Digital Communications
Telecommunications Engineering

Chapter 1

Introduction

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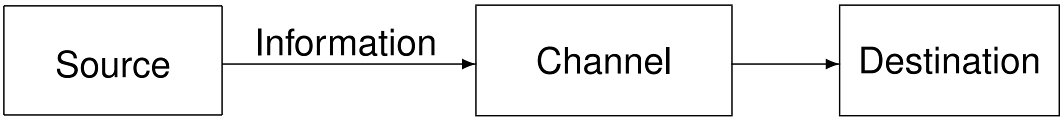
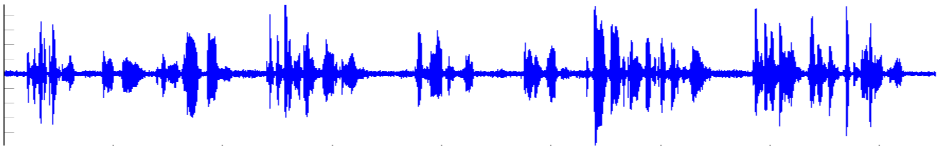
Definition: Communication system

- Purpose of a communication system: *transmission*
- Transmission: *The process of sending (transporting) information from one point (source) to another point (destination) through some transmission structure (channel)*



- Physical representation of information during transmission
  - The usual case: electrical signal
    - ★ Information / electrical signal conversion: Transducer
      - Example: output of a microphone (speech signal)



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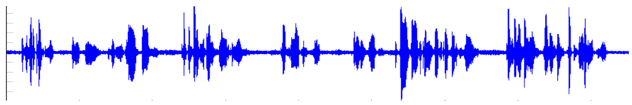
Analog and digital communication systems

- Analog communication system
  - Designed to send information over a continuous waveform



- Digital communication system
  - Designed to send information over a sequence of symbols pertaining to a finite alphabet (M possible values for each symbol)
    - ★ Example: Bits ($M = 2$): $\{0, 1\}$
      - Information:011000110111001101010111001001101 0...
  - Transmission at a given rate (symbol rate) R_s symbols/s
    - ★ A symbol is transmitted every $T = \frac{1}{R_s}$ seconds
  - Symbols have to be converted in electrical signals for transmission
    - ★ Each symbol is mapped onto a waveform
    - ★ Simplest case: waveforms of $T = \frac{1}{R_s}$ seconds
- Digital systems are predominant over analog ones

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Advantages of digital systems

- Possibility of regeneration
- Existence of techniques for error detection and correction
- Information can be encrypted
- Channel distortion can be compensated (equalization)
- Information format is independent of the nature of information (voice, data, TV, etc.)
- For multiplex/medium access, CDM/CDMA and TDM/TDMA can be used (as well as FDM/FDMA)
- In general, circuits are
  - More reliable
  - Cheaper
  - More flexible (programmable)

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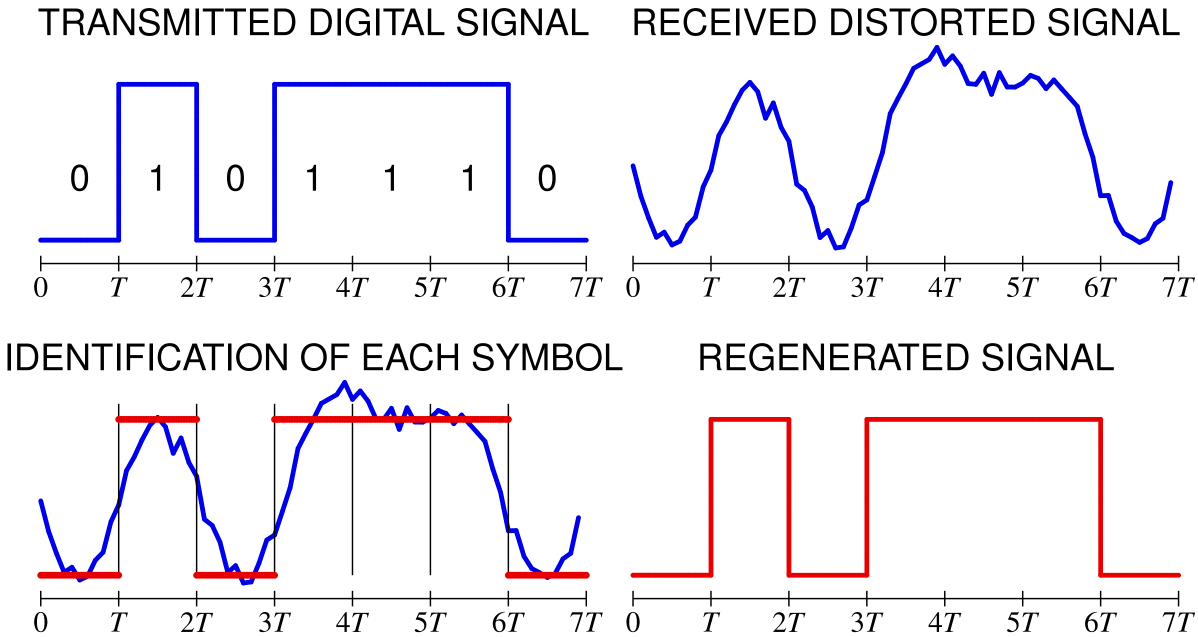
Digital Regeneration

BIT ENCODING - Binary system using squared pulses

$1 \equiv$ High level

$0 \equiv$ Low level



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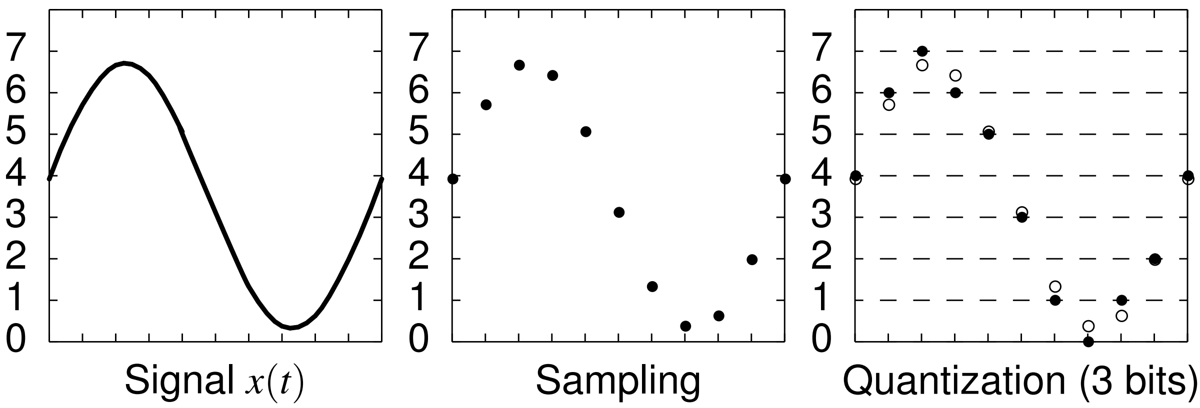
Disadvantages of digital systems

