



# AUTOMATA THEORY AND FORMAL LANGUAGES

## UNIT 7: TURING MACHINE



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# OUTLINE

- Definition of Turing Machine
- Variations of Turing Machines
- Universal Turing Machine
- Additional issues



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# OUTLINE

- **Definition of Turing Machine**
- Variations of Turing Machines
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# Definition of Turing Machine

# Alan Turing



- 23 June 1912 - 7 June 1954
  - Contributions:
    - Mathematics,
    - Cryptanalysis,
    - Logic,
    - ...
    - Artificial Intelligence.

<http://www.alanturing.net>



# 2012 THE ALAN TURING YEAR

## A Centenary Celebration of the Life and Work of Alan Turing

**Centenary Events**

- ATY Events Calendar
- ATY Resources
- ATY Resources - Computing Subjects
- TYCOM Media Group
- TYCOM Timeline
- Alan Turing's Jade 2012
- TYCOM Summary
- ACM Turing Award
- ACM Turing Centenary Event
- AMSI-Turing, Monash
- ANU-Turing, ANU
- AMSI-Joint Math Meeting
- ANU-Turing, ANU
- Bletchley Park
- Barriers in Complexity Workshop
- Brazilian Alan Turing Year
- Cambridge Alan Turing Year
- ICITCS 2012, Manchester
- British Logic Colloquium 2012
- Computability in Europe 2012
- Complexity in Europe 2012
- CORR/Randomness workshop
- Computability Theory
- Computing Research '12
- CONCUR 2012, Newcastle
- Computing Research '12
- DCG 2012 in Cambridge

• To link to this webpage please use the url: <http://www.turingcentenary.eu/> - and the ATY logo your webpage - or monochrome version [here](#)

• If you wish to be included in the Turing Centenary email list, please enter your email address here and press Submit:

The Alan Turing Year on Facebook - and on Twitter  
ATY Press and Media Contact - Daniela Derbyshire - email: [turing@live.co.uk](mailto:turing@live.co.uk)

**June 23, 2012, is the Centenary of Alan Turing's birth in London.** During his relatively brief life, Turing made a unique impact on the history of computing, computer science, artificial intelligence, developmental biology, and the mathematical theory of computability.

**2012 will be a celebration of Turing's life and scientific impact,** with a number of major events taking place throughout the year. Most of these will be linked to places with special significance in Turing's life, such as Cambridge, Manchester and Bletchley Park.

**The Turing Year is coordinated by the Turing Centenary Advisory Committee (TCAC),** representing a range of expertise and organisational involvement in the 2012 celebrations. Organisations and individuals wanting to contribute ideas or support for the Turing Year are invited to contact any of the current TCAC members

**Turing Centenary Advisory Committee (TCAC)**

• Sir John Dermott Turing (Honorary President)  
120 Queen's Gate, Putney Common, London SW15 8TQ, United Kingdom  
Sir Alan Turing at the Open University

• S Barry Cooper (Leeds, Chair TCAC)

• Martin H. Hyland (Cambridge, British Logic Colloquium)  
LMU General Semantics Preceptor, FELICE Young College, Head of Pure Maths (DPhilMS)  
• Michael Herndl (Birmingham, BRAIN)

• Alan Turing Institute (London, UK) - to be confirmed in 2012

• Simon Lavington (Suffolk, Computer Conservation Soc)

**News**

Grant a plaque to Alan Turing  
Computing and Petition started

Chancellor of Britain's Greatest Computer gets widespread praise and media attention

Plaque for the Brazilian Alan Turing has been installed

New ATY Calendar of Events goes live

<http://www.mathcomp.leeds.ac.uk/turing2012/>



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# Definition of Turing Machine

## Alan Turing

- <http://www.alanturing.net>
- [http://en.wikipedia.org/wiki/Alan\\_Turing](http://en.wikipedia.org/wiki/Alan_Turing)
- Bibliography:
  - "On Computable Numbers, with an Application to the Entscheidungsproblem" 1936
  - Turing, A.M. Computing machinery and intelligence. Mind, 59, 433-460. 1950
    - "I propose to consider the question, "Can machines think?"
    - [http://en.wikipedia.org/wiki/Computing\\_machinery\\_and\\_intelligence](http://en.wikipedia.org/wiki/Computing_machinery_and_intelligence)
    - <http://blog.santafe.edu/wp-content/uploads/2009/05/turing1950.pdf>
- Charles Petzold. The Annotated Turing: A Guided Tour Through Alan Turing's Historic Paper on Computability and the Turing Machine. Wiley. 2008.  
[www.theannotatedturing.com](http://www.theannotatedturing.com)



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# Definition of Turing Machine

Can machines do everything?

## □ We have studied:

- Simple languages.
- Application of simple languages to solve restricted problems:
  - Analysis.
  - Pattern recognition.
  - Syntax analysis.

## □ In this unit, we want to answer:

- Which languages can be defined by means of any computational device?
- Which problems can be solved/computed?





# Definition of Turing Machine

## Can machines do everything?

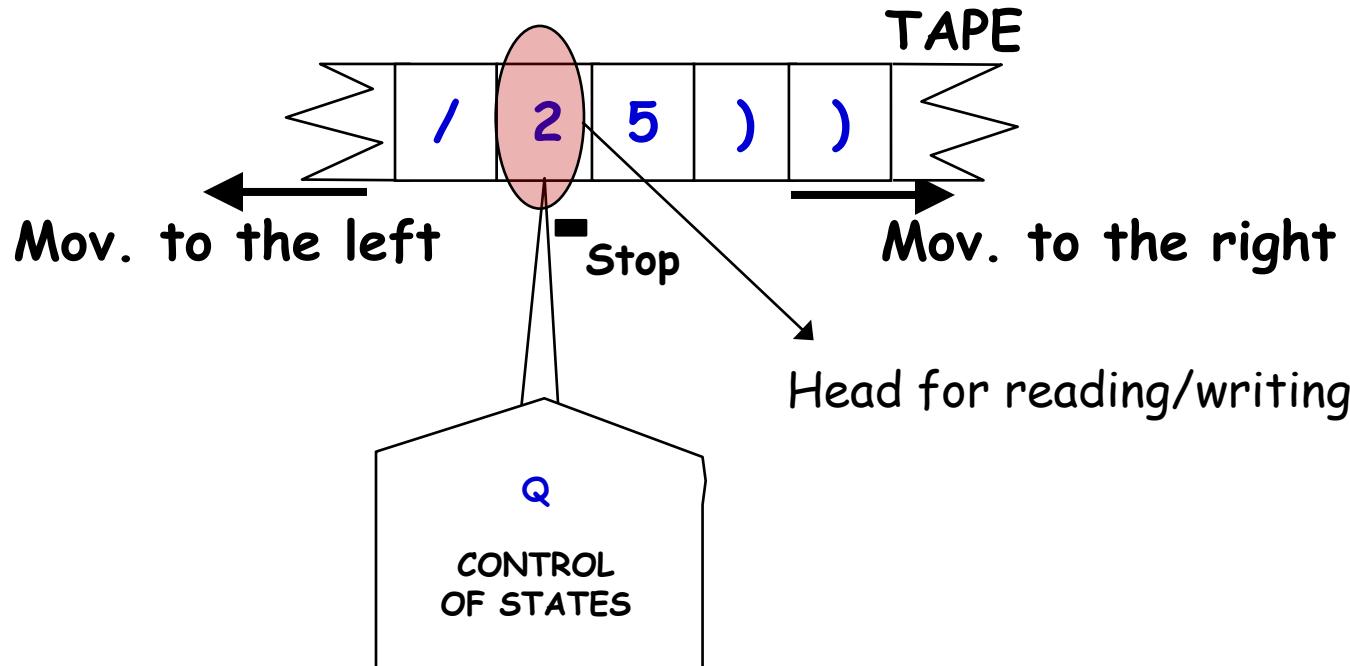
- Which is the solution?
  - He says that she always say the truth.
  - She says that he always lies (he never says the truth).
- Can a computer answer to any possible question in a dialog system?
- Is a grammar ambiguous?
- Is there a solution for  $X^N + Y^N = Z^N$ , with  $N > 2$ ?



# Variations of Turing Machines

## What is a Turing Machine?

- Mechanical device:
  - Infinite tape divided into cells with a head for reading/writing.
  - This head can be moved from left or right or stay in the same cell.



