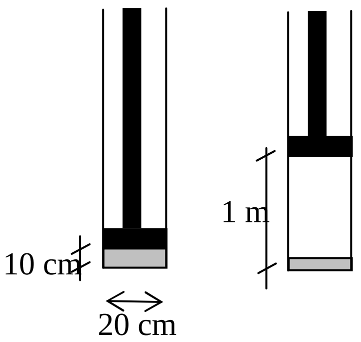
Worked exercise Topic 2

Assume you have a vertical cylinder of radius 10 cm and height L m, equipped with a piston. The piston is separated from the bottom of the cylinder a distance of 10 cm and this cavity is completely filled with liquid water at 80ºC. The piston begins to move vertically very slowly and the whole ensemble is maintained at 80 ºC. The piston stops when reaches a distance of 1 m from the bottom. Look in the web for the equilibrium water vapor pressure at 80 ºC and 10 ºC and for the density at 80 ºC and answer the following questions: (a) What would be the pressure inside? (b) What would be the level of water and how many moles of water have passed to the gas phase? (c) What distance should the piston be displaced to completely evaporate all the water? (d) At the distance set in section c) the piston is fixed and temperature is decreased to 20 ºC. Would water condense? How much?

Solution:



a) As the piston moves a vacuum is created in the cavity the piston leaves below. Water evaporates to fill the vacuum until water vapor pressure equals the equilibrium value at 80 ºC. You can look in the web http://www.engineeringtoolbox.com/water-thermal-properties-d_162.html

finding $P^0(H_2O, 80\ ºC) = 47.5\ kN/m^2 = 47.5\cdot 10^3\ N/m^2 = 47.5\cdot 10^3$ Pa

b) The amount of water in the gas phase can be calculated assuming ideal gas behavior, but for doing so you will need the gas constant in appropriate units. You can look in the web http://en.wikipedia.org/wiki/Gas_constant finding 8.314 $m^3 Pa K^{-1} mol^{-1}$.

$$n = \frac{47.5 \cdot 10^3 V_{gas}}{8.314 \times (273+80)} \frac{Pa}{Pa \cdot m^3 \cdot K^{-1} \cdot mol^{-1} \cdot K} = 0.0173 V_{gas} \quad (1)$$

where V_{gas} is the volume of the gas cavity.

Now the problem is calculating the volume of the gas cavity since the evaporated water decreases the liquid water level. We can set out the problem in the following way:

Initially, before the piston begins to move, the water volume in the liquid phase is $V_{liq}(init) = \pi\cdot 0.1^2\cdot (0.1) = 3.142\cdot 10^{-3}\ m^3$. After the piston moves, the total cylinder volume is $V_{cyl} = \pi\cdot 0.1^2\cdot (1) = V_{gas} + V_{liq}(after)$, where V_{gas} is the volume of the gas cavity and $V_{liq}(after)$ is the volume of liquid water after evaporating n moles.

But V_{liq}(after) = V_{liq} (init) – V_n, that is to say, to calculate the liquid water volume after the evaporation we need to subtract the volume of evaporated water from the initial liquid volume. To calculate the volume of liquid water that has passed to the gas phase we need the density of water at 80 ºC. You can look for it in the web http://www.engineeringtoolbox.com/water-thermal-properties-d_162.html finding ρ = 0.972 $g\cdot cm^{-3}$.

$$V_n = \frac{n \times M_{H_2O}}{\rho_{H_2O,80^oC}} = \frac{n \times 18 g \cdot mol^{-1}}{0.972 g \cdot cm^{-3}} = n \cdot 18.52 cm^3 = n \cdot 18.52 \cdot 10^{-6} m^3 \qquad (2)$$

Therefore, the volume of the gas cavity is

$$\begin{aligned} V_{gas} &= V_{cyl} - V_{liq}(after) = V_{cyl} - V_{liq}(init) + V_n = 3.142 \cdot 10^{-2} - 3.142 \cdot 10^{-3} + n \cdot 18.52 \cdot 10^{-6} = \\ &= 0.028278 + 0.0173 \times 18.52 \cdot 10^{-6} V_{gas} \approx 0.028278 m^3 \end{aligned} \qquad (3)$$

Replacing this value in equation (1) we arrive to the number of moles of water in the gas phase:

n = 0.0173×0.028278 = 4.892· 10^{-4} moles.

To calculate the level of water after evaporation we need the volume of liquid water V_{liq}(after) = V_{liq} (init) – V_n = V_{liq} (init) - n×18.52· 10^{-6} = 3.142· 10^{-3} - 4.892· 10^{-4}×18.52· 10^{-6} ~3.142· 10^{-3}. This means that the volume variation is insignificant.

c) To completely evaporate all the water, the volume of the gas phase must accommodate all the water at a pressure equal to the vapor pressure of water at 80 ºC (= 47 500 Pa). The number of moles of water in the gas phase would be

$$n = \frac{V_{liq}(init)\rho_{H_2O,80^oC}}{M_{H_2O}} = \frac{3.142 \cdot 10^{-3} m^3 \times 10^6 cm^3 \cdot m^{-3} \times 0.972 g \cdot cm^3}{18 g \cdot mol^{-1}} = 169.668\ moles \qquad (4)$$

Therefore,

$$L = \frac{V}{\pi \cdot 10^{-2}} = \frac{nRT}{P\pi \cdot 10^{-2}} = \frac{169.668 \times 8.314 \times 353}{47{,}500 \times \pi \times 10^{-2}} = 333.7\ m \qquad (5)$$

d) The pressure inside the cylinder is 47,500 N/m^2 which corresponds to the equilibrium vapor pressure of water at 80 ºC. If temperature is decreased to 20 ºC, water will condense up to the point at which vapor pressure equals the equilibrium value at 20 ºC, which is

$P^0(H_2O$, 20 ºC) = 2.3 kN/m^2 = 2.3· 10^3 N/m^2 = 2.3· 10^3 Pa

Now the question is how much water can be accommodated inside a cylinder of 333.7 m length at 22,300Pa?. The answer is easy assuming ideal gas behavior

$$n = \frac{PV}{RT} = RT = \frac{2.3 \cdot 10^{3}\,\pi \cdot 10^{-2} \cdot 333.7}{8.314 \times 293} = 9.898\ moles$$

The amount of condensed water will be the difference between the moles in gas phase at 80 ºC and a 20 ºC = 169.668 – 9.898 = 159.77 moles.