- Need for synchronism
  - Identification of the interval for each symbol
- Higher bandwidth
- Many information sources are analog in nature
  - A/D conversion
    - Sampling
    - Quantization → Quantization error
  - D/A conversion
    - Interpolation
    - Low pass filtering

Analog to digital (A/D) conversion

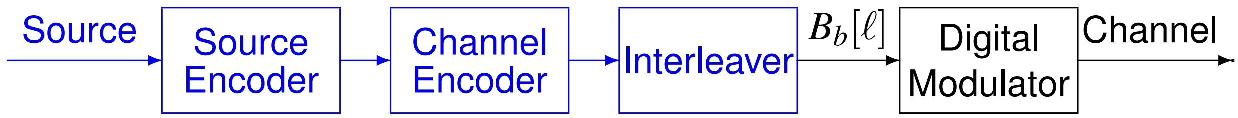
- Analog sources: continuous amplitude, continuous time
- Analog/digital conversion:
  - Discrete time : Sampling at rate f_s samples/s
  - Discrete amplitudes: Quantization with n bits/sample
    - Quantization noise: only 2^n quantization levels
      - Difference between sampled value and quantized value
    - Lower as n increases



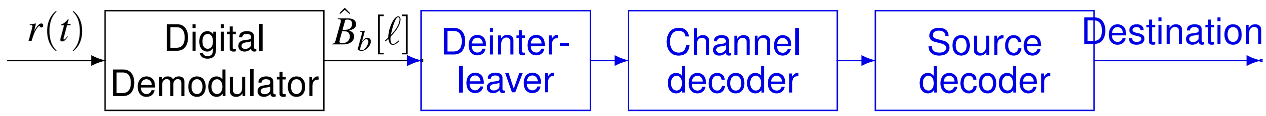
- Binary rate (bits/s): $R_b = f_s$ (samples/s) $\times\, n$ (bits/sample)

Digital Transmitter/Receiver - Basic functional blocks

- Digital transmitter



- Digital receiver



Source and channel coding

- Source coding
  - Reduction of redundancy (compression)
  - Lower binary data rate requirements for transmission
- Channel coding
  - Error detection and/or correction
  - Introduction of redundancy (structured)
  - Capability of detection/correction depends on complexity
  - Simplest example: repetition code
    - Repetition code of order 1: $0 \rightarrow 00$ $1 \rightarrow 11$
      - Detects 1 error over a two-bits block
    - Repetition code of order 2: $0 \rightarrow 000$ $1 \rightarrow 111$
      - Detects 2 errors or corrects 1 error (correction based on majority decision) over a three-bits block

Interleaving

- Protection for burst errors
  - In combination with channel encoder
- Re-arrangement of data in a non-contiguous way
  - Goal: to transform burst error in several isolated errors
    - ★ Channel decoder deals with relatively few errors per block
- Kinds of interleavers
  - Block interleavers
  - Convolutional interleavers

Interleaving - Some example

Uncoded Data

1 0 1 1 1 0

Channel Code

Repetition code (order 2)

Coded Data

111 000 111 111 111 000

Interleaved data

101 110 101 110 101 110

Transmission

101 110 101 1 10 101 1 10

Burst of errors

De-interleaved data

11 1 00 0 11 1 11 1 1 1 1 0 0 0

Isolated errors

1	0	1	1	1	0
1	0	1	1	1	0
1	0	1	1	1	0

Interleaver

$N_c \times N_b$

Block interleaver

Data input: per column

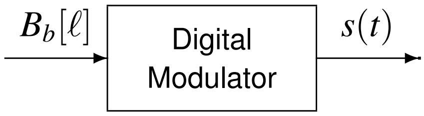
Data output: per row

1	1	1
0	0	0
1	1	1
1	1	1
1	1	1
0	0	0

De-interleaver

$N_b \times N_c$

Digital modulator - Conversion of bits ($B_b[\ell]$) in a signal



- Transmission of sequence of bits $B_b[\ell]$ at rate $R_b = \frac{1}{T_b}$ bits/s
  - Conversion in electrical signal $s(t)$
- Block-wise bit transmission - Sequence of symbols
  - Segmentation of sequence $B_b[\ell]$ in blocks of m bits
  - Each block of m bits is a symbol
    - ★ 1 symbol $\equiv m$ bits
    - ★ Alphabet of possible symbols: $M = 2^m$ symbols: $B \in \{b_i\}_{i=0}^{N-1}$
  - Sequence of symbols $B[n]$
    - ★ Symbol rate $R_s = \frac{1}{T}$ symbols/s (bauds)
    - ★ Relationship between rates R_b / R_s: $R_b = m \cdot R_s$ (or $T = m \cdot T_b$)
  - Transmission of a symbol (block of m bits) each T seg.
- Simplest conversion from bits/symbols sequence to a signal $s(t)$
  - Piecewise generation: "pieces" of T seconds (corresponding to 1 symbol)
    - ★ Symbol interval for $B[n]$: interval $nT \leq t < (n+1)T$

Symbol / signal conversion - Simplest model

- Transmission of the first symbol of the sequence
  - $B \equiv B[0]$
  - Symbol interval: $0 \leq t < T$
- Symbol / signal conversion
  - Alphabet of M possible symbols: $B \in \{b_0, b_1, \cdots, b_{M-1}\}$
  - Definition of M waveforms of T seconds

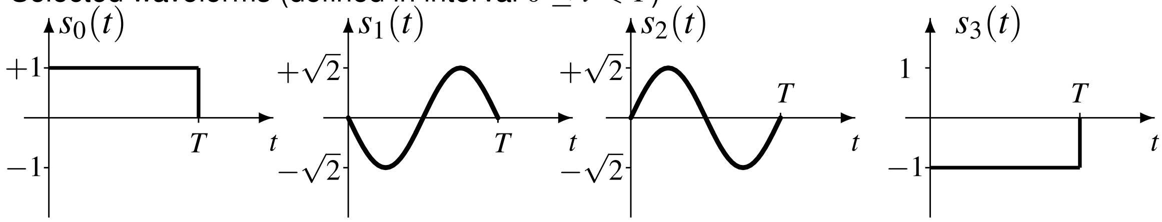
$$\{s_0(t), s_1(t), \cdots, s_{M-1}(t)\}, \text{ definidas en } 0 \leq t < T$$

