DIGITAL COMMUNICATIONS
THEORY
(Time: 60 minutes. Grade 4/10)

	Grade	
Last Name(s):	1	
First (Middle) Name:	2	
ID number: .... Group	3	
Signature	T	

Question 1

A baseband PAM modulation transmits a sequence of data $A[n] \in \{0, A\}$ with probabilities $P(A[n] = 0) = 1 - p$ and $P(A[n] = A) = p$. Autocorrelation function is given by:

$$R_A[k] = \mathbb{E}\{A[n]A[n+k]\} = A^2 p(1-p) \cdot \delta[k] + A^2 p^2$$

a) Calculate the mean energy per symbol of the transmitted sequence.

b) Obtain the power spectral density of PAM signal, and plot it with proper axis labels, if the shaping filter is a normalized root-raised cosine filter with roll-off factor $\alpha = 0.25$, and sequence is transmitted at symbol rate $R_s = 1$ kbauds. Provide also the bandwidth of the PAM signal.

NOTE: Some transformations

$$x[n] = \delta[n] \stackrel{TF}{\longleftrightarrow} X(e^{j\omega}) = 1$$

$$x[n] = 1,\ \forall n \stackrel{TF}{\longleftrightarrow} X(e^{j\omega}) = 2\pi \sum_k \delta(\omega + 2\pi k)$$

(1 point)

Question 2

A differential PSK modulator generates the sequence of transmitted symbols in the following way: the symbols of the constellation are

$$A[n] = R \cdot e^{j\phi[n]}$$

where the phase that is transmitted at instant n, $\phi[n]$, is obtained taking into account the phase transmitted in the previous instant, $\phi[n-1]$, and the phase associated to the new binary information $\Delta_\phi[n] = \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$ radians

$$\phi[n] = \phi[n-1] + \Delta_\phi[n]$$

a) Plot the block diagram taking into account that the input is the sequence of data bits $B_b[\ell]$ and the output is the complex baseband signal $s(t)$.

b) If the initial reference phase is $\phi[-1] = \frac{\pi}{4}$, obtain the alphabet of transmitted symbols, plot the constellation, and calculate the mean energy per symbol if all symbols are transmitted with the same probability.

c) Provide an optimal binary assignment to minimize the bit error rate (BER).

d) If carriers used in the transmitter to generate the bandpass signal have frequency ω_c rad/s and a null phase, and carriers used at the receiver have the same frequency but a different phase $\theta_c \neq 0$ radians (non-coherent receiver), obtain the received noiseless constellation.

e) Design a proper receiver to deal with the previous situation with a non-coherent receiver (you have to provide the analytical expressions or the block diagram to obtain the estimation of the received bits from the observation $q[n]$).

(1.5 points)

Question 3

In a digital communication system that uses a 4-PAM constellation with normalized levels, the binary rate is 4 kbits/s, and the initial symbols of the sequence to be transmitted are

$$A[0] = +1,\ A[1] = -3,\ A[2] = +1,\ A[3] = +3,\ A[4] = +1,\ A[5] = -1,\ A[6] = +1,\ A[7] = -1.$$

To implement the system two modulations are considered: direct sequence spread spectrum and OFDM modulation.

The equivalent discrete channel $d[m]$ (equivalent discrete channel at chip period for spread spectrum, or at the time of an OFDM symbol divided by the number of carriers or carriers plus cyclic prefix for an OFDM modulation) is

$$d[m] = \delta[m] - \frac{1}{4} \cdot \delta[m-1] + \frac{2}{3} \cdot \delta[m-2].$$

a) If the direct sequence spread spectrum modulation is used, with spreading factor $N = 3$, spreading sequence

$$x[m] = \delta[m] - \delta[m-1] + \delta[m-2],$$

carrier frequency $\omega_c = 2\pi \times 10^6$ radians/s, and using a root-raised cosine transmitter filter with roll-off factor $\alpha = 0.25$, obtain the sequence of samples of the modulated signal at chip rate, $s[m]$, which is associatted to the first 3 symbols of the data sequence given above and calculate the bandwidth of the bandpass modulated signal.

b) If an OFDM modulation with $N = 4$ carriers and carrier frequency $\omega_c = 2\pi \times 10^6$ radians/s is used, obtain the ordered sequence of samples of the signal to be transmitted associatted to the 8 initial symbols of the data sequence given above, and calculate the bandwidth of the modulated signal in the following cases:

   i) The sistem is designed to transmit using the minimal possible bandwidth.

   ii) The system is designed to transmit without intersymbol and intercarrier interferences, using the minimum bandwidth which is necessary to avoid these two effects.