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## Turing Machine: Operation

**Operations that a TM carries out:**

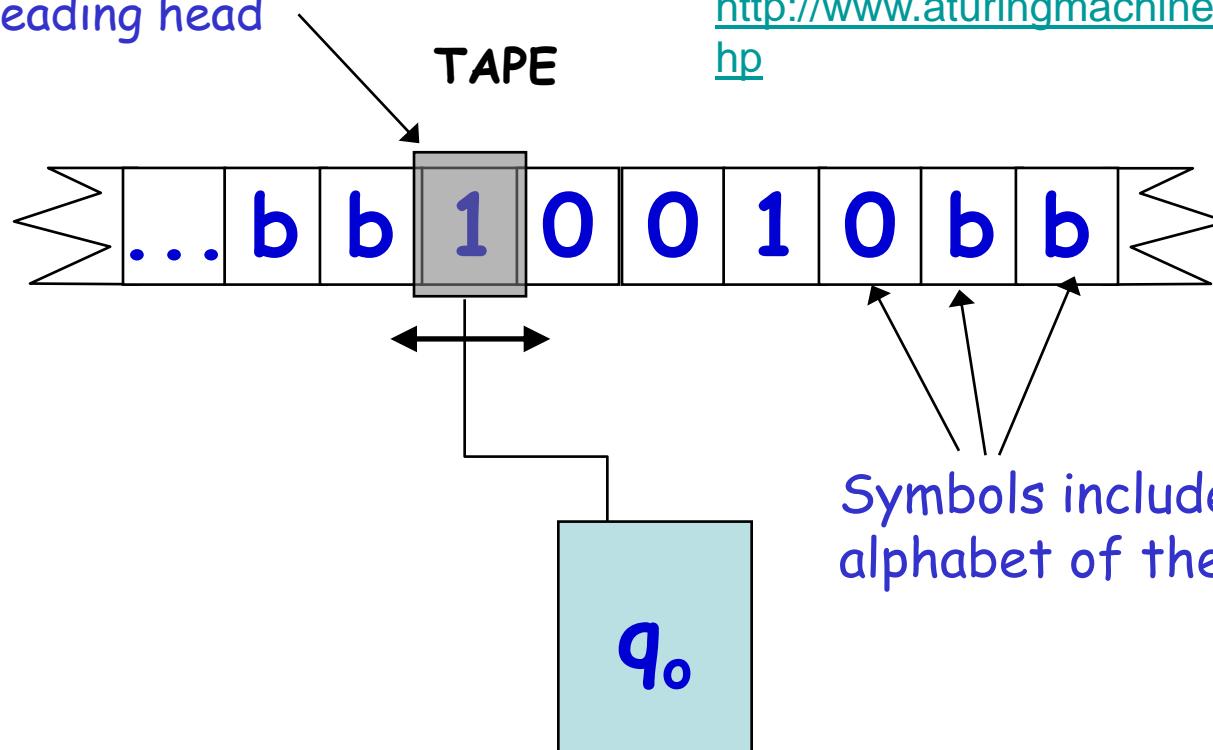
- Being in a state  $p$  and reading a symbol in the cell on which the R/W head is, it carries out three actions:
  - Transits to a new state.
  - Writes a new symbol in the tape, in the same cell where the current symbol has been read. This symbol replaces the previously read (unless it is the same).
  - Moves the R/W head to the left, to the right or stops in the same position.



# Variations of Turing Machines

## Turing Machine: Initial Situation

Initial position of  
the reading head



Example:

<http://www.aturingmachine.com/index.php>

Symbols included in the  
alphabet of the tape,  $\Gamma$

Initial State



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## Turing Machine: Operation

Characteristics of the Tape:

- Infinite tape.
- It can contain a character in each cell.
- It can be read.
- It can be written.
- Initially it is considered with infinite blank symbols to the right and left of the word.
- It can be moved to the left or right (a cell each time) or not move.





## Turing Machine: Operation

- There are many variants of Turing Machines...
- ... but they all are equivalent.
- The tape is one-dimensional and infinite by both sides.
- Initially:
  - The tape contains the word, and the rest of elements of the tape (left and right of the word) are the blank symbol (b or □).
  - At the beginning, the R/W head is located on the left-most element of the word.





## Turing Machine: Formal Definition

- Septuple:  $(\Sigma, \Gamma, b, Q, q_0, f, F)$ , where:
  - $\Gamma$ : alphabet of symbols in the tape
  - $\Sigma \subset \Gamma$ : alphabet of input symbols
  - $b \in \Gamma, b \notin \Sigma$  : is the blank symbol (the only symbol allowed to be in the tape infinitely at any step during the computation). It indicates empty cell ( $\square$ ).
  - $Q$ : set of states (finite).
  - $q_0 \in Q$ : initial state.
  - $F \subset Q$ : set of final states.
  - $f$ : transition function

$$Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R,S\}$$

L or -: left  
R or +: right  
S or =: stay





# Variations of Turing Machines

## Turing Machine: Transition Function

f: Transition function, table with double input

$$Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

L: mov. to the left  
R: mov. to the right  
S: stop

$\downarrow Q / \Gamma \rightarrow$	Symbol	Symbol
Input	State, symbol, movement	...
Input	State, symbol, movement	
...	...	

Empty cells in the table:

- Transitions that are NOT possible → The machine stops.





# Variations of Turing Machines

## Turing Machine: Example

$$\mathcal{M} = (\Sigma=\{0,1\}, \Gamma=\{0,1,\square\}, \delta, Q = \{q_0, q_1, q_F\}, q_0, F=\{q_F\}, f)$$

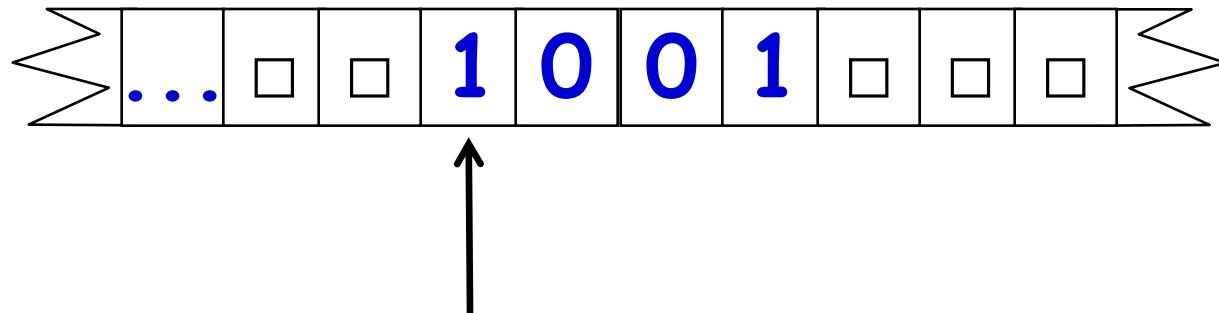
f	0	1	$\square$
$\rightarrow q_O$	$(q_0, 0, +)$	$(q_1, 1, +)$	$(q_F, 0, =)$
$q_1$	$(q_1, 0, +)$	$(q_0, 1, +)$	$(q_F, 1, =)$
$* q_F$			



# Variations of Turing Machines

## Turing Machine: Example

f	0	1	□
-> qO	(q0, 0, +)	(q1, 1, +)	(qF, 0, =)
q1	(q1, 0, +)	(q0, 1, +)	(qF, 1, =)
* qF			

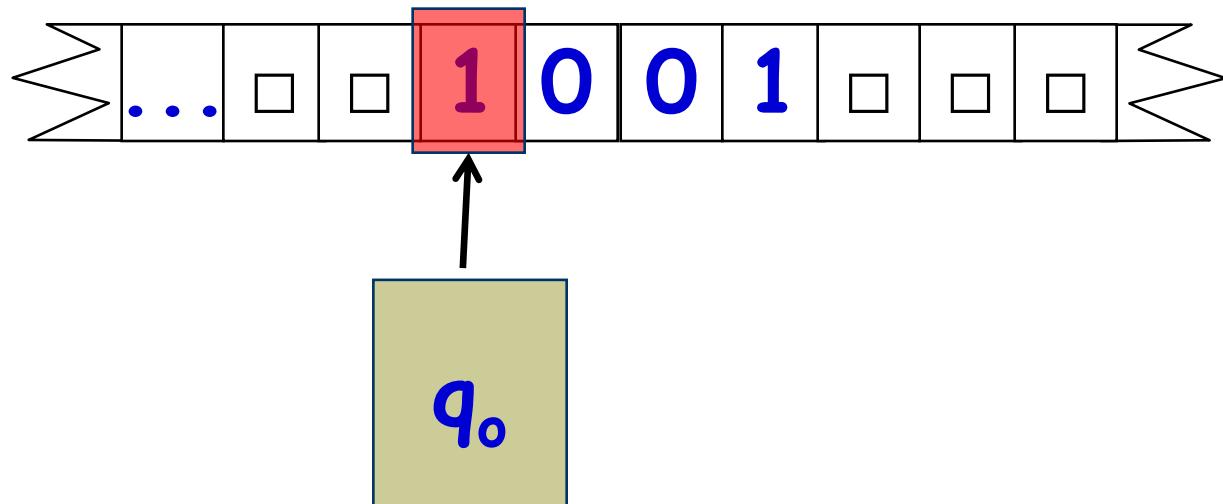


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# Variations of Turing Machines

