Exercises for chapter 5

1. Consider a discrete time OFDM modulation with $N = 4$ carriers.

   a) Assuming that cyclic prefix is not used, find the conditions in order to avoid ICI and ISI.

   b) Using a cyclic prefix of length $C = 2$, and assuming that the discrete channel at sampling time $T/(N+M)$ is $d[m] = \delta[m] + \frac{1}{3}\delta[m-2]$, obtain the discrete equivalent channels $p_{k,i}[n]$. From this result, discuss and explain whether there is ICI and ISI or not.

2. Consider an OFDM modulation with $N = 4$ carriers and a cyclic prefix of 2 samples. Each carrier is modulated with equiprobable QPSK symbols. Moreover, assume the following equivalent discrete channel, sampled at time $T/6$,

   $$d[m] = \delta[m] - \frac{1}{3}\delta[m-1],$$

   with additive Gaussian complex white noise (the variance is N_0). In the receiver, a ML decoder, designed for a QPSK modulation, is used.

   a) Obtain the equivalent discrete channels $p_{k,i}[n]$ corresponding to the 16 sub-channels.

   b) Obtain the SNR for each carrier.

   c) Determine the probability of error per carrier and the mean probability of error.

3. A communication system uses a discrete time OFDM modulation with an even number N of carriers.

   a) Assume that the cyclic prefix is not used. The baseband equivalent discrete channels, sampled at time T, are denoted as $p_{k,i}[n]$ and $d[m]$ is the equivalent discrete channel sampled at time T/N.

      i) Write and discuss about the conditions that $p_{k,i}[n]$ should have to fulfill to avoid inter-symbol interference (ISI).

      ii) Write and discuss about the conditions that $p_{k,i}[n]$ should have to fulfill to avoid inter-carrier interference (ICI).

   b) Assume now that a cyclic prefix is used.

      i) Determine the length of the cyclic prefix needed to eliminate the interference for a given channel $d[m]$.

      ii) What is the loss in transmission rate compared to the system without cyclic prefix.

4. A communication system uses a direct sequence spread spectrum modulation with spreading factor $N = 4$, and spreading sequence $x[0] = +1$, $x[1] = -1$, $x[2] = +1$ and $x[3] = -1$. The transmitter filter at chip time $g_c(t)$ is a root-raised cosine pulse with roll-off factor $\alpha = 0.25$. In the receiver a matched filter to $g_c(t)$ is employed. The continuos time channel impulse response is $h(t) = \delta(t) + \frac{1}{2}\delta\left(t - \frac{T}{2}\right)$, hence the discrete equivalent channel at chip time is $d[m] = \delta[m] + \frac{1}{2}\delta[m-2]$.

   a) Explain how the samples $s[m]$ are generated, given the symbols $A[n]$ and the spread sequence $x[m]$, and then calculate the values of $s[m]$ for $0 \leq m \leq 11$ when $A[0] = +1$, $A[1] = -1$, $A[2] = -1$.

   b) Obtain the sequence $v[m]$ at the output of the receiver filter $g_c(-t)$ when $A[0] = +1$, $A[1] = -1$, $A[2] = -1$ when the spread spectrum signal is transmitted through $h(t)$ and in the absence of noise (assume $A[n] = +1$ for $n < 0$).

c) Explain how the observations at symbol time $q[n]$ are obtained given the sequences $v[m]$ and $x[m]$. For that you can either plot the receiver diagram or explain in a detailed form the process. Calculate the values $q[n]$ for $0 \leq n \leq 2$ in the absence of noise.

5. A communication system uses a direct sequence spread spectrum modulation with spreading factor $N = 5$. The symbol sequence $A[n]$ is white with mean energy E_s. Finally, the spreading sequence is

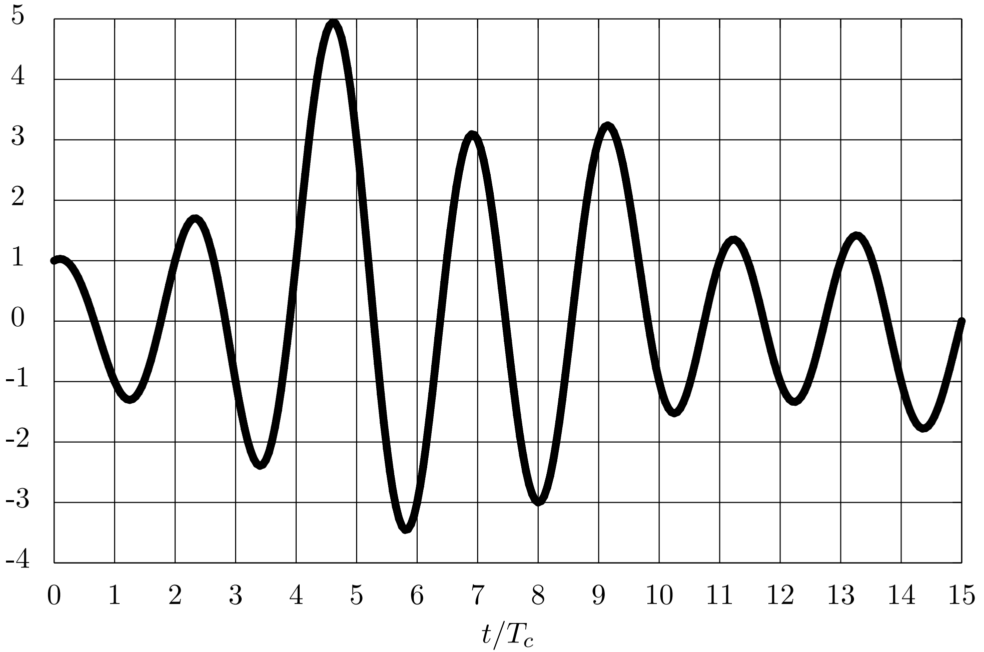
$$x[0] = +1,\ x[1] = -1,\ x[2] = +1,\ x[3] = -1,\ x[4] = +1.$$

