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SYNTHESIS OF MECHANISMS

PROBLEM 1:

A four bar path generation mechanism has to be designed. Three prescribed position for the tracer point (path point) are given by the customer according to the reference system in figure below:

P_1= (34,-38) P_2= (0,-27) P_3= (-36,-29).

The crank angle for each position is also a mandatory requirement.

β_1= Not applicable $\beta_2=30^0$ $\beta_3=60^0$

Due to application requirements, the prescribed relative angle between the initial A_1P_1 linkage and the other positions ($A_2P_2=\alpha_2$ and $A_3P_3=\alpha_3$) are also given:

α_1=Not applicable $\alpha_2=20^0$ $\alpha_3=45^0$

Using analytical synthesis design a suitable four bar mechanism and give the length

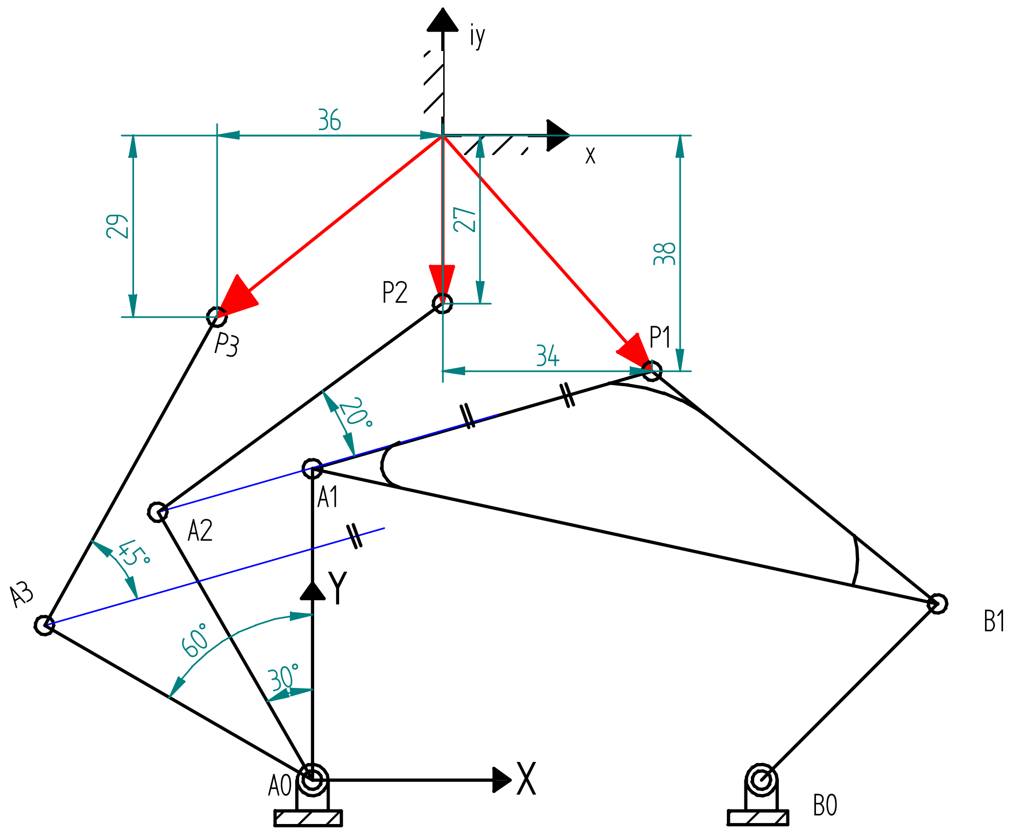


Figure 1. Sketch of the mechanisms design.

We divide the problem in to Dyads:

Dyad A. Contains vector which involve the A fixed position.

Dyad B: B fixed position.

Both Dyads are solved following the same reasoning.

DYAD A.

The equation of the system is:

$$\overrightarrow{W_A} + \vec{\dot{Z}}_A - \overrightarrow{W_A} - \dot{Z}_A = \delta_2$$

$$\overrightarrow{W_A} + \dot{Z}_A - \overrightarrow{\ddot{W}} - \ddot{Z} = \delta_3$$

