Exercises for chapter 2

1. A baseband PAM system uses as receiver filter $f(t)$ a matched filter, $f(t) = g(-t)$, having two choices for transmission filter $g(t)$

$$g_a(t) = \frac{1}{\sqrt{T}}\Pi\left(\frac{t}{T}\right) = \begin{cases} \frac{1}{\sqrt{T}}, & |t| \leq \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases},$$

and

$$g_b(t) = \frac{1}{\sqrt{T}}\text{sinc}\left(\frac{t}{T}\right)$$

where T is the symbol period.

   a) Compute the equivalent discrete channel $p[n]$ condidering both transmitter filters, and demonstrate that the system fulfills the Nyquist ISI criterion when transmission is performed through a Gaussian channel (linear distortion is negligible).

   b) Represents the spectral power density of the transmitted signal, and of the received signal (at the output of the receiver filter) when $A[n]$ is white and distortion and noise are negligible.

   c) For a 2-PAM constellation, represent the eye diagram at the output of the receiver filter. The distortion and noise introduced by the channel can be considered negligible.

   d) For a 4-PAM constellation, represent the eye diagram at the output of the receiver filter. The distortion and noise introduced by the channel can be considered negligible.

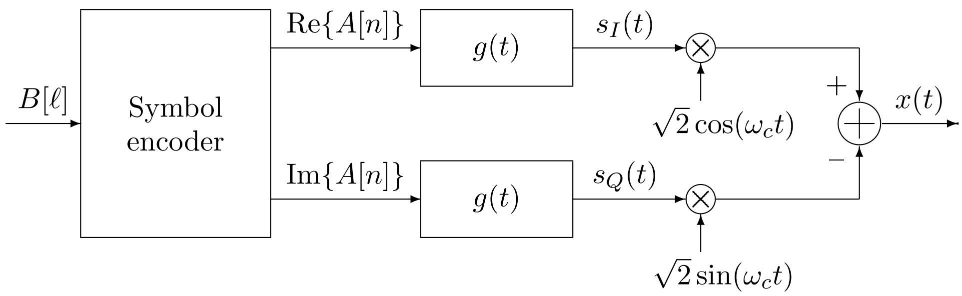
   e) If now transmission is performed through a linear channel with impulse response $h(t) = \delta(t) + \frac{1}{3} \cdot \delta\left(\frac{T}{2}\right)$, the equivalent discrete channel $p[n]$ condidering both transmitter filters, discuss about ISI, and compare the length of $p[n]$ in both cases.

2. A 2-PAM constellation is transmitted through the equivalent discrete channel

$$p[n] = \delta[n] + \frac{1}{2} \cdot \delta[n-1] + \frac{1}{4} \times \delta[n-2].$$

The discrete time noise $z[n]$ is white, Gaussian, with variance σ_z^2. Calculate the exact expression of the probability of error with a memoryless symbol-by-symbol detector.

3. A digital communication system uses the following modulator:



The system works in 1-4 kHz bandwidth. In this bandwidth the channel behaves like a AWGN. Design the transmitter: symbol encoder (using a QAM constellation), shaping filter at the transmitter $g(t)$ and carrier frequency ω_c to make a transmission without ISI to a binary rate of 9600 bits/s and using the whole bandwidth.

4. A baseband communication system uses a BPSK constellation, $A[n] \in [\pm 1]$, and the following shaping filter

$$g(t) = \begin{cases} \frac{1}{\sqrt{T}}, & -\frac{T}{2} < t < 0 \\ -\frac{1}{\sqrt{T}}, & 0 \leq t < \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}$$

The modulated signal is transmitted through a linear channel with impulse response