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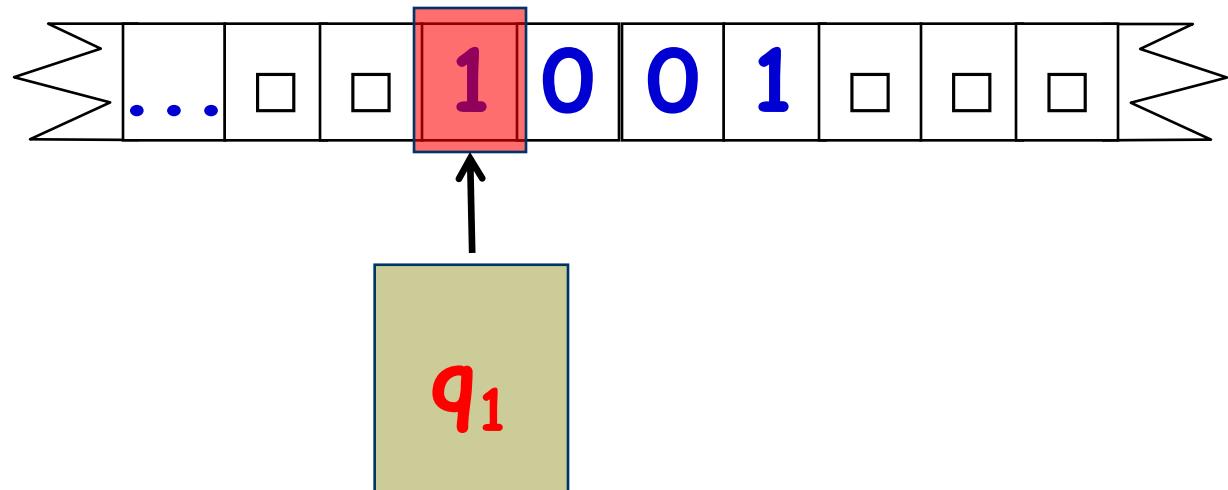


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# Variations of Turing Machines

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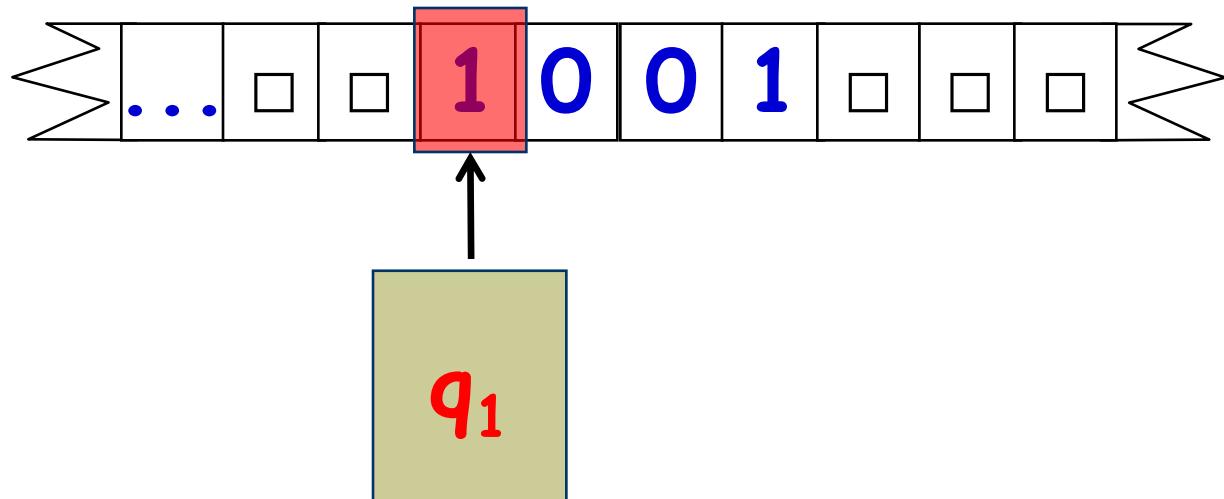


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$* q_F$			

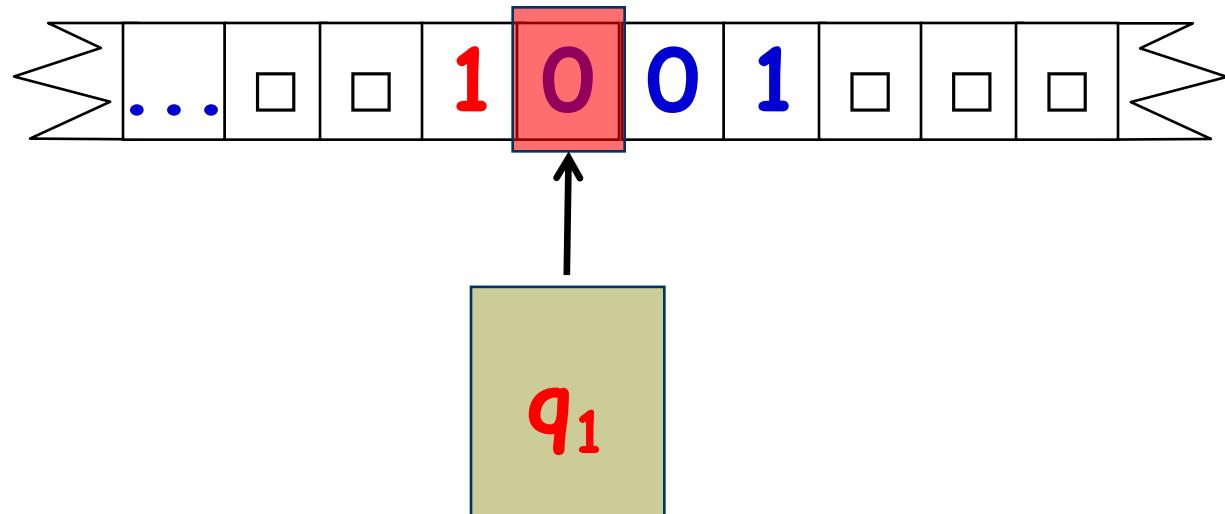


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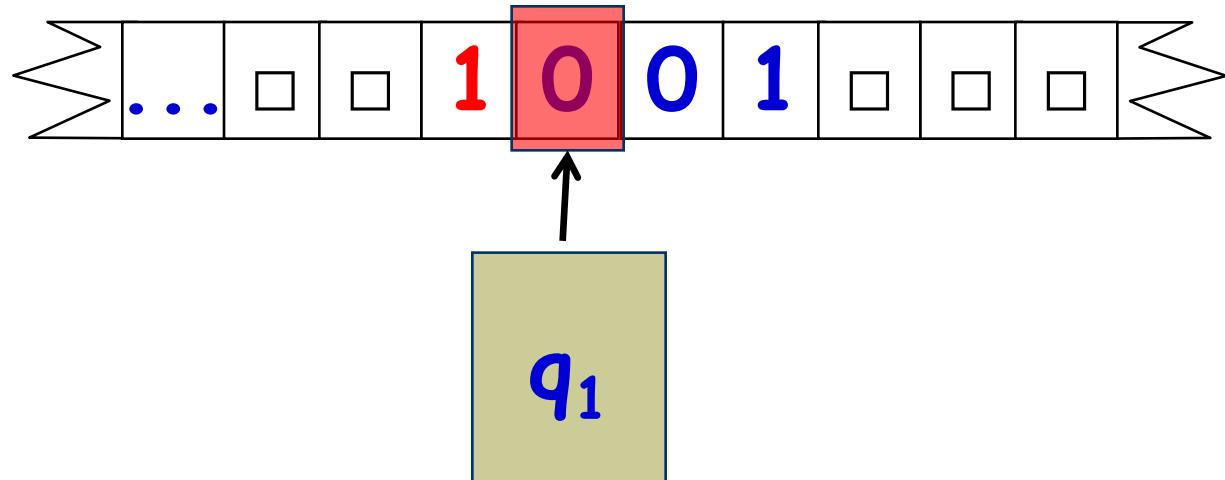