  - Symbol / waveform association: $b_i \leftrightarrow s_i(t)$
  - Generation of the piece of signal to be transmitted
    - ★ If $B = b_i$, then $s(t) = s_i(t)$
- Transmission of symbol $B[n]$
  - Symbols interval: $nT \leq t < (n+1)T$
  - Value of the symbol: $B[n] = b_j$
    - ★ The waveform associated to b_j is moved to symbol interval

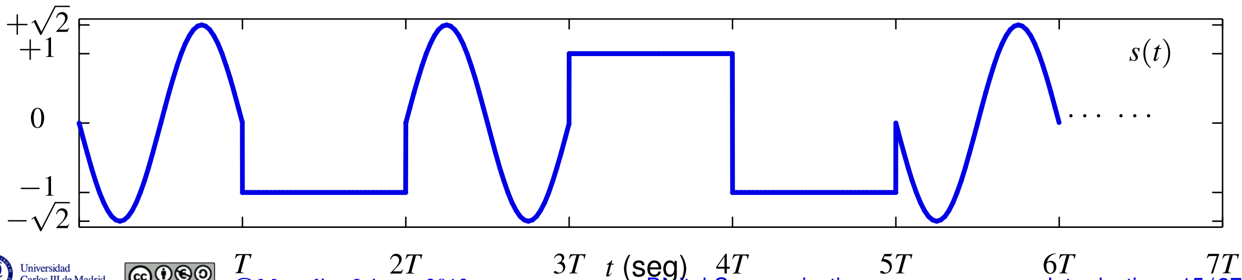
$$s(t) = s_j(t - nT), \text{ in } nT \leq t < (n+1)T$$

Example $M = 4$

- Number of bits per symbol: $m = 2 \rightarrow M = 4$ symbols
- Symbols: $b_0 \equiv 00$, $b_1 \equiv 01$, $b_2 \equiv 10$, $b_3 \equiv 11$
- Selected waveforms (defined in interval $0 \leq t < T$)

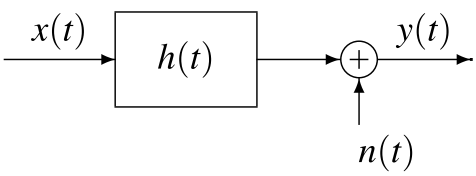


- Sequence to be transmitted: $B_b[\ell] = 011110001101\cdots$
- Sequence of symbols
  - Segmentation of $B_b[\ell]$: $01 \mid 11 \mid 10 \mid 00 \mid 11 \mid 01 \mid \cdots$
  - Sequence $B[n] = b_1 \mid b_3 \mid b_2 \mid b_0 \mid b_3 \mid b_1 \mid \cdots$
- Transmitted signal
  - Piecewise generation:
    $s(t) = \{s_1(t) \mid s_3(t-T) \mid s_2(t-2T) \mid s_0(t-3T) \mid s_3(t-4T) \mid s_1(t-5T) \mid \cdots\}$



Transmission through the channel

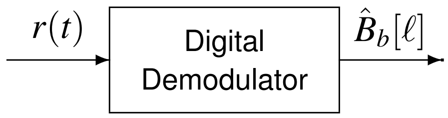
- Received signal at the output of the channel: $r(t)$
  - Signal suffers distortion during transmission
  - Received signal is different from transmitted signal:
    $r(t) \neq s(t)$
- Channel model - Effects of distortion that are considered
  - Linear distortion
    - Model: linear time invariant system, $h(t)$, $H(j\omega)$
  - Thermal noise
    - Model: random process $n(t)$, stationary, ergodic, white, Gaussian, with power spectral density $S_n(t) = \frac{N_0}{2}$, being $N_0 = k \times T_{emp}$ (Boltzmann constant times temperature)



  - Received signal

$$r(t) = s(t) * h(t) + n(t)$$

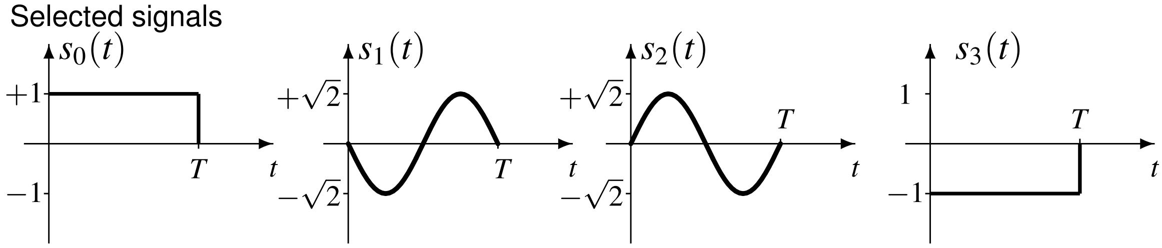
Demodulador digital



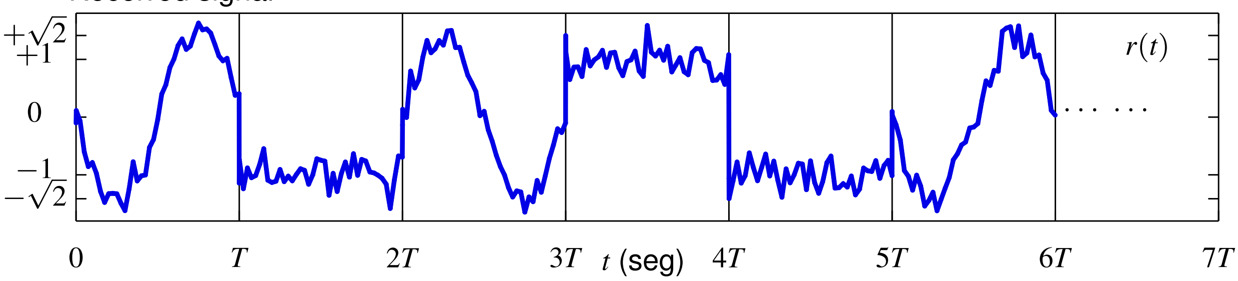
- Estimation of bit sequence $B_b[\ell]$ from received signal $r(t)$
  - Signal is distorted during transmission: $r(t) \neq s(t)$
- Process to recover bits from $r(t)$
  - Piecewise process: splitting in symbol intervals
  - Estimation of symbol (m bits) transmitted in each interval
- Estimation of first symbol: $\hat{B} \equiv \hat{B}[0]$
  - Observation of $r(t)$ in first interval: $0 \leq t < T$
  - Comparison with M possible waveforms
    - ★ If the "*more similar*" is $s_k(t)$, then $\hat{B} = b_k$
- Estimation of symbols with index n: $\hat{B}[n]$
  - Observation of signal $r(t)$ in interval $nT \leq t < (n+1)T$
  - Comparison with M possible waveforms
    - ★ If the "*more similar*" is $s_v(t)$, then $\hat{B} = b_v$