a) If a causal rectangle pulse of duration T_c (with normalized energy) is used as a shaping filter, represent the modulated signal corresponding to the sequence

$$A[0] = +1,\ A[1] = +3,\ A[2] = -1.$$

b) Get the analytic expression of the power spectral density of the baseband signal, $S_s(j\omega)$, if the filter used at chip time is $g_c(t) = \frac{1}{\sqrt{T_c}} \cdot \operatorname{sinc}\left(\frac{t}{T_c}\right)$.

c) Obtain the values $q[n]$ for $0 \leq n \leq 2$ if the output signal of the matched filter $g_c(t)$, $v(t)$, is shown in the following figure (note that the horizontal axes is scaled by T_c).



6. An OFDM modulation uses a bandwidth of 4 kHz in the band of 5 kHz - 9 kHz. This modulation is used to provide a wireless communication service to a certain number of users that goes from 4 to 10 users. The communication system is composed of a base station (transmitter) that sends the OFDM modulation and a certain number of receivers (one for each user that is being served) that are physically separated. The information sequence addressed to each one of the users is sent in each of the carriers that defines the OFDM modulation.

a) Get the maximum and minimum service rate (symbol rate) that each user could get if the modulation is not using cyclic prefix. Take into account that the rate will depend on the number of users that are being served.

b) Assuming that we are giving service to 4 users ($N = 4$) and that the binary rates required by each user are $R_{u0} = 8$ kbit/s, $R_{u1} = 4$ kbit/s, $R_{u2} = 2$ kbit/s and $R_{u3} = 1$ kbit/s, get the modulation order that each user will need.

c) The discrete time OFDM signal $s[m]$ that transports the information of the 4 users is transmitted through the wireless channel. Each user i receives the signal $s[m]$ though a different channel $d_i[m]$ $(i = 0 \dots 3)$ due to the different propagation channels:

$$d_i[m] = \delta[m] + a_i\delta[m-1] \text{ if } i = 0, 1$$
$$d_i[m] = \delta[m] + a_i\delta[m-2] \text{ if } i = 2, 3$$

Get the length of the cyclic prefix that should be added to the signal $s[m]$ so that all the users can recover their information with no ISI and ICI.

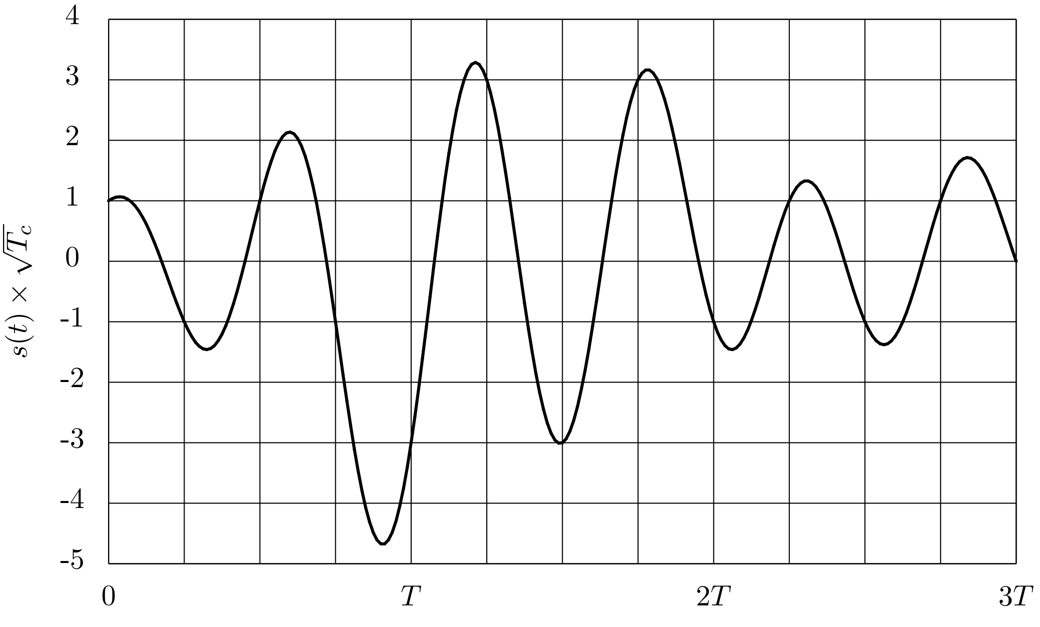
d) Design the demodulator that each user should have to recover their information sequence.