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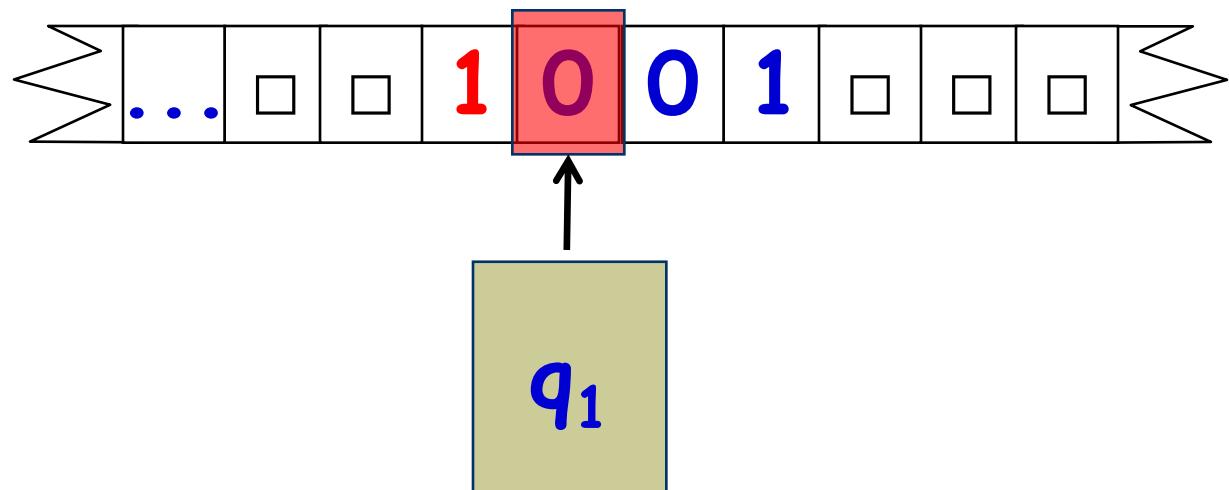


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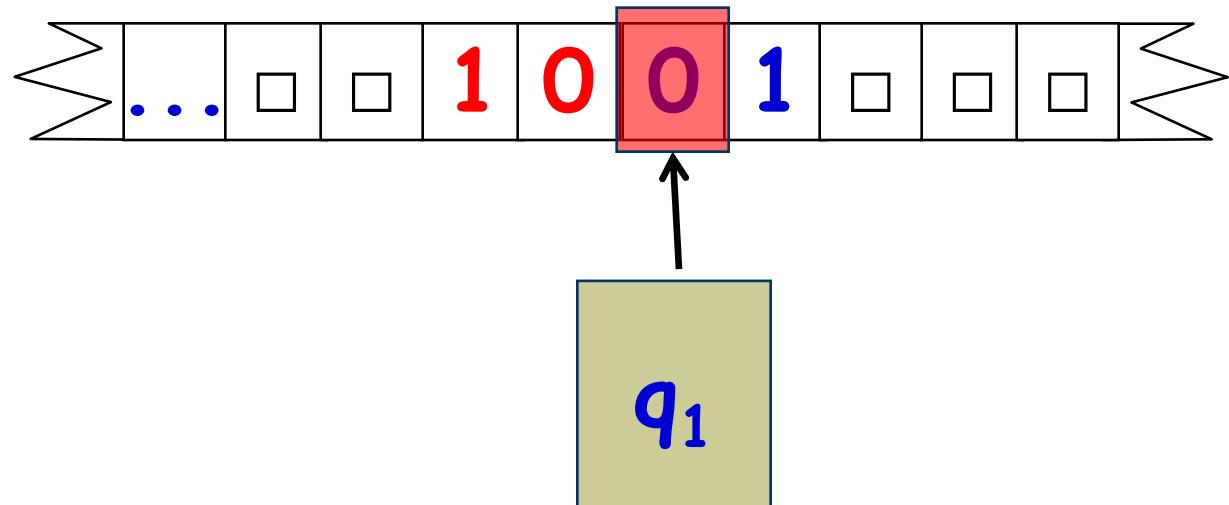


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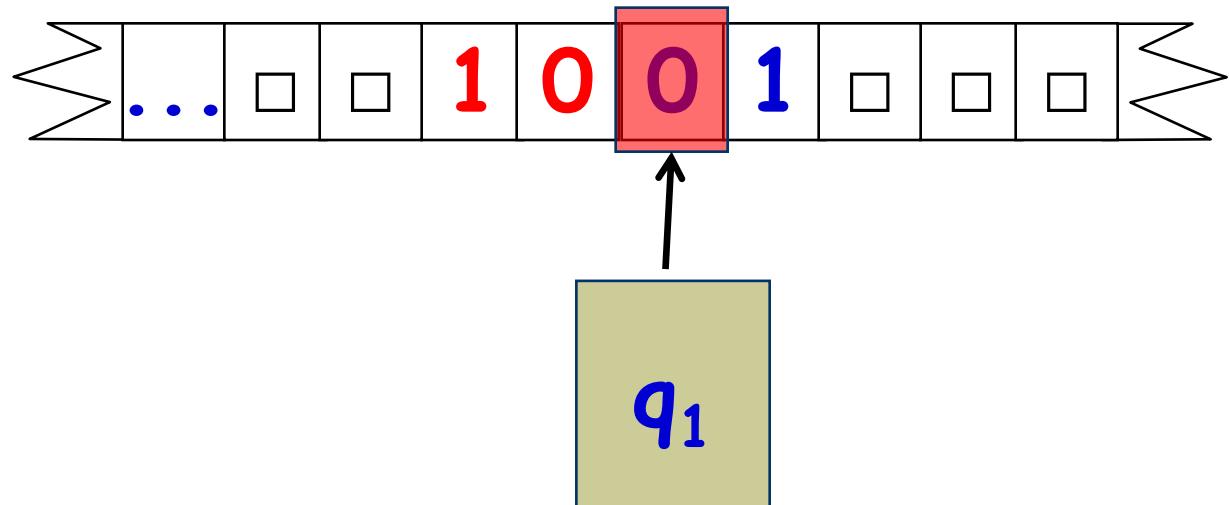


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# Variations of Turing Machines

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$* qF$			

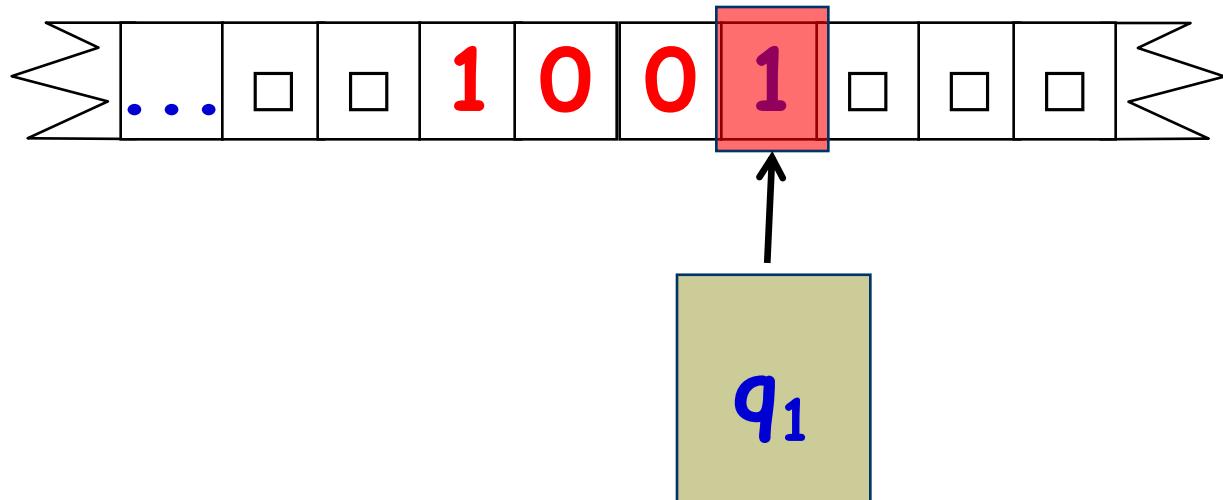


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# Variations of Turing Machines

## Turing Machine: Example

$\dagger$	0	1	$\square$
$\rightarrow q_0$	$(q_0, 0, +)$	$(q_1, 1, +)$	$(q_F, 0, =)$
$q_1$	$(q_1, 0, +)$	$(q_0, 1, +)$	$(q_F, 1, =)$
$* q_F$			

