DIGITAL COMMUNICATIONS
THEORY
(Time: 60 minutes. Grade 4/10)

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First (Middle) Name:	2	
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Question 1

M-ary (with M symbols) frequency modulations CPFSK and MSK make use of pulses with the following shape

$$g_i(t) = \sin(\omega_i t) \cdot w_T(t), \text{ para } i = 0, 1, \cdots, M-1,$$

where $w_T(t)$ is a causal squared pulse of unit amplitude and duration T seconds. For a given communication system, the usable range for the frequencies of each pulse is limited between two frequencies, such that $\omega_a \leq \omega_i \leq \omega_b$, where $\omega_a = 2\pi f_a$ and $\omega_b = 2\pi f_b$, with $f_a = 950$ MHz and $f_b = 1250$ MHz.

For the case $M = 4$, obtain the maximum possible symbol rate, and the values of the four frequencies (ω_i or f_i, $i = 0, 1, 2, 3$) when using:

a) A MSK modulation.

b) A CPFSK modulation.

(1 point)

Question 2

A digital communication system has assigned for its use the band of frequencies (channel) between 800 MHz and 950 MHz. In this frequency band, the behavior of the channel is considered ideal (it does not introduce linear distortion, but only introduces white, Gaussian additive noise with power spectral density $N_0/2$). The transmitted modulation is a 16-QAM with normalized levels, and sequence $A[n]$ is white. The transmission filter is a square-root raised cosine with roll-off factor α. For the reception filter there are two possibilities:

- Filter $f_a(t)$, a square-root raised cosine with roll-off factor α.
- Filter $f_b(t)$ given by $f_b(t) = \frac{1}{\sqrt{T}}$ for $|t| \leq T/2$ and $f_b(t) = 0$ for $|t| > T/2$.

a) For $\alpha = 0.25$ and receiver filter $f(t) = f_a(t)$, obtain the maximum symbol rate and maximum bit rate, indicating the value for the carrier frequency ω_c that has to be used to achieve these rates transmitting in the specified frequency band.

b) For $\alpha = 0.25$, plot the power spectral density of the modulated transmitted signal $x(t)$. Proper labels, with numerical values, have to be included in both axes, and only the positive range of frequencies ($\omega \geq 0$ rad/s.) has to be plotted.

c) For $\alpha = 0$, demonstrate on one hand if there is or not intersymbol interference (ISI) in the transmission, and on the other hand if the sampled noise at the receiver, $z[n]$, is or not white, in the following cases:

   i) Receiver filter is $f(t) = f_a(t)$.

   ii) Receiver filter is $f(t) = f_b(t)$.

(1.5 points)

Question 3

With respecto to the multipulse modulations (OFDM and direct sequence spread spectrum), answer to the following questions:

a) Explain the motivation in both cases, OFDM and spread spectrum, to work in the transmitter in discrete time generating the sequence of samples $s[m]$. Present arguments individually for each case.

b) For channels $h_{eq}(t) \neq \delta(t)$ where the equivalent discrete channel at period $\frac{T}{N}$ introduces ISI (i.e., $d[m] \neq \delta[m]$) explain what desing options allow to eliminate ISI at symbol rate T. Again, explain the procedures individually for each case.

c) In a direct sequence spread spectrum modulation, with spreading sequence $x[m]$, the receiver filter at symbol period, $f(t)$, instead of being as usual a filter matched to the transmitter filter at symbol period $g(t)$, is a filter matched to the joint response of both the transmitter filter $g(t)$ and the channel, whose impulse response is $h_{eq}(t) = h_1\delta(t)+h_2\delta(t-\tau_2)$, with $\tau_2 = C\cdot T_c$, with C being and integer number. For this case, neglecting the noise effect, formulate mathematically how the observation at symbol rate, $q[n]$, can be obtained by processing samples at the receiver at chip rate T_c, providing expressions in terms of the spreading sequence $x[m]$, the transmitter filter at chip period, $g_c(t)$, and the received baseband signal $v(t)$.

(1.5 points)

DIGITAL COMMUNICATIONS
EXERCISES
(Time: 120 minutes. Grade 6/10)

Last Name(s): .... First (Middle) Name: .... ID Number: .... Group .... Signature	Grade	
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Exercise 1

In a digital communication system three linear block codes are available, and in some cases two codes can be concatened to improve the performance of individual codes.

First block code is a repetition code of rate 1/3. Second encoder is a systematic linear block code, systematic by the beginning (the first k bits of the n encoded bits replicate the k uncoded bits with information), whose encoded words of n bits are $\mathcal{C}_2 = \{0000, 1001, 0101, 0011, 1100, 1010, 0110, 1111\}$. Third linear block code has the following parity check matrix:

$$\mathbf{H}_3 = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

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a) Obtain the generating matrices for the three codes.

b) Obtain the detection and correction capabilities for code 2 and for code 3, individually, and discuss which one is better in terms of its correction capability.

c) Obtain the resulting encoded words for the concatanation of codes 1 and 2 (in this order), and provide the rate parameters (parameters k and n) for this concatenated code. Does this concatenation provide any advantage over the individual codes?

d) Obtain the generating matrix for the concatenated code obtained by the concatenation of codes 2 and 3 (in this order), and the rate parameters (parameters k and n) for this concatenated code. Compare the performance of this code with the one of the previous section.

e) Obtain the syndrome table for code 3, and decode, by providing the k uncoded information bits, explaining clearly each step of the symdrome based decoding technique, the following received word:

$$\boldsymbol{r} = 11111.$$

(3 puntos)

Exercise 2

A digital communication system transmits a 2-PAM modulation, $A[n] \in \{\pm 1\}$, through the following equivalent discrete channel

$$p[n] = \frac{3}{4} \cdot \delta[n] + \frac{5}{4} \cdot \delta[n-1].$$

Discrete-time noise sampled at the demodulator output, $z[n]$, is Gaussian with variance $N_0/2$. At the receiver, two possible configurations can be used, depending on if a linear equalizer, with discrete response

$$w[n] = -\frac{12}{25} \cdot \delta[n] + \frac{4}{5} \cdot \delta[n-1]$$

is or not introduced before the detector.

a) When the equalizer is NOT used, and if detector is a memoryless symbol-by-symbol detector, select the optimal delay for decision, d, and calculate the probability of error that is obtained with this optimal delay.

b) In the structure WITH EQUALIZER, if detector is a memoryless symbol-by-symbol detector, select the optimal delay for decision, d, and calculate the probability of error that is obtained with this optimal delay (for this simple case, the exact probability of error can be obtained, without requiring the typical approximation for linear equalizers).

c) If now detector is a maximum likelihood sequence detector, used with the structure WITH EQUALIZER, i.e., applied to the equalizer output, $u[n]$, plot the trellis diagram for this detector, and obtain the approximated probability of error assuming that the sequence $A[n] = +1, \forall n$ has an erroneous event at minimum euclidean distance.

d) In the structure WITH EQUALIZER and maximum likelihood sequence detector, obtain the maximum likelihood sequence of length $L = 3$ symbols, $\mathbf{A} = [A[0], A[1], A[2]]$, by applying the optimal deconding algorith if the sequence that is received at the output of the equalizer is

$$u[0] = +1.11, \; u[1] = +1, \; u[2] = +1.26, \; u[3] = -1.16, \; u[4] = -1$$

by assuming that between each block of $L = 3$ symbols with information, a cyclic header of two symbols $+1$ is transmitted to cyclicly reset the state of the system.
REMARK: Clear evidence of the development of the applied algorithm has to be provided.

(3 points)