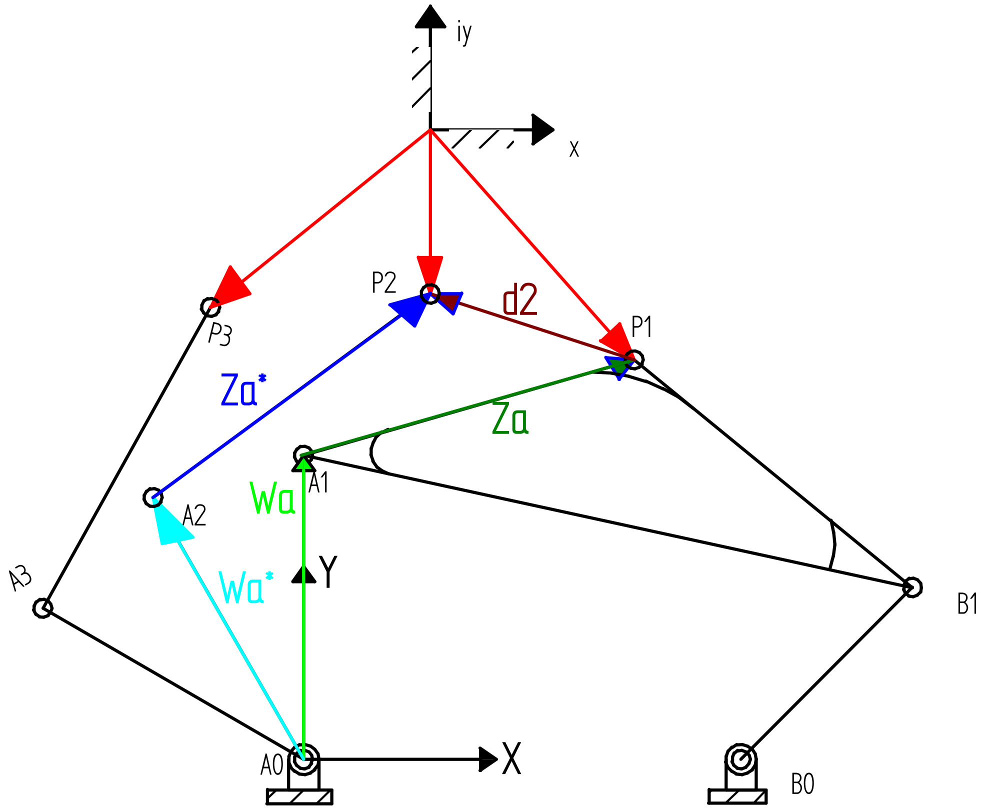
Using the Euler condition, we can express equations above as follows:

$$W \cdot \left(e^{i\beta_2} - 1\right) + Z\left(e^{i\beta_2} - 1\right) = \delta_2$$

$$W \cdot \left(e^{i\beta_3} - 1\right) + Z\left(e^{i\beta_3} - 1\right) = \delta_3$$

$$e^{i\beta_j} = cos\beta_j + isin\beta_j$$

Where W and Z are the module of the corresponding vectors of the links A_0A and AP.



Therefore, the solution for the equation system is:

$$W_A = \frac{\begin{vmatrix} \delta_2 & e^{i\alpha_2} - 1 \\ \delta_3 & e^{i\alpha_3} - 1 \end{vmatrix}}{\begin{vmatrix} e^{i\beta_2} - 1 & e^{i\alpha_2} - 1 \\ e^{i\beta_3} - 1 & e^{i\alpha_3} - 1 \end{vmatrix}}$$

$$Z_A = \frac{\begin{vmatrix} e^{i\beta_2} - 1 & \delta_2 \\ e^{i\beta_3} - 1 & \delta_3 \end{vmatrix}}{\begin{vmatrix} e^{i\beta_2} - 1 & e^{i\alpha_2} - 1 \\ e^{i\beta_3} - 1 & e^{i\alpha_3} - 1 \end{vmatrix}}$$

Where:

$e^{i\beta_j} = cos\beta_j + isin\beta_j$

and

$\vec{\partial_j} = \vec{P_j} - \vec{P_1}$

Solving the equations, we obtain:

$$W_A = \frac{(1.037 - 2.78i)}{(-0.048 + 0.017i)} = \frac{(1.037 - 2.78i)\cdot(0.048 + 0.017i)}{(-0.048 + 0.017i)\cdot(-0.048 - 0.017i)} = \frac{(\approx 0 + 0.153i)}{0.00265}$$

$$Z_A = \frac{(-2.60 - 1.26i)}{(-0.048 + 0.017i)} = \frac{(-2.60 - 1.26i)\cdot(0.048 + 0.017i)}{(-0.048 + 0.017i)\cdot(-0.048 - 0.017i)} = \frac{(0.148 + 0.014i)}{0.00265}$$