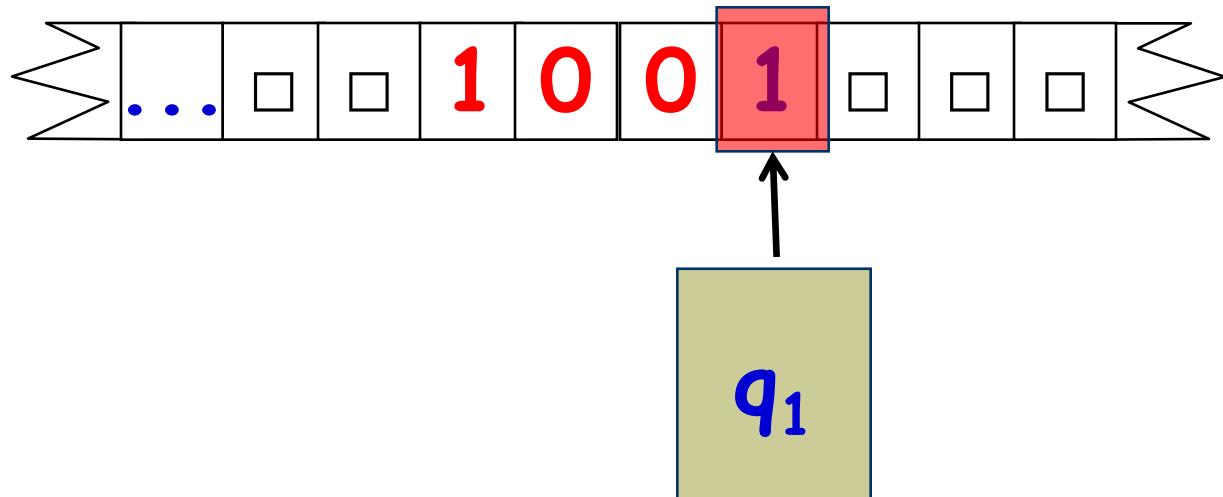


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# Variations of Turing Machines

## Turing Machine: Example

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$* q_F$			

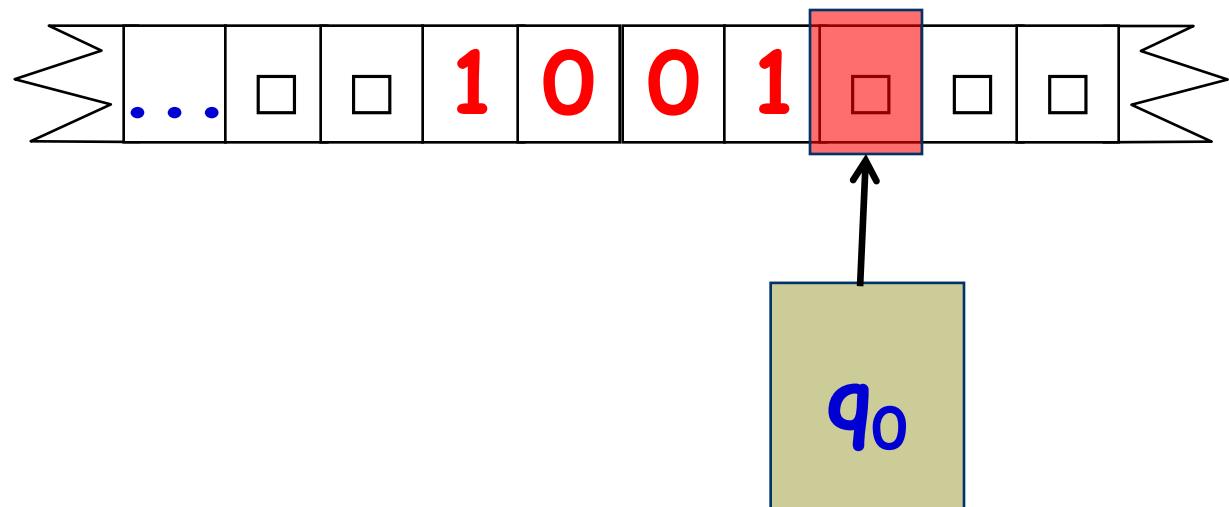


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# Variations of Turing Machines

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$* qF$			

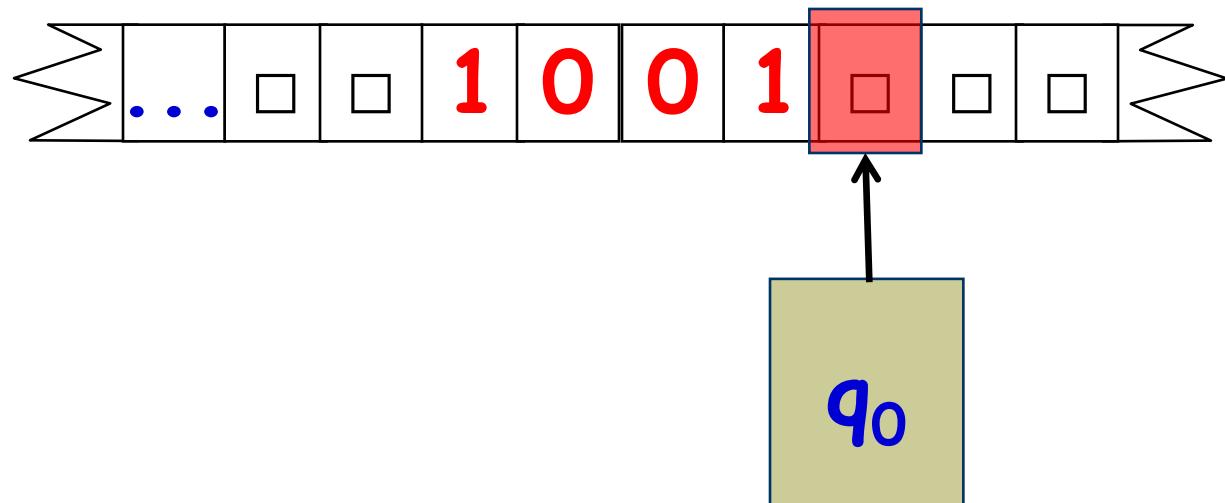


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# Variations of Turing Machines

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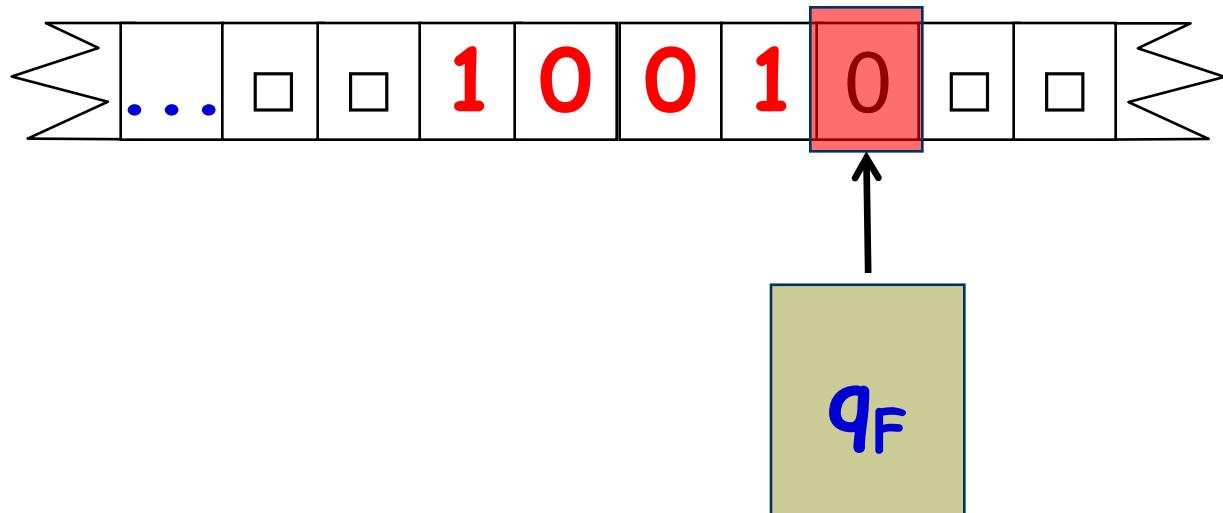


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# Variations of Turing Machines

## Turing Machine: Example

T	U	I	D
$\rightarrow q_0$	$(q_0, 0, +)$	$(q_1, 1, +)$	$(q_F, 0, =)$
$q_1$	$(q_1, 0, +)$	$(q_0, 1, +)$	$(q_F, 1, =)$
$* q_F$			



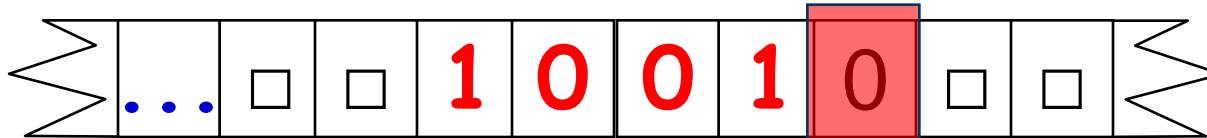
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# Variations of Turing Machines

## Turing Machine: Example

$\rightarrow qO$	$(q0, 0, +)$	$(q1, 1, +)$	$(qF, 0, =)$
$q1$	$(q1, 0, +)$	$(q0, 1, +)$	$(qF, 1, =)$
* $qF$			

0 → even number of 1s  
1 → odd number of 1s



$q_F$

Situations without transitions → Stop



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## Turing Machine: Graph Representation