Example $M = 4$

- Symbols: $b_0 \equiv 00$, $b_1 \equiv 01$, $b_2 \equiv 10$, $b_3 \equiv 11$
- Selected signals



- Received signal



- Symbol detection
  - Segmentation of signal in symbol intervals
    - ★ $n = 0$, interval $0 \leq t < T$ - "*More similar*" signal: $s_1(t) \to \hat{B}[0] = b_1$
    - ★ $n = 1$, intervalo $T \leq t < 2T$ - "*More similar*" signal: $s_3(t) \to \hat{B}[1] = b_3$
    - ★ Following the same procedure: $\hat{B}[2] = b_2$, $\hat{B}[3] = b_0$, $\hat{B}[4] = b_3$, $\hat{B}[5] = b_1$
- Sequence of bits: $B[n] = b_1 | b_3 | b_2 | b_0 | b_3 | b_1 | \cdots \Rightarrow B_b[\ell]$: $01|11|10|00|11|01|\cdots$

Selection of the M waveforms - Constraints

- Energy of the transmitted signal
  - Mean energy per transmitted symbol
    - ★ Probability of each symbol: $p_B(b_i) = P(B[n] = b_i)$
    - ★ Mean energy per symbol

$$E_s = \sum_{i=0}^{M-1} p_B(b_i) \cdot \mathcal{E}\{s_i(t)\}$$

- Adaptation to the channel
  - To minimize the distortion that signal suffers during transmission
  - Ideal situation: $s(t) * h(t) = s(t) \rightarrow r(t) = s(t) + n(t)$
    - ★ Is fulfilled if: $s_i(t) * h(t) = s_i(t)$ for $i = 0, 1, \cdots, M-1$
- Performance: probability of error in the detection
  - Depends on "*similarity*" between signals
  - Figure of merit for "*similarity*": distance between signal

$$d(s_i(t), s_k(t)) = \sqrt{\mathcal{E}\{s_i(t) - s_k(t)\}} = \sqrt{\int_{-\infty}^{\infty} |s_i(t) - s_k(t)|^2 \, dt}$$

    - ★ Reducing errors: Increasing distance between signals

Selection of the M waveforms - Discrete representation

- To simultaneously consider these 3 constraints is difficult in the continuous time domain
- Discrete representation of signals
  - Points in a N-dimensional Hilbert space
    - ★ Coordinates: N-dimensional vector (Encoder)

$$s_i(t) \rightarrow \boldsymbol{a}_i = \begin{bmatrix} a_{i,0} \\ a_{i,1} \\ \vdots \\ a_{i,N-1} \end{bmatrix}$$

    - ★ Orthonormal base: N orthonormal signals (Modulator)

$$\{\phi_0(t), \phi_1(t), \cdots, \phi_{N-1}(t)\}, \quad \mathcal{E}\{\phi_j(t)\} = 1, \quad \int_{-\infty}^{\infty} \phi_j(t) \cdot \phi_k^*(t) \, dt = 0, \text{ if } k \neq j$$

    - ★ Definition of signals in this discrete representation

$$s_i(t) = \sum_{j=0}^{N-1} a_{i,j} \cdot \phi_j(t), \; 0 \leq t < T$$

Basic digital communication model

- Two step conversion
  - Encoder: Converts each m bits block (b_i) in a point in a N-dimensional space ($\boldsymbol{a}_i$)
    - ★ Energy and similarity can be measured from discrete representation

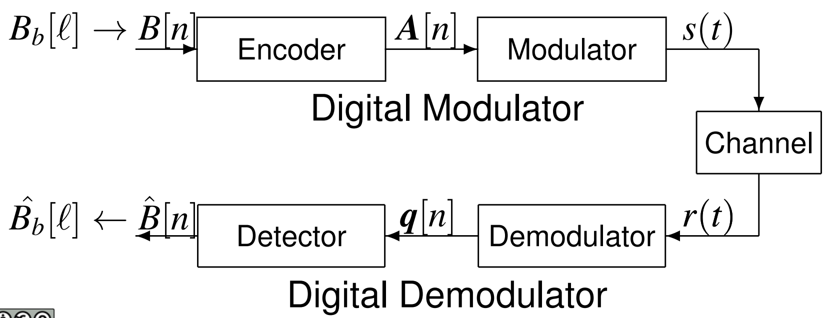
$$\mathcal{E}\{s_i(t)\} \equiv \mathcal{E}\{\boldsymbol{a}_i\} = ||\boldsymbol{a}_i||^2$$

$$d(s_i(t), s_k(t)) \equiv d(\boldsymbol{a}_i, \boldsymbol{a}_k) = ||\boldsymbol{a}_i - \boldsymbol{a}_k||$$

  - Modulator: Generates the waveform associated to each symbol ($s_i(t)$) by using the base
    - ★ Adaptation to the channel requires adaptation of each element in the base

$$\text{Ideal adaptation : } \phi_j(t) * h(t) = \phi_j(t), \ \forall j$$

- Basic digital communication model



Encoder

- Converts a sequence of bits, $B_b[\ell]$, in a sequence of discrete representation of signals (symbols), $A[n]$
  - Possible values for $A[n]$: Constellation: $\{\boldsymbol{a}_i\}_{i=0}^{M-1}$, $\boldsymbol{a}_i \in \boldsymbol{R}^N$
- $M = 2^m$ symbols - $m = \log_2 M$ bits per symbol
- Binary rate: R_b - Symbol (baud) rate: R_s

$$R_b = \frac{1}{T_b} \text{ bits/s - } R_s = \frac{1}{T} \text{ symbols/s (bauds) - } R_b = m \cdot R_s$$

- Constellation design constraints: P_e, BER, E_s
  - Performance: distance between symbols (minimum distance)

