}\cdot\text{s}$

$$f = 2\pi^2 \left( \frac{\mu n}{P} \right) \left( \frac{R}{c_r} \right) = 2\pi^2 \left( \frac{0.1 \cdot 12}{500000} \right) \left( \frac{0.06}{0.0001} \right) = 0.028$$

$$\text{Increase in friction coefficient: } \frac{|0.017 - 0.028|}{0.017} \cdot 100\% = 65\%$$

$$\text{Power loss: } H = 2\pi T_f n = 2\pi \cdot 10.08 \cdot 12 = 760\text{ W}$$

$$T_f = f \cdot W \cdot R = 0.028 \cdot 6000 \cdot 0.06 = 10.08\text{ N}\cdot\text{m}$$

2. If  $T = 100\text{ °C} \rightarrow \mu = 13\text{ mPa}\cdot\text{s} = 0.013\text{ Pa}\cdot\text{s}$

$$f = 2\pi^2 \left( \frac{\mu n}{P} \right) \left( \frac{R}{c_r} \right) = 2\pi^2 \left( \frac{0.013 \cdot 12}{500000} \right) \left( \frac{0.06}{0.0001} \right) = 0.0037$$

$$\text{Decrease in friction coefficient: } \frac{|0.017 - 0.0037|}{0.017} \cdot 100\% = 78\%$$

$$\text{Power loss: } H = 2\pi T_f n = 2\pi \cdot 1.33 \cdot 12 = 100.3\text{ W}$$

$$T_f = f \cdot W \cdot R = 0.0037 \cdot 6000 \cdot 0.06 = 1.33\text{ N}\cdot\text{m}$$

### Problem 2.

A 100 mm diameter shaft is supported by a bearing of 100-mm length with a diametral clearance of 0.075 mm. It is lubricated by SAE 20 oil at the operating temperature of 70°C. The shaft rotates 3000 rpm and carries a radial load of 5000 N. Estimate the bearing coefficient of friction and power loss.

#### Data:

$$D = 100\text{ mm} = 0.1\text{ m}$$

$$L = 100\text{ mm} = 0.1\text{ m}$$

$$c_d = 0.075\text{ mm} = 0.075 \cdot 10^{-3}\text{ m}$$

$$T = 70\text{ °C}$$

SAE 20

$$n = 3000\text{ rpm} = 3000/60 = 50\text{ rev/s}$$

$$\underline{W = 5000\text{ N}}$$

$$f - ?$$

$$H - ?$$



**Solution:**

SAE 20 oil at  $T = 70^\circ\text{C} \rightarrow \mu = 12.5 \text{ mPa}\cdot\text{s} = 0.0125 \text{ Pa}\cdot\text{s}$

The radial load per unit of projected bearing area [Pa]:

$$P = \frac{W}{2RL} = \frac{5000}{2 \cdot 0.05 \cdot 0.1} = 500000 \text{ Pa}$$

Friction coefficient:

$$f = 2\pi^2 \left( \frac{\mu n}{P} \right) \left( \frac{R}{c_r} \right) = 2\pi^2 \left( \frac{0.0125 \cdot 50}{500000} \right) \left( \frac{0.05 \cdot 2}{0.075 \cdot 10^{-3}} \right) = 0.033$$

Friction torque:

$$T_f = f \cdot W \cdot R = 0.033 \cdot 5000 \cdot 0.05 = 8.25 \text{ N}\cdot\text{m}$$

Power loss:

$$H = 2\pi T_f n = 2\pi \cdot 8.25 \cdot 50 = 2591.8 \text{ W}$$

**Problem 3**

A journal bearing of 200-mm diameter, 100-mm length, and 0.1-mm radial clearance carries a load of 20 kN. The shaft rotates at 1000 rpm. The bearing is lubricated by SAE 20 oil and the average temperature of the oil film is estimated at  $70^\circ\text{C}$ . Determine the minimum oil film thickness, bearing coefficient of friction, maximum pressure within the oil film, angles  $\phi$ ,  $\theta_{p_{\max}}$  and  $\theta_{p_0}$ .

**Data:**

$D = 200 \text{ mm} = 0.2 \text{ m}$

$L = 100 \text{ mm} = 0.1 \text{ m}$

$C_r = 0.1 \text{ mm} = 0.1 \cdot 10^{-3} \text{ m}$

SAE 20

$T = 70^\circ\text{C}$

$n = 1000 \text{ rpm} = 1000/60 = 16.67 \text{ rev/s}$

$W = 20 \text{ kN} = 20000 \text{ N}$

$h_0 - ?$

$f - ?$

$P_{\max} - ?$

$\phi - ?$

$\theta_{p_{\max}} - ?$

$\theta_{p_0} - ?$

**Solution:**

SAE 20 oil at  $T = 70^\circ\text{C} \rightarrow \mu = 12.5 \text{ mPa}\cdot\text{s} = 0.0125 \text{ Pa}\cdot\text{s}$

The radial load per unit of projected bearing area [Pa]:

$$P = \frac{W}{DL} = \frac{20000}{0.2 \cdot 0.1} = 1000000 \text{ Pa}$$



Sommerfeld number:

$$S = \frac{\mu n}{P} \left( \frac{R}{c_r} \right)^2 = \frac{0.0125 \cdot 16.67}{1000000} \left( \frac{0.1}{0.1 \cdot 10^{-3}} \right)^2 = 0.208$$

The following parameters are determined on the diagrams according to the Sommerfeld

number and to the ratio:  $\frac{L}{D} = \frac{0.1}{0.2} = \frac{1}{2}$ .

Minimum thickness variable (graph):

$$\frac{h_0}{c_r} \cong 0.32 \Rightarrow h_0 = 0.32 c_r = 0.32 \cdot 0.0001 = 3.2 \cdot 10^{-5} \text{ m}$$

$$h_0 = 0.032 \text{ mm}$$

Coefficient of friction variable (graph):

$$\frac{R}{c_r} f \cong 6 \Rightarrow f = \frac{6 \cdot c_r}{R} = \frac{6 \cdot 0.0001}{0.1} = 0.006$$

Maximum film pressure ratio (graph):

$$\frac{P}{p_{\max}} \cong 0.325 \Rightarrow p_{\max} = \frac{P}{0.325} = \frac{1000000}{0.325} = 3076923 \text{ Pa} = 3.08 \text{ Mpa}$$

Position of minimum film thickness (graph):

$$\phi = 43^\circ$$

Terminating position of oil film (graph):

$$\theta_{p0} = 57^\circ$$

Position of maximum film pressure (graph):

$$\theta_{p0} = 17.5^\circ$$

### Lubrication – Diagrams and Graphs