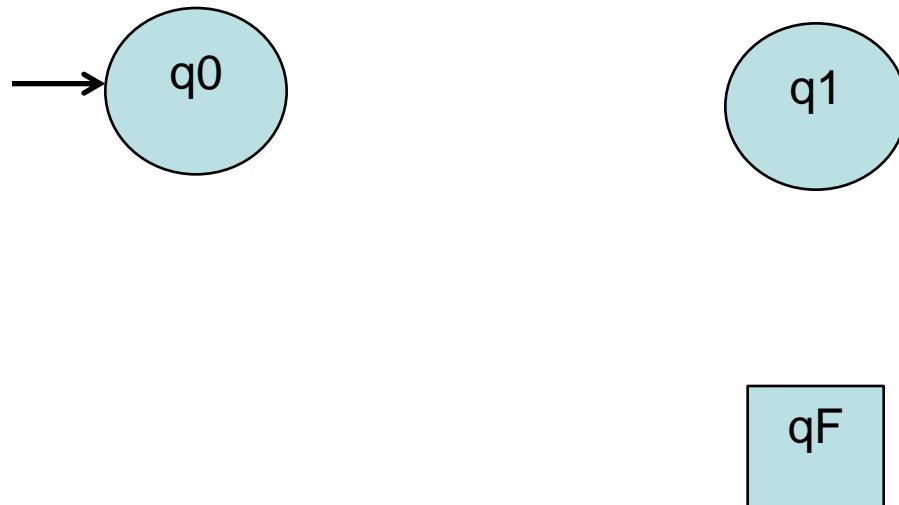
- The transition function can be described using also a diagram with states, i.e. a graph in which:
  - Nodes represent states.
  - Arches represent transitions between states.
  - Each arch is labelled including the requisites and effects of each transition (initial symbol, symbol that is rewritten, and direction to move the input header).



# Variations of Turing Machines

## Turing Machine: Graph Representation

$\tau$	$u$	$l$	$\square$
$\rightarrow q_0$	$(q_0, 0, +)$	$(q_1, 1, +)$	$(q_F, 0, =)$
$q_1$	$(q_1, 0, +)$	$(q_0, 1, +)$	$(q_F, 1, =)$
$* q_F$			

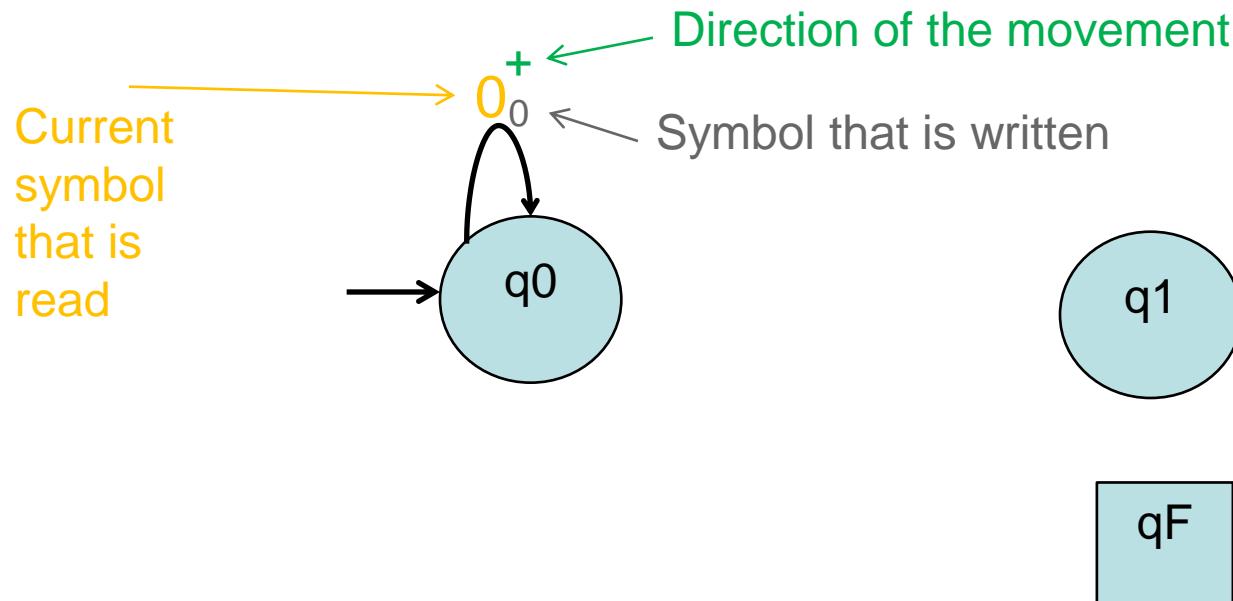


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# Variations of Turing Machines

## Turing Machine: Graph Representation

-> q0 q1 * qF	(q0, 0, +) (q1, 0, +)	(q1, 1, +) (q0, 1, +)	(qF, 0, =) (qF, 1, =)
---------------------	--------------------------	--------------------------	--------------------------

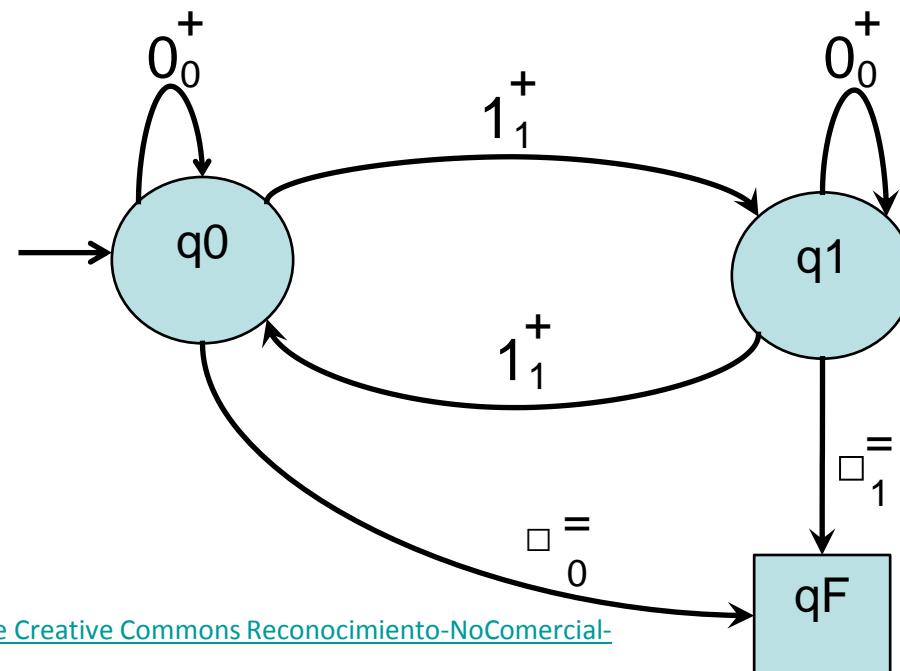


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# Variations of Turing Machines

## Turing Machine: Graph Representation

$t$	$o$	$i$	$\square$
$\rightarrow q_0$	$(q_0, 0, +)$	$(q_1, 1, +)$	$(q_F, 0, =)$
$q_1$	$(q_1, 0, +)$	$(q_0, 1, +)$	$(q_F, 1, =)$
$* q_F$			

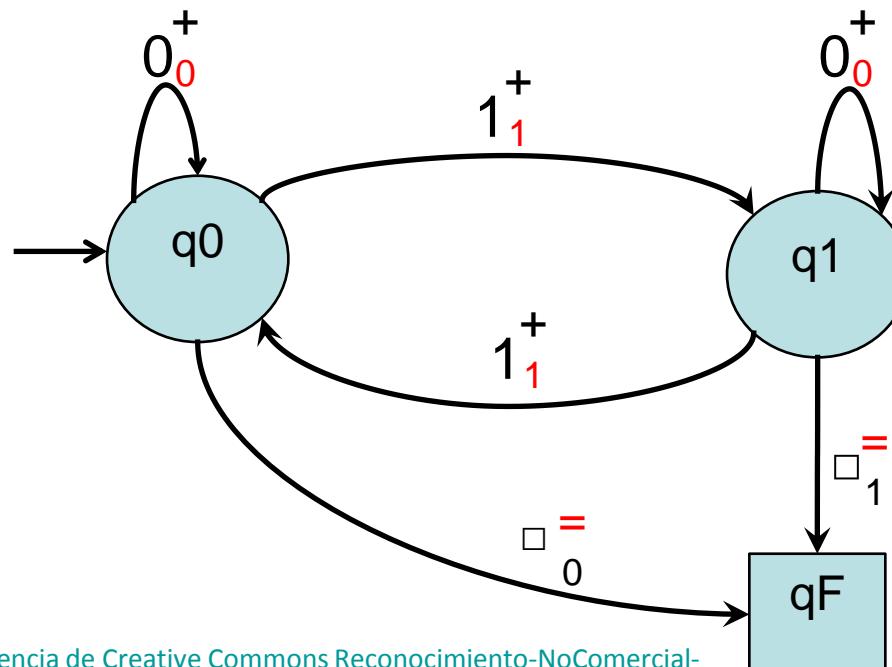


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# Variations of Turing Machines