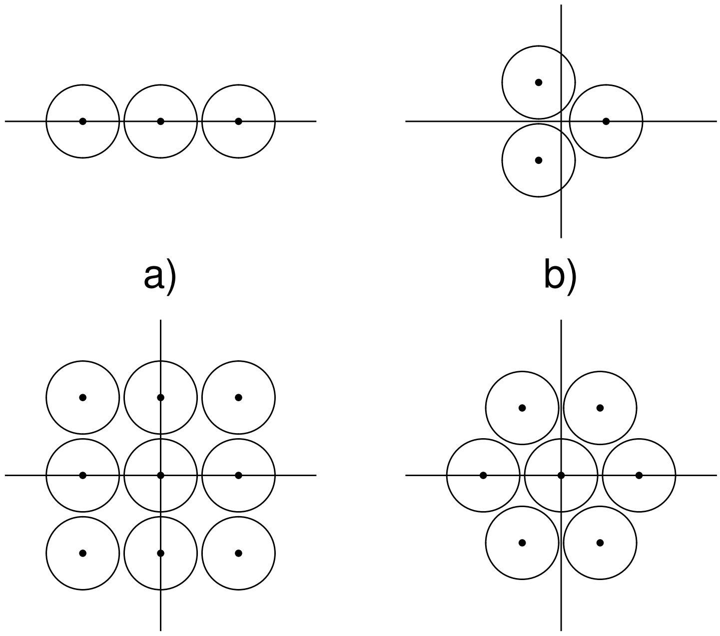
$$d(\boldsymbol{a}_i, \boldsymbol{a}_j) = \sqrt{\sum_{k=0}^{N-1} (a_{i,k} - a_{j,k})^2}$$

  - Symbol energy: squared norm

$$\mathcal{E}\{\boldsymbol{a}_i\} = ||\boldsymbol{a}_i||^2 = \sum_{k=0}^{N-1} (a_{i,k})^2$$

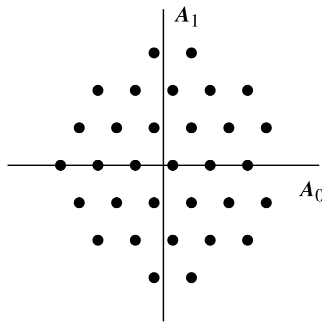
Encoder (II)

- Design constraints: P_e, BER, E_s
- Optimal design: Sphere packing a), b), d)
  - Provides minimum distance with symbols being as close as possible to the origin



Encoder (III)

- Sphere packing
  - Optimal: minumum P_e for a given E_s
  - Hexagonal constellations

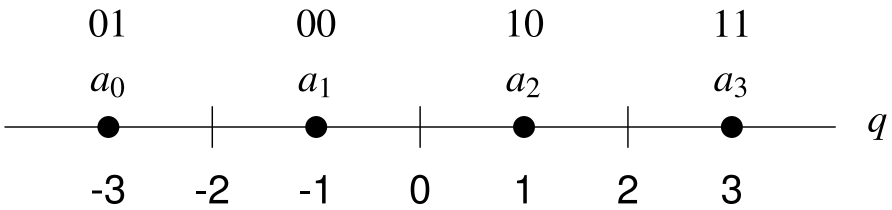


- Practical considerations
  - Simplicity of the transmitter
  - Peak eenrgy limitation
  - Peak to average power ratio
  - Simplicity of the receiver

  $\Rightarrow$ Constellations: QAM, PSK, unipolar, orthogonal, $\cdots$
- Bit assignment
  - M symbols $\rightarrow m = \log_2 M$ bits/symbol
  - Gray coding (minimizes BER)

Gray coding

- The m bits assigned to adjacent symbols (at minimum distance) differ in only one bit



  - For high signal to noise ratio

$$BER \approx \frac{1}{m} \cdot P_e$$

$m = \log_2(M)$: number of bits per symbol

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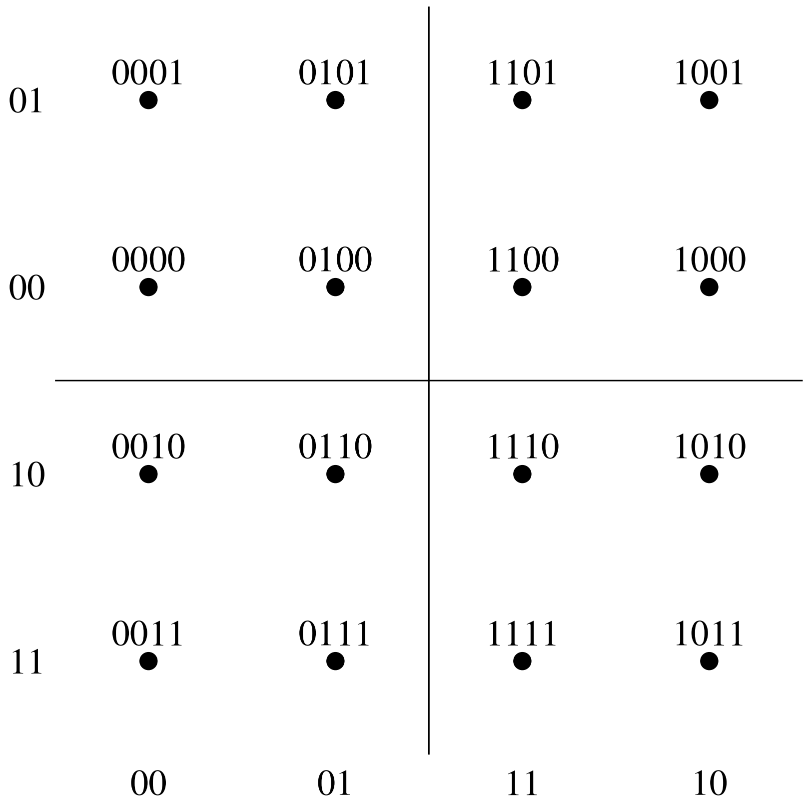
Gray code for QAM