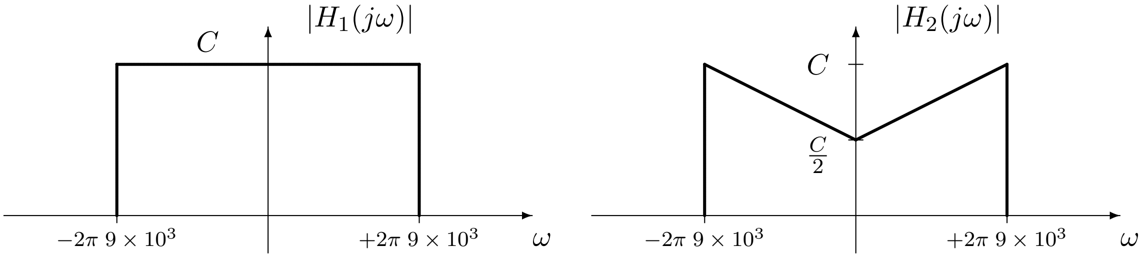
$$h(t) = \delta(t) + \delta(t - T/2),$$

and the receiver uses a matched filter. Noise at the input of the receiver is white, Gaussian, with power spectral density $N_0/2$ W/Hz.

a) Calculate the equivalent discrete channel $p[n]$.

b) Obtain the power spectral density of the discrete time noise $z[n]$ present at the output of the sampler at the receiver, explaining the procedure to obtain the result.

5. A baseband transmission system sends the modulated signal through one of these channels. The receiver filter will be matched to the transmitter filter.



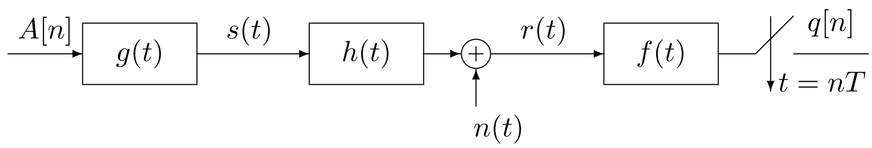
a) Design for Channel 1 the shaping filters at the transmitter and receiver in order to get no ISI and the noise at the output of the sampler be white.

b) Design for Channel 2 the shaping filters at the transmitter and receiver in order to get no ISI.

c) Design for Channel 2 the shaping filters at the transmitter and receiver in order to get white noise at the output of the sampler.

d) Get the maximum symbol rate in both channels.

6. Next figure shows the block diagram for a baseband PAM system. There, $A[n]$ is the sequence of transmitted symbols, ($A[n] \in \{\pm 1\}$), $g(t)$ is a squared-root raised cosine filter, $h(t)$ is the channel impulse response, $n(t)$ is the AWGN with PSD $N_0/2$, $f(t)$ is the receiver filter, T is the symbol period and $q[n]$ are the samples at the output of the receiver.



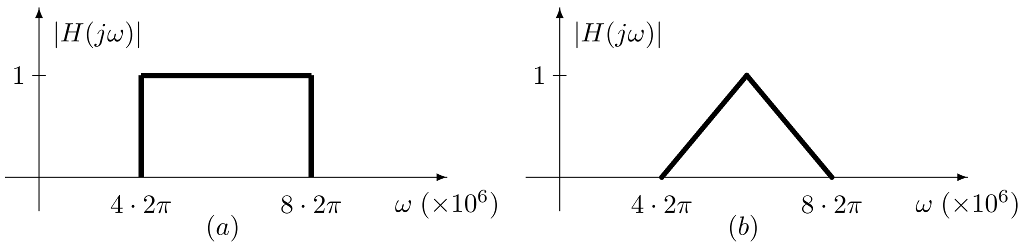
a) If $f(t)$ is designed for $k(t) = g(t) * f(t)$ to match Nyquist criteria, get $f(t)$ as a function of $g(t)$ and get the PSD for the discrete filtered noise $z[n]$.

b) If $h(t) = \delta(t) - \frac{1}{10}\delta(t-2T)$, get the impulse response of the discrete equivalent channel $p[n]$.

c) Under (b) conditions, is there ISI in the system?

d) Assuming $g(t) = f(t) = 1/\sqrt{T}$ if $|t| \leq T/2$ and $f(t) = g(t) = 0$ otherwise (that is, $f(t)$ and $g(t)$ are identical normalized squared pulses defined in $[-T/2, T/2]$), get the eye diagram in the absence of noise.