7. A direct-sequence spread spectrum system with spreading factor of $N = 4$, has a spreading sequence as

$$x[m] = \delta[m] + a \cdot \delta[m-1] + b \cdot \delta[m-2] + c \cdot \delta[m-3],$$

with $\{a, b, c\} \in \{\pm 1\}$. The transmit filter at chip time, T_c, is $g_c(t) = \frac{1}{\sqrt{T_c}}\text{sinc}\left(\frac{t}{T_c}\right)$.

a) If the transmission of the information symbols, $A[n]$, generates the baseband modulated signal, $s(t)$, that is shown in next figure



where T is the symbol time of the transmitted sequence, $A[n]$, and where we should notice that the signal amplitude, $s(t)$, is scaled by a factor of $\sqrt{T_c}$, get the values of a, b, and c, and the three initial values of the transmitted sequence $A[n]$.

b) The signal $s(t)$ is transmitted through an ideal channel, without noise and in the receiver is filtered with a matched filter to the transmitter at chip time, that is, $f(t) = g_c(-t)$. The output of this filter is $v(t)$. Get $v(t)$ from $s(t)$, draw the block diagram of the spread-spectrum receiver and compute the observations at the filter output $q[n]$, for $n \in \{0, 1, 2\}$.

HINT: for this transmission filter at chip time, we have $r_{g_c}(t) = g_c(t) * g_c(-t) = \text{sinc}\left(\frac{t}{T_c}\right)$.

8. Consider a direct sequence spread spectrum modulation with spreading factor $N = 10$, $\tilde{x}[m] = (-1)^m$, and $g_c(t) * g_c^*(-t)$ fulfills the Nyquist ISI criterio at chip period, $T_c = T/N$. The equivalent baseband channel is $h_{eq}(t) = \delta(t-\tau)$. Determine the equivalent discrete channel, $p[n]$ (if it nos possible to symplify the resulting expression, provide it in terms of $g_c(t)$). Discuss the existence of intersymbol interference (ISI) in the following cases:

a) $\tau = T$

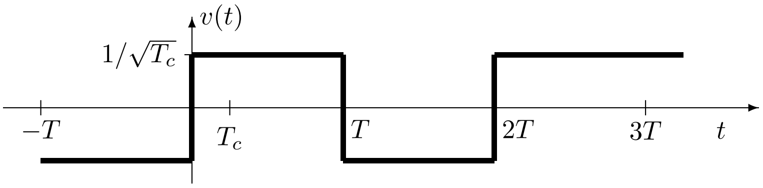
b) $\tau = T/2$

c) $\tau = T/4$

9. A direct sequence spread spectrum modulation is used with a spreading factor of $N = 4$. The baseband shaping pulse used is as follows

$$g(t) = \sum_{m=0}^{N-1} x[m] g_c(t - mT_c),$$

where the spreading sequence used is $x[m] = +1, -1, +1, -1$, for $m = 0, 1, 2, 3$, and $g_c(t)$ is a causal pulse with duration T_c (chip length) and unit energy.



(a) Using the corresponding baseband spread spectrum receiver, get the output of the demodulator $q[n]$ for $n = 0, 1, 2$, if the receiver input $v(t)$ is the signal in previous figure.

(b) Get the discrete equivalent channel and the error probability for the conventional receiver if the transmitted constellation is a 2-PAM (or BPSK) $A[n] \in \{\pm 1\}$, and the channel is $h_{eq}(t) = \delta(t) + \delta(t - T/2)$, compare these results with the ones obtained by using an alternative sequence $x_r[m] = +1, -1, +1, +1$, in the receiver for the despreading. Is there ISI?

(c) For the previous sequence used in the transmitter $x[m]$ and the channel given in the previous section, is it possible to eliminate the ISI modifying the spreading sequence in the transmitter? If so, identify such a sequence.

10. A direct sequence spread spectrum modulation has a spreading factor of $N = 10$. A 2-PAM constellation with normalized levels, $A[n] \in \{\pm 1\}$, is transmitted. The equivalent discrete channel at chip period T_c is

$$d[m] = \delta[m] - 0.5\delta[m - 4]$$

and additive noise is white, Gaussian with power spectral density $\frac{N_0}{2}$.

The spreading sequence $x[m]$ is $\{-1, -1, +1, +1, -1, +1, +1 - 1, +1, +1\}$.

a) Plot the shaping pulse at symbol period, $g(t)$, if the shaping pulse at chip period, $g_c(t)$, is a causal and nomalized rectangular pulse.

b) Get the discrete equivalent channel at symbol time T. Determine if there is ISI and get the error probability if you used a memoryless symbol by symbol detector.