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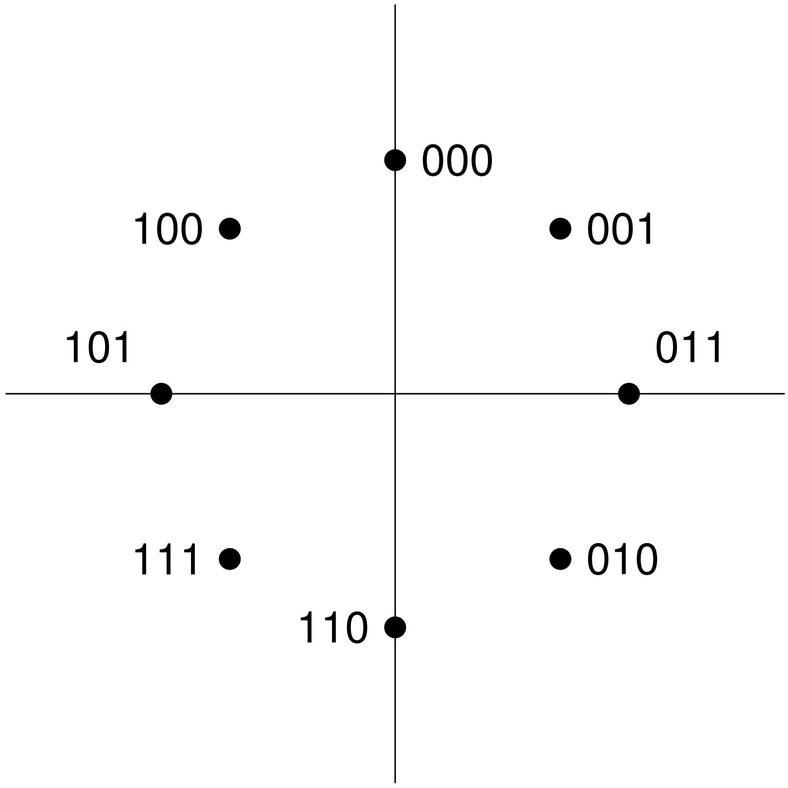
Gray code for PSK



Modulator

- Transforms symbols to analog (continuous) waveforms
  - Determines the spectral characteristics of the signal
  - Requirement: to be well-matched with channel characteristics
- Orthonormal basis in a signal space of dimension N

$$\{\phi_j(t)\}, \quad j = 0, \cdots, N-1$$

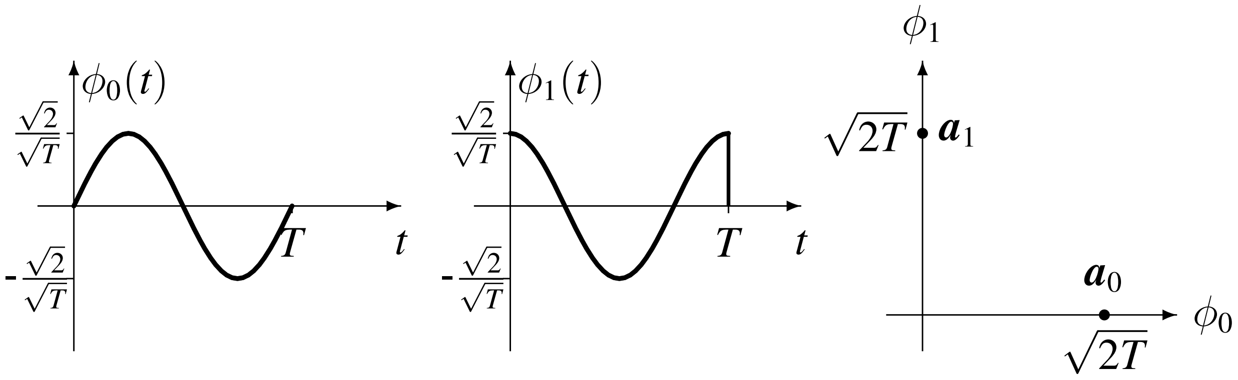
  Simplest approach: time-limited (to symbol period T) waveforms $\rightarrow \phi_j(t)$ defined on $0 \leq t < T$
  - Example $N = 2$

$$\phi_0(t) = \sqrt{\frac{2}{T}} \sin\left(\frac{2\pi}{T}t\right), \quad \phi_1(t) = \sqrt{\frac{2}{T}} \cos\left(\frac{2\pi}{T}t\right)$$

- Generation of the waveform associated to symbol $\boldsymbol{a}_i$ (T seconds)

$$s_i(t) = \sum_{j=0}^{N-1} a_{i,j} \cdot \phi_j(t), \; 0 \leq t < T$$

Base and constellation - Example $N = 2$



$$\boldsymbol{a}_0 = \begin{bmatrix} a_{0,0} \\ a_{0,1} \end{bmatrix} = \begin{bmatrix} \sqrt{2T} \\ 0 \end{bmatrix}, \quad \boldsymbol{a}_1 = \begin{bmatrix} a_{1,0} \\ a_{1,1} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2T} \end{bmatrix}$$

$$s_0(t) = a_{0,0} \cdot \phi_0(t) + a_{0,1} \cdot \phi_1(t)$$

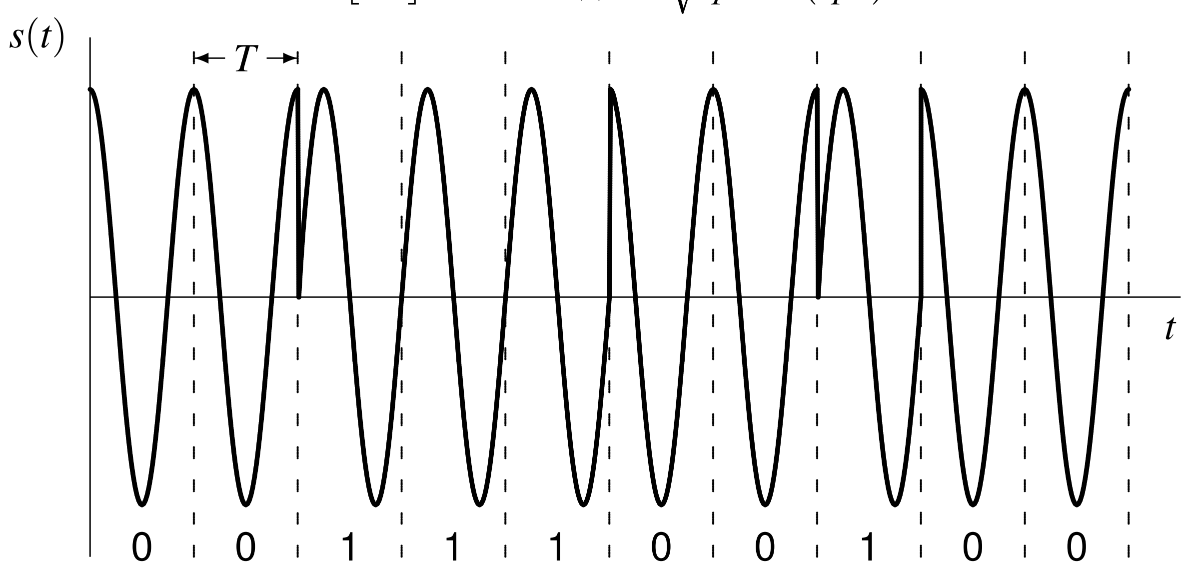
$$s_1(t) = a_{1,0} \cdot \phi_0(t) + a_{1,1} \cdot \phi_1(t)$$