7. Consider the following frequency response of two different communication channels.



a) Show if it is possible, using a QAM modulation a transmission without ISI and with white noise at the output of the sampler if we use in the receiver a matched filter to the transmitter. If your answer is positive, get the maximum transmission rate and get the shaping pulses that you would use on channel in fig. (a) and on channel in fig. (b).

b) You would like to transmit to a 10 Mbits/s bit rate using a PSK modulation over channel in fig. (a) with squared-root raised cosine filters in the transmitter and receiver. Get the minimum number of symbols needed M in the PSK modulation and the obtained symbol rate.

c) Given the constellation of previous section, obtain the feasible range of values for the roll-off factor α of the shaping filters taking into account the available bandwidth and from the range of α values get the one minimizing the effect of deviations from optimal sampling instants at the receiver.

8. A communication system uses a squared-root raised cosine filter in the transmitter for a baseband PAM modulation with roll-off factor α. In the receiver there is a matched filter to the transmitter. Assume that the channel is AWGN with an impulse response $h(t)$ and noise PSD $N_0/2$. The channel bandwidth is 4 kHz.

a) Show if the sampled noise at the output of the matched filter is white.

b) Get the maximum symbol transmission rate without ISI and get the roll-off factor needed for this rate.

c) Draw the PSD of the transmitted signal in these two cases:

i) Sequence $A[n]$ is white with mean symbol energy E_s.

ii) Sequence $A[n]$ has a PSD $S_A(e^{j\omega}) = 1 + 1 \cdot \cos(\omega)$.

d) If the roll-off factor used is $\alpha = 0.25$ transmitting at the maximum symbol rate possible without ISI, get the number of symbols M needed to get a binary rate of 19200 bits per second.

9. A digital communications system uses as transmitter filter $g(t)$ a root-raised cosine pulse with roll-off factor α. The receiver employs a matched filter.

a) If the transmission is performed through the linear channel with response $h(t) = \delta(t) + \frac{1}{4} \cdot \delta(t-2T)$, calculate the equivalent discrete channel.

b) If the channel is a baseband channel with bandwidth $B = 10$ kHz, and the desired binary rate is 54 kbits/s, using a baseband M-PAM

(i) Calculate the minimum order of the constellation (number of symbols M) allowing to achieve the desired rate.

(ii) Calculate the symbol rate, R_s, which is necessary to obtain such binary rate with this constellation.

(iii) Calculate, for this M, the value of α that allows to completely fill the available bandwidth.

c) Repeat the previous question if the channel is a bandpass channel and the modulation is a bandpass PAM using a M-QAM constellation.

10. A linear baseband modulation uses a normalized rectangular pulse of duration T. This modulation is transmitted through a linear channel with impulse response $h(t) = \delta(t) - 0.5\delta\left(t - \frac{T}{2}\right)$.

In the receiver, we consider two different scenarios. In the first case, the receiver employs a matched filter to the transmitter. In the second case, the receiver employs a matched filter to the rectangular pulse shown in Figure 1.

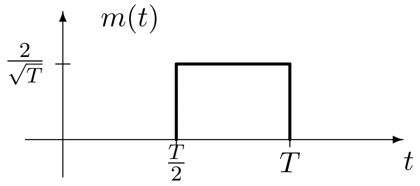


Figure 1: Rectangular pulse corresponding to exercise number 13.

a) Calculate the equivalent discrete channel in both cases.

b) In the second scenario, analyze if the sampled noise at the output of the (second) matched filter is white.

c) Explain, from the point of view of the ISI and sampled noise at the output of the receiver, what is the best option for the receiver.

11. A digital communication system has assigned to its use the frequency range between 30 and 40 MHz. A M-QAM modulation will be used. Both transmitter and receiver will employ root-raised cosine filters with roll-off factor α.

a) Obtain the maximum symbol rate allowing to transmit without intersymbol interference (ISI), and determine the value for α that is used to achieve such maximum rate.

b) If a transmission rate of 36 Mbits/s is desired, obtain the minimum constellation order M (number of symbols in the M-QAM constellation) that is required.

c) When transmitting at the maximum symbol rate without ISI, plot the power spectral density of the transmitted signal in two cases:

(i) Sequence of data, $A[n]$, is white.