<u>NOTE</u>: Expressions for the DFT of N points for a sequence $x[m]$, and the corresponding inverse DFT

$$X[k] = \sum_{m=0}^{N-1} x[m] \cdot e^{-j\frac{2\pi k}{N}m}, \quad x[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot e^{j\frac{2\pi k}{N}m}$$

(1.5 points)

DIGITAL COMMUNICATIONS
EXERCISES
(Time: 120 minutes. Grade 6/10)

	Grade	
Last Name(s):	1	
First (Middle) Name:	2	
ID Number: .... Group	T	
Signature		

Exercise 1

Two users of a communication system wish to transmit the following sequence of equiprobable symbols $A_1[n] \in \{\pm 1\}$ and $A_2[n] \in \{\pm 2\}$, respectively. Each user transmits its sequence through a different channel ($p_1[n]$ and $p_2[n]$, respectively) and the transmitted signals are added at the receiver, where each sequence will be intended to be separated. The received observation is:

$$q[n] = A_1[n] * p_1[n] + A_2[n] * p_2[n] + z[n]$$

where $z[n]$ is white and Gaussian noise with $\sigma_z^2 = 1$, $p_1[n] = 0.9\delta[n] - 0.1\delta[n-1]$ and $p_2[n] = 0.8\delta[n] - 0.2\delta[n-1]$. If the receiver tries to recover the signal of user 1, the signal due to user 2 is an interference for user 1 (it has the same effect than intercarrier interference in an OFDM modulation). The same thing happens when the receiver tries to recover signal of user 2.

a) Obtain the noiseless received constellation and plot the trellis diagram of the whole system if you pretend to detect simultaneously both sequences (you have to plot all branches, but only have to include labels for branches starting from a single state, indicating clearly the symbols generating the transition and the output generated by the system in that transition).

b) If the receiver uses a memoryless symbol-by-symbol detector, obtain the probability of error for user 1 with delay $d = 0$, obtain the probability of error for user 2 with delay $d = 0$, compare the results explaining which user has better performance and why.

c) Design the 2 coefficients equalizer designed to recover data of user 1, unknowing the existence of user 2. Use the ZF design criterion for delay $d = 0$. Obtain the expression for the residual ISI along with the interference generated by symbols of user 2, and using them obtain an approximated expression for the probability of error for user 1.

d) Design a 2 coefficients equalizer with delay $d = 0$ using ZF criterion to recover the data of user 1, but taking now into account the existence of user 2 (extend the development of the ZF equalizer to include this information), and obtain an approximated expression for the probability of error of user 1.

NOTE: Inverse of a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(3 points)

Exercise 2

a) A linear block code has the following dictionary

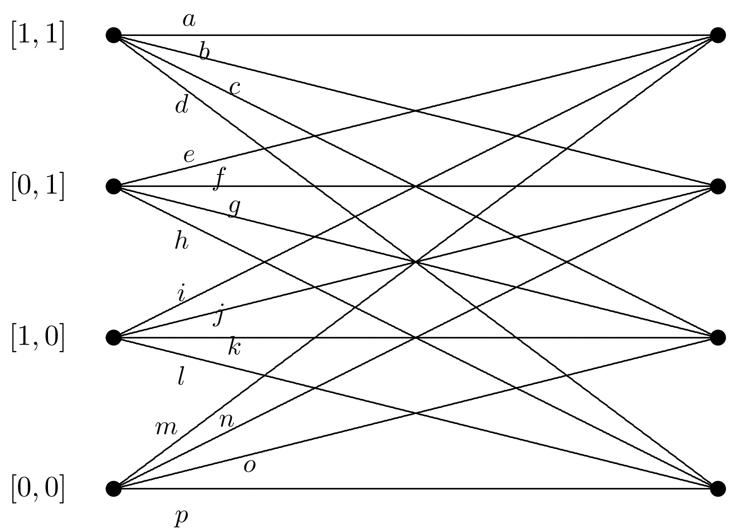
i	$\mathbf{b}_i$	$\mathbf{c}_i$
0	0 0 0	x x x x x x
1	0 0 1	1 1 1 0 0 0
2	0 1 0	0 1 1 1 1 0
3	0 1 1	1 0 0 1 1 0
4	1 0 0	1 0 1 1 0 1
5	1 0 1	0 1 0 1 0 1
6	1 1 0	x x x x x x
7	1 1 1	0 0 1 0 1 1

i) Obtain coded words $\mathbf{c}_0$ and $\mathbf{c}_6$ if the code is linear, say if the code is or not systematic explaining why, and calculate the detection and correction capabilities of the channel code.

ii) Obtain the generating matrix and the parity check matrix of the code.

iii) Obtain the syndrome table, and decode (providing the estimation of the uncoded word $\mathbf{b}_i$), indicating each step of the syndrome based decoding algorithm, if the received word is $\mathbf{r} = 1\ 1\ 1\ 0\ 1\ 1$.

b) A convolutional encoder has the following trellis diagram



$a \equiv 11|1001$
$b \equiv 01|0011$
$c \equiv 10|1100$
$d \equiv 00|0110$

$e \equiv 11|1101$
$f \equiv 01|0111$
$g \equiv 10|1000$
$h \equiv 00|0010$

$i \equiv 11|1011$
$j \equiv 01|0001$
$k \equiv 10|1110$
$l \equiv 00|0100$

$m \equiv 11|1111$
$n \equiv 01|0101$
$o \equiv 10|1010$
$p \equiv 00|0000$

i) Obtain the generating matrix with D polynomials, and plot the schematic representation of the encoder.

ii) Assuming that all the previously transmitted bits are zeros, encode the following binary sequence 10110100 and calculate the approximated probability of error if hard decoding is used and the bit error rate of the modulation used to transmit is $BER = \varepsilon$.

iii) Decode, by applying the optimal decoding algorithm (clear evidence of its application has to be provided), the first four information bits, $\hat{B}[m]$, $m \in \{0, 1, 2, 3\}$, for the following received sequence, assuming that $B[m] = 0$ for $m < 0$ and for $m \geq 4$

$$\mathbf{r} = 101100011010$$

(3 points)