Modulator - continuous transmission

$$s(t) = \sum_n \sum_{j=0}^{N-1} A_j[n] \cdot \phi_j(t - nT)$$

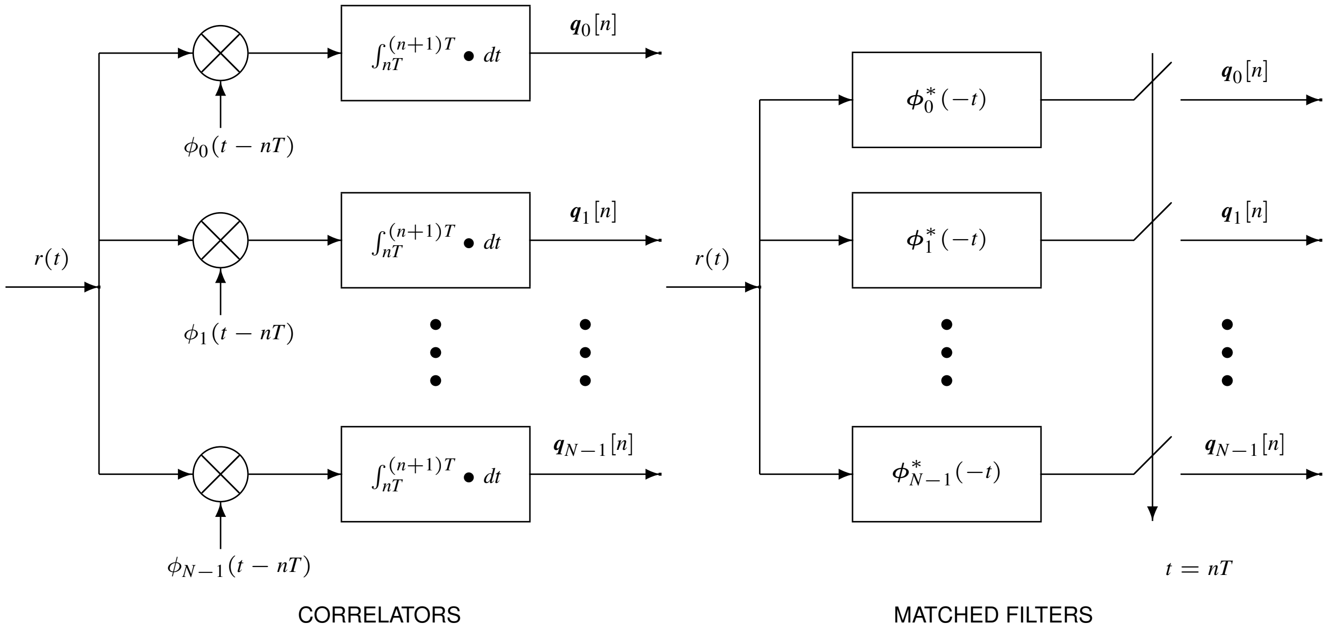
$$1 \rightarrow \boldsymbol{a}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow s_0(t) = \sqrt{\frac{2}{T}} \sin\left(\frac{2\pi}{T} t\right), \quad 0 \leq t < T$$

$$0 \rightarrow \boldsymbol{a}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow s_1(t) = \sqrt{\frac{2}{T}} \cos\left(\frac{2\pi}{T} t\right), \quad 0 \leq t < T$$



Demodulator

- Discrete time representation of the received continuous waveform



Detector

- Decision regions: $\hat{B} = b_j$ si $\boldsymbol{q}_0 \in I_j$
- Minimization of the probability of symbol error P_e
  - Assignment of $\boldsymbol{q}_0$: decision region of symbol maximizing posterior probability $p_{B|\boldsymbol{q}}(b_j|\boldsymbol{q}_0)$
    - ★ Symbol $\boldsymbol{a}_j$ maximizing $p_A(\boldsymbol{a}_j) \cdot f_{\boldsymbol{q}|A}(\boldsymbol{q}_0|\boldsymbol{a}_j)$ (MAP criterion)
  - Symbol equiprobability
    - ★ Symbol $\boldsymbol{a}_j$ maximizing $f_{\boldsymbol{q}|A}(\boldsymbol{q}_0|\boldsymbol{a}_j)$ (ML criterion)
- Gaussian channel model

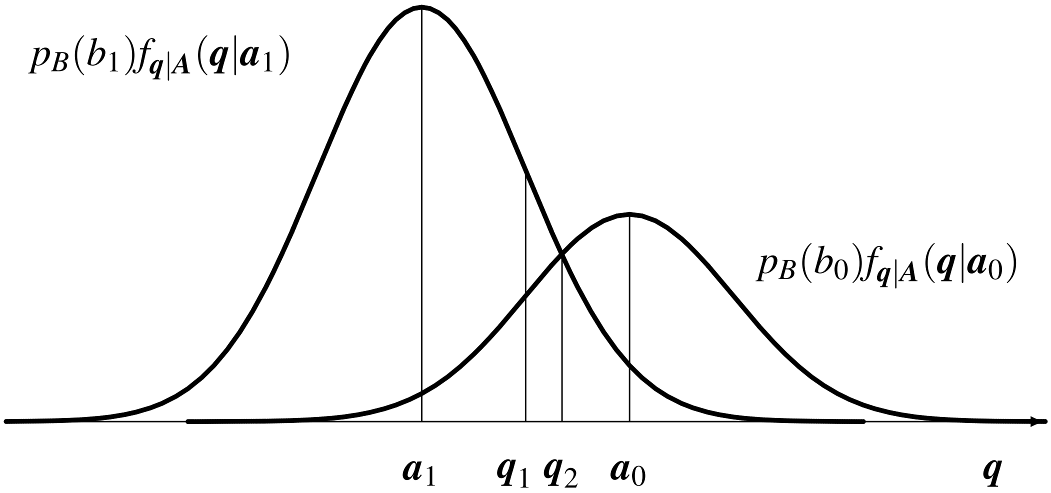
$$\boldsymbol{q}[n] = \boldsymbol{A}[n] + \boldsymbol{z}[n]$$