(ii) Sequence of data, $A[n]$, has the following autocorrelation function

$$R_A[k] = 2 \cdot \delta[k] + \delta[k-1] + \delta[k+1].$$

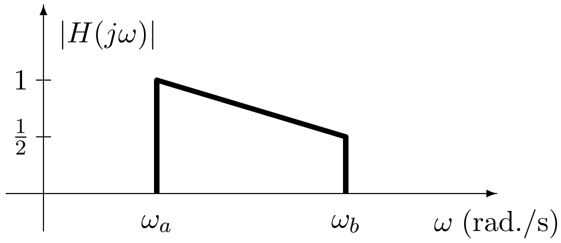
12. A digital communication system has been assigned the frequency range of 10 - 15 MHz. The modulation that will be used is a 16-QAM.

    a) If the transmitter uses a square-root raised-cosine (SRRC) shaping pulse with a roll-off factor of $\alpha = 0.25$, the receiver is a matched filter to the transmitter and assuming that the channel frequency response is flat in the rage of frequencies used for the transmission:

        i) Get the maximum symbol rate and the maximum binary rate without ISI.

        ii) Get the power spectral density of the modulated signal $x(t)$ if the information sequence $A[n]$ is white.

    b) If in the range of frequencies assigned the channel behaves as in next figure (with $\omega_a = 2\pi \times 10 \times 10^6$ and $\omega_b = 2\pi \times 15 \times 10^6$) and the transmit and receive filter are as defined before:



        i) Show if it is possible or not the transmission without ISI.

        ii) Discuss if the discrete noise at the output of the receiver $z[n]$ is white. Explain your answer.

    c) For the channel of previous section and still assuming that the receiver is a matched filter to the transmitter:

        i) Get the transmitter filter so that there is no ISI. The filter can be given in the time domain $g(t)$ or in the frequency domain $G(j\omega)$.

        ii) Discuss if in this case the discrete noise at the output of the receiver is white or not.

13. A digital communication system uses a causal square pulse of length T that is normalized in energy. The receiver uses a matched filter (matched to $g(t)$). The modulated signal is transmitted through a channel whose complex equivalent baseband response is:

$$h_{eq}(t) = \delta(t) + j\delta\left(t - \frac{T}{2}\right).$$

    a) Without taking into account the channel effect (i.e., $h_{eq}(t) = \delta(t)$), do the selected transmitter and receiver filters fulfill the ISI Nyquist criterion?

    b) Obtain the equivalent discrete channel and the constellation at the receiver when the transmitted constellation is an orthogonal constellation with symbols $A[n] \in \{+1, +j\}$.

    c) Repeat the previous section if now

$$h_{eq}(t) = j\delta(t - T).$$

    Explain if in that case ISI will be present or not.

14. Two digital communication systems are available. The first one is a baseband system and the second one is a bandpass system. The available range of frequencies for the first system is between 0 and 20 kHz, and the constellation is a M-PAM. The second system has been allotted the frequency range between 20 and 40 kHz, and uses a M-QAM constellation. In both systems, transmitter and receiver filters will be matched, and the transmitter filter is a root-raised cosine filter with roll-off factor α.

a) Obtain the maximum symbol rate that can be achieved in a transmission without intersymbol interference (ISI) if the channel has an ideal behavior in its specified frequency band. Indicate the value or set of values of α that can be used to obtain such maximum rate:

   i) In the baseband system.

   ii) In the bandpass system.

b) If a roll-off factor $\alpha = 0.25$ is used, represent the power spectral density of the transmitted signal, properly labeling each axis of the picture:

   i) In the baseband system, using a 2-PAM constellation.

   ii) In the bandpass system, using a 4-QAM constellation.

c) In the bandpass system, if you pretend an ISI free transmission at binary rate of 64 kbits/s:

   i) Select the carrier frequency, ω_c, that you would use for transmission.

   ii) Obtain the minimum required constellation order (number of symbols, M, in the constellation) that allows to transmit at the specified binary rate.

   iii) Obtain the symbol rate used to transmit at the required binary rate when the constellation obtained in the previous section is used.