$$\boldsymbol{W_A = 57.60i}$$

$$\boldsymbol{Z_A = 55.78 + 5.39i}$$

The ground pivot A_0 is located at:

$$A_o = P_1 - Z_A - W_A = (34 - 38i) - (57.60i) - (55.78 + 5.39i)$$

$$\boldsymbol{A_0 = -21.71 - 101i}$$

And the first position of A:

$$\boldsymbol{A_1 = A_0 + W = -21.71 - 43.39i}$$

The same procedure should be repeated for the DyadB,

P_1= (34,-38) P_2= (0,-27) P_3= (-36,-29). ARE CONSTANT VECTORS

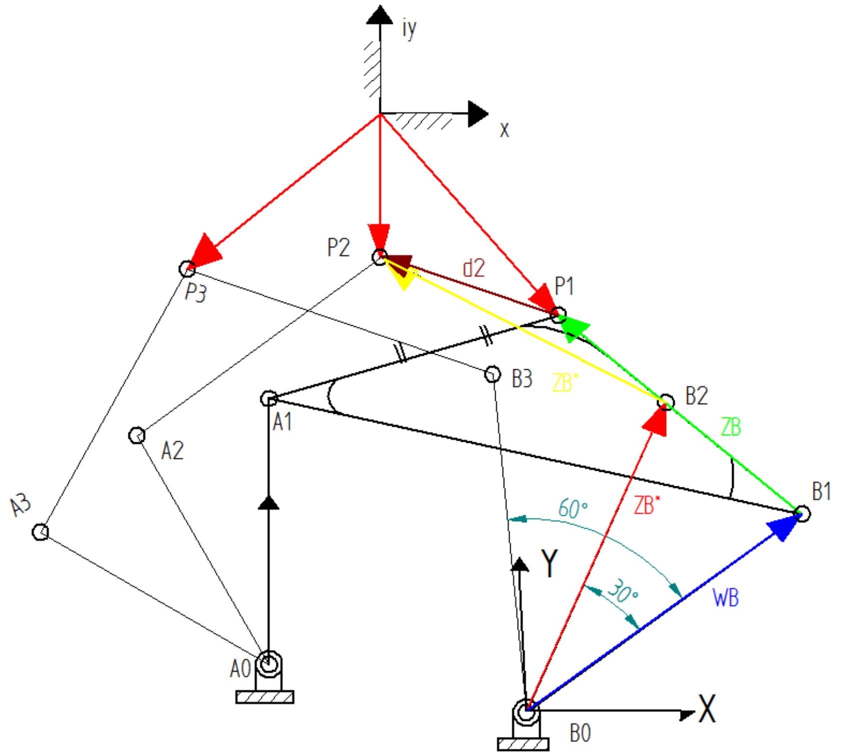
α_1=Not applicable $\alpha_2=20^0$ $\alpha_3=45^0$. ***The angular orientation of the coupler link is also constant. If AP rotates 40^0 so does BP because is a solid rigid.***

but now, you must assume some values. This is part of the mechanisms synthesis. DESIGN is an open problem.

There are multiple solutions depending on the values you select for φ

In order to calculate, assume these values for Dyad B:

φ_1= Not applicable $\varphi_2=-30^0$ $\varphi_3=-52.5^0$



$$W_B = \frac{(1.04 - 2.78i)}{(0.097 + 0.138i)} = \frac{(1.04 - 2.78i)\cdot(0.097 - 0.138i)}{(0.097 + 0.138i)\cdot(0.097 - 0.138i)} = \frac{(-0.28 + 0.415i)}{0.0285}$$

$$Z_B = \frac{(-8.15 - 11.12i)}{(0.097 + 0.138i)} = \frac{(-8.15 - 11.12i)\cdot(0.097 - 0.138i)}{(0.097 + 0.138i)\cdot(0.097 - 0.138i)} = \frac{(-0.734 + 2.21i)}{0.0285}$$

$$\boldsymbol{W_B = -9.86 - 14.53i}$$

$$\boldsymbol{Z_B = 25.72 + 77.48i}$$

The ground pivot B_0 is located at:

$$B_o = P_1 - Z_B - W_B = (34 - 38i) - (25.72 + 77.48) - (-9.86 - 14.53i)$$

$$\boldsymbol{B_0 = 18.13 - 101i}$$

And the first position of B:

$$\boldsymbol{B_1 = P_1 - Z_B = (34 - 38i) - (25.72 + 77.48i) = 8.27 - 115.48i}$$

The rest of the points can be located using both, graphical synthesis or analytical synthesis.