## Turing Machine: Graph Representation

$f$	0	1	$\square$
$\rightarrow qO$	$(q0, 0, +)$	$(q1, 1, +)$	$(qF, 0, =)$
$q1$	$(q1, 0, +)$	$(q0, 1, +)$	$(qF, 1, =)$
$* qF$			

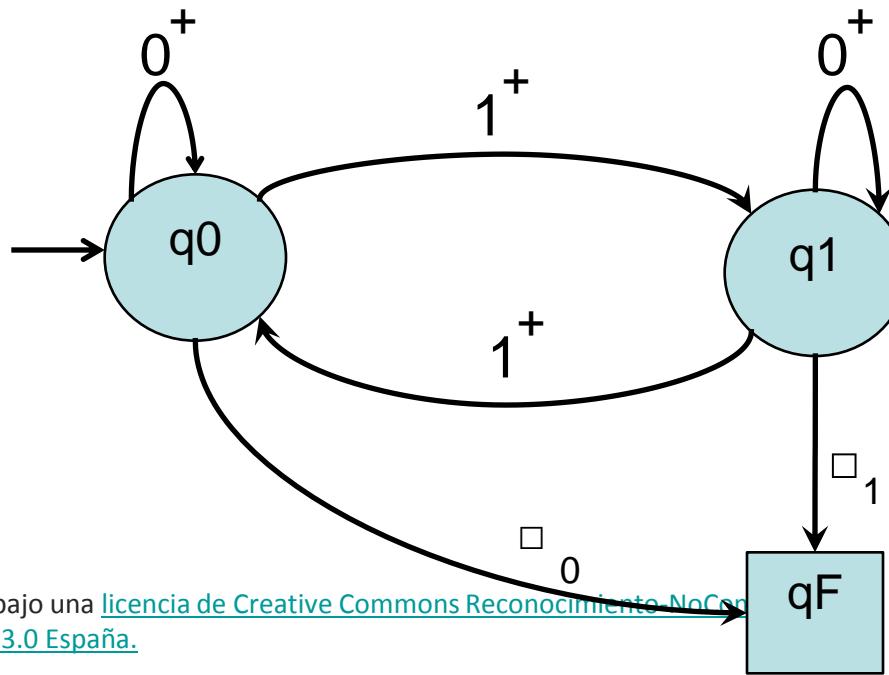


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# Variations of Turing Machines

## Turing Machine: Graph Representation

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## Turing Machine and Languages

- Turing Machine as TRANSDUCER:
  - It modifies the content of the tape,
  - Examples: TM that replaces digits by zeros.  
TM that appends a parity bit to the input.
- Turing Machine as RECOGNIZER:
  - TM that recognizes a language.
  - TM that accepts a language.





## Turing Machine and Languages

- Turing Machine as TRANSDUCER:
  - **Objective:** transform the input ( $\rightarrow$  Provide the result of an operation).
  - It verifies:
  - If the input is well-formed, it must finish in a final state.
  - If the input is NOT well-formed, it must finish in a nonfinal state (shows an error in the input word).





## Turing Machine and Languages

- Turing Machine as RECOGNIZER
  - **Objective:** Decide if the input string is valid or not, following a specific criterion.
- Two main concepts: RECOGNIZE, ACCEPT
  - A TM **RECOGNIZES** a language L, if for any input in the tape, w, it stops in a final state iff  $w \in L$ .
  - A TM **ACCEPTS** a language L if, when analyzing a word w, it stops in a final state iff  $w \in L$ .
    - If the word does not belong to the language, the TM does not need to stop.





# OUTLINE

- Definition of Turing Machine
- **Variations of Turing Machines**
- Universal Turing Machine
- Additional issues



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# Variations of Turing Machines

- We have defined a generic Turing Machine.
- The definition of TM admits many variations, but having the same computation capacity, i.e. **all the variations are equivalent**.
- Two Turing Machines,  $TM_1$  y  $TM_2$  are equivalent if **both carry out the same action for all their inputs.**
  - What does equivalent mean for a Turing Machine?
    - ✓ **TM as transducers:** for each possible input, at the beginning of the process, **at the end of this the contents of the tape must be the same.**
    - ✓ **TM as recognizers:** both TM **accepts the same words.**
    - ✓ If for some input  $w$ , one TM does not stop, the second one will not stop for such input  $w$ .





# Variations of Turing Machines

- Using the generic TM it is possible to impose restrictions, without they suppose limitations in the computation capacity.
- These restrictions can be imposed on:
  - **The alphabet of the tape: binary TM.**  $\Gamma = \{0,1\}$ . (~~Not  $\Sigma = \{0,1\}$~~   $\rightarrow$   ~~$F = \{0,1, b\}$~~ )
  - **The structure of the tape: Limited TM.**
    - E.g.: infinite tape only by the right (limited by the left using the input word).
    - **Movements (to write, move and change the state)**
      - E.g.: not to allow that the head remains quiet; not to allow that it writes and changes simultaneously of state; etc.





# Variations of Turing Machines

## Turing Machine with Binary Alphabet

### Theorem:

Given a generic TM , there is an equivalent TM with binary alphabet in the tape,  $\Gamma = \{0,1\}$

### Turing Machine with Binary Alphabet

$$M \rightarrow M_{(2)}$$

**IMPORTANT:** it is  $\Gamma = \{0,1\}$  and  $\Sigma \subseteq \Gamma$  , ~~not  $\Sigma = \{0,1\}$  and  $\Gamma = \{0,1, b\}$~~

Different descriptions in the bibliography

(According to section 2.2.3, Alfonseca 2007)





## Turing Submachines (Subroutines)

- Same concept that functions, methods, procedures (subprograms).
- A TM can, during its execution, invoke to another one.
- **DEFINITION:**
  - A submachine of Turing is a TM that can receive arguments (i.e. symbols) to carry out a specific task specify, and can be invoked with another TM.





## Turing Submachines (Subroutines)

- When a TM  $M_1$  invokes another TM  $M_2$  with an argument  $\sigma$ , the parameter in the definition of the submachine is replaced by the symbol by means it has been invoked:  $M_2(\sigma)$ .
  - $M_2(\sigma)$  is executed:
    - Using the same tape that  $M_1$ .
    - From the initial state of  $M_2$ .
    - With the R/W head of  $M_2$  on the same position where it was in  $M_1$ .





## Turing Submachines (Subroutines)

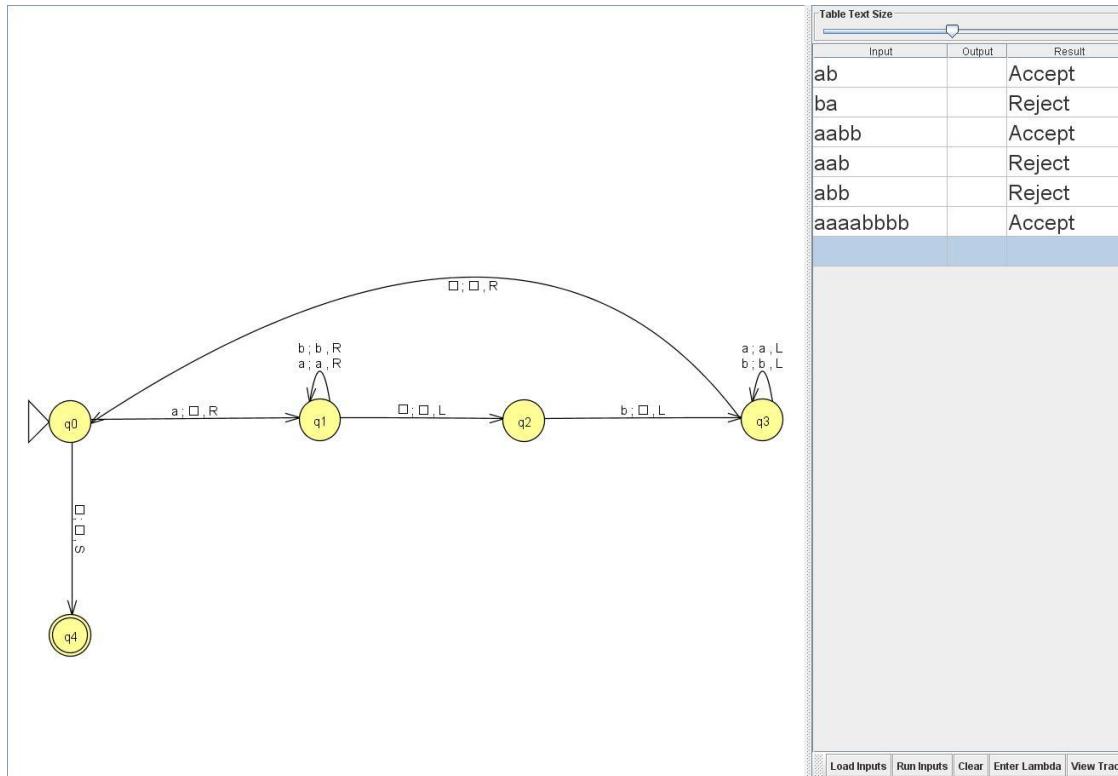
- To use Turing Submachines does not mean that the computability capacity (calculation) of TM is increased.
- Example:
  - Design a TM (using submachines) that recognizes the words of the language  $L=\{a^n b^n, n \geq 0\}$