Observation is equal to the transmitted symbol plus noise

  - Assumes perfect channel adaptation (ideal choise of the orthonormal base $\{\phi_j(t)\}_{j=0}^{N-1}$
  - Noise $z[n]$: N-dimensional Gaussian distribution

$$f_{\boldsymbol{q}|A}(\boldsymbol{q}|\boldsymbol{a}_i) = \frac{1}{(\pi N_o)^{N/2}} e^{-\frac{||\boldsymbol{q}-\boldsymbol{a}_i||^2}{N_0}}$$

MAP criterion



$$I_1 = (-\infty, q_2), \quad I_0 = [q_2, \infty)$$

$$p_B(b_0) < p_B(b_1) \Rightarrow d(q_2, a_0) < d(q_2, a_1)$$

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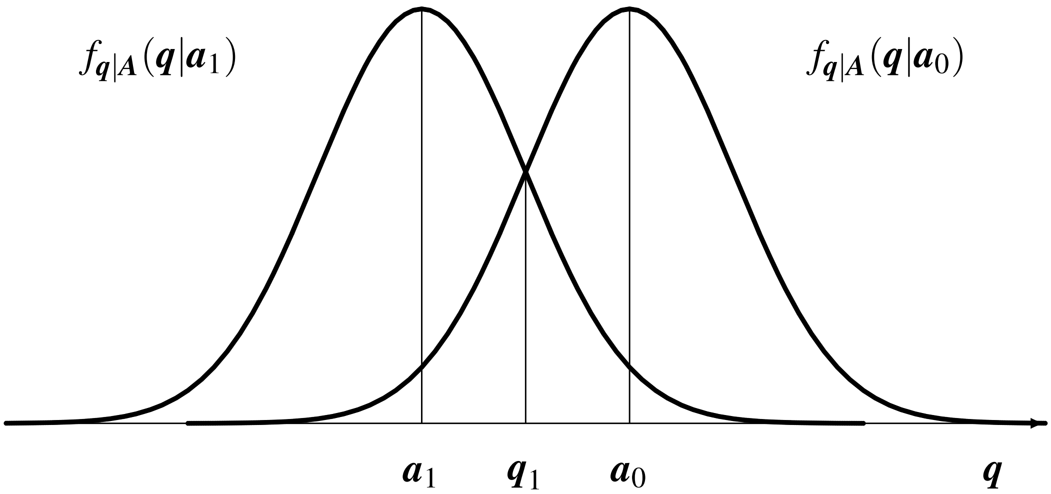
ML criterion (a_i equiprobable)



$$q_1 = \frac{a_0 + a_1}{2}, \quad I_1 = (-\infty, q_1), \quad I_0 = [q_1, \infty)$$

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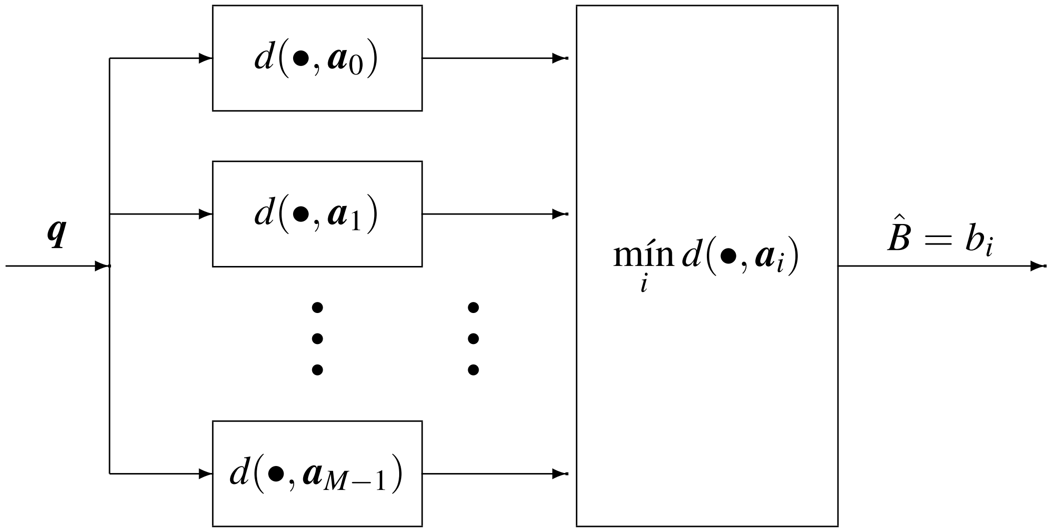
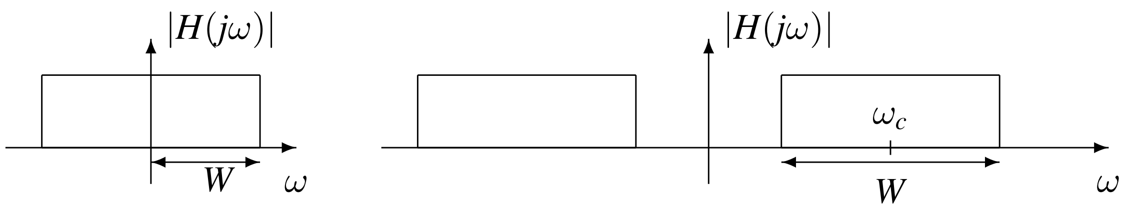
Minimum distance detector

- Gaussian channel (Gaussian noise)
- Symbol equiprobability



Characteristics of real channels

- Limited bandwidth
  - The assigned channel typically has a limited available bandwidth (B Hz, $W = 2\pi B$ rad./s)
    - ★ Baseband channels
    - ★ Bandpass channels (central frequency ω_c rad./s)
  - Signals limited in time ($\phi_j(t)$ of length T) are not appropriate



- Introduction of distortions (non ideal channels)
  - Noise (Gaussian)
  - Linear distortion: linear time invariant (LTI) model: $h(t)$, $H(j\omega)$

$$\boldsymbol{q}[n] \neq \boldsymbol{A}[n] + \boldsymbol{z}[n]$$

  - Non linear distortion (not considered here): intermodulation distortion (IMD)

Main objectives of *Digital Communications*

- To extend the basic digital communication model to consider these realistic constraints introduced by real channels
  - To analyze the mechanisms that are necessary to generate band-limited signals
    - Baseband
    - Bandpass
  - To analyze the effect of linear distortion and the necessary mechanisms to handle it at the receiver
    - Optimum receiver
    - Sub-optimum receivers (with lower implementation requirements)
  - To analyze techniques allowing to control the probability of error of the system