# Variations of Turing Machines

## Turing Submachines (Subroutines)

**Example:**  $L = \{a^n b^n, n \geq 0\}$

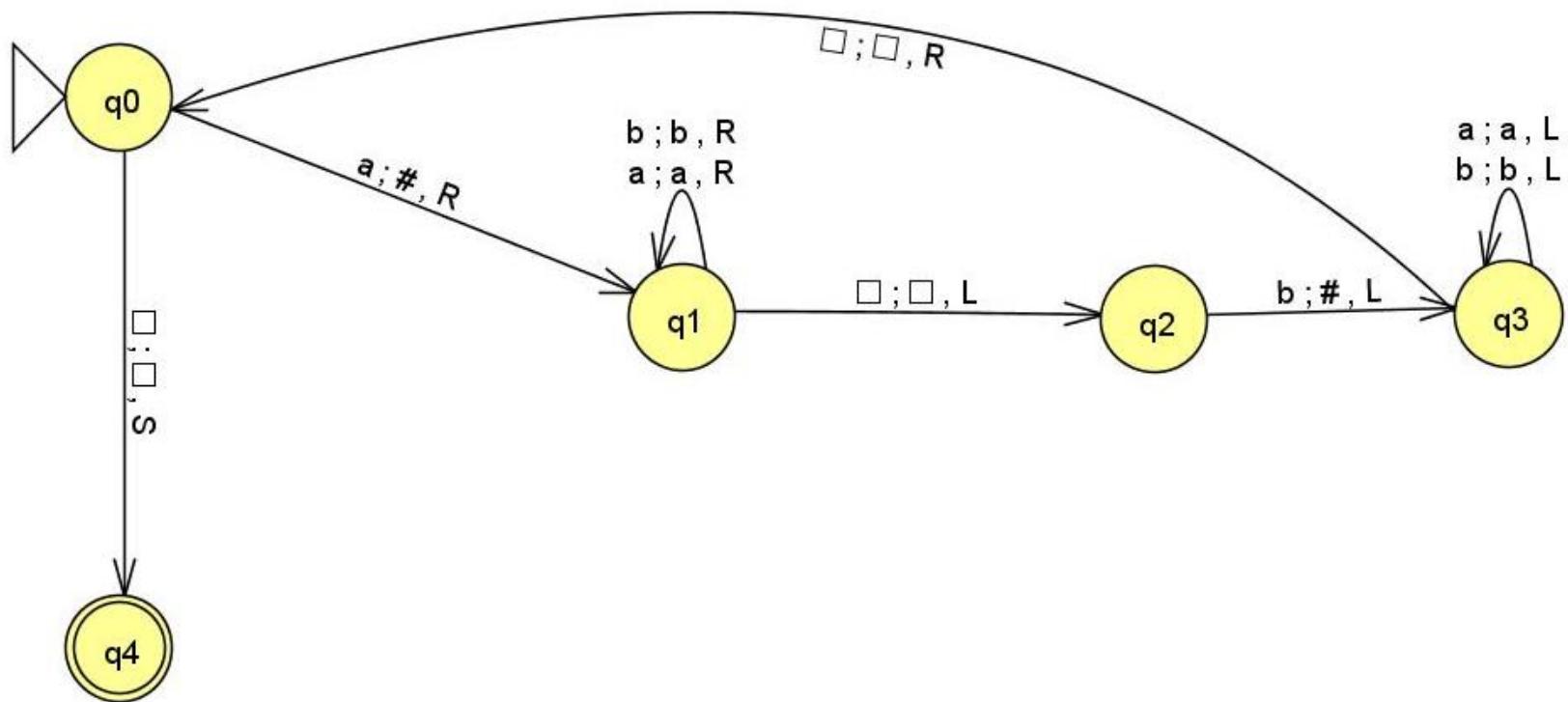


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# Variations of Turing Machines

## Turing Submachines (Subroutines)

Example:  $L=\{a^n b^n, n \geq 0\}$

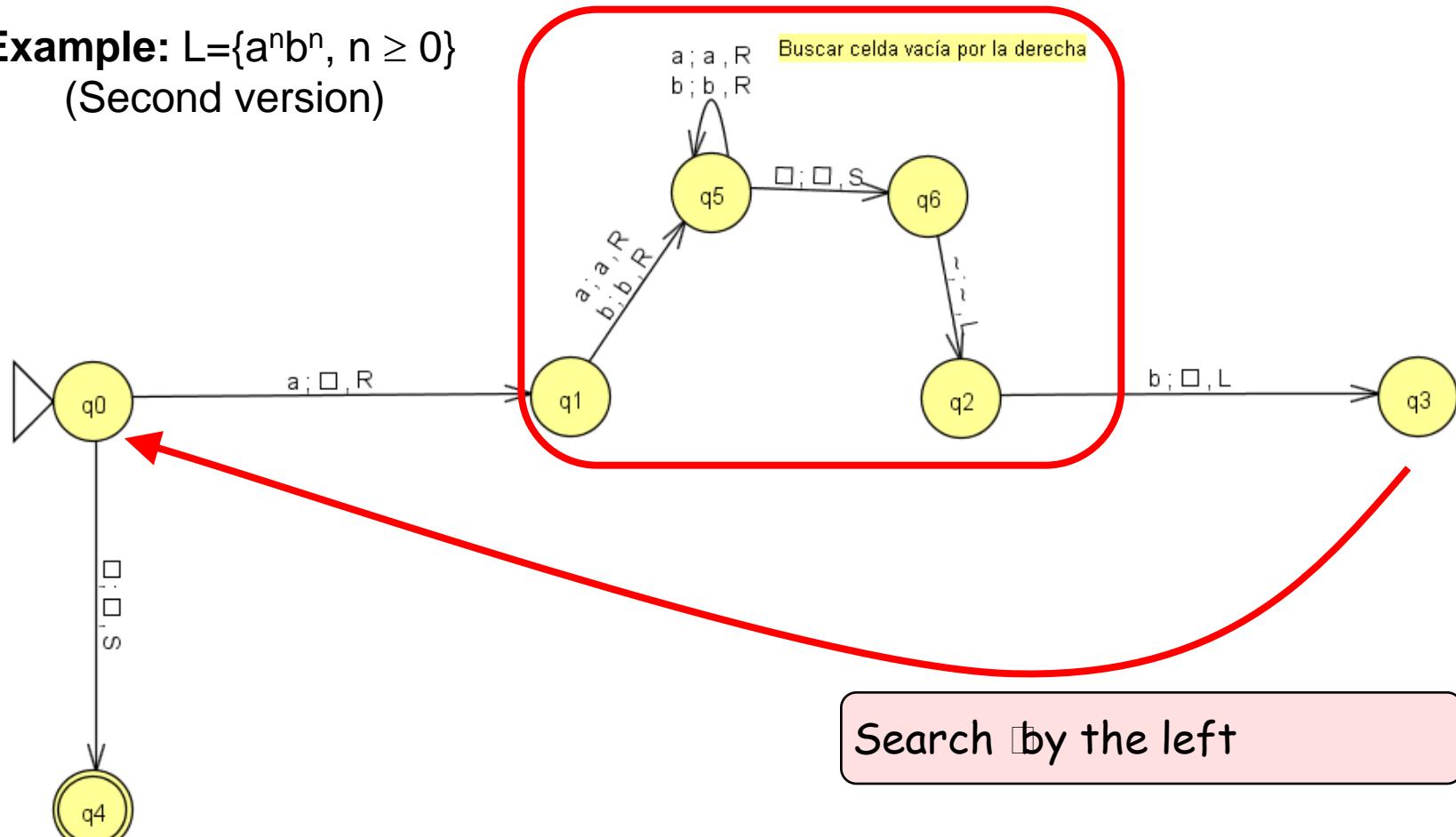


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# Variations of Turing Machines

## Turing Submachines (Subroutines)

**Example:**  $L=\{a^n b^n, n \geq 0\}$   
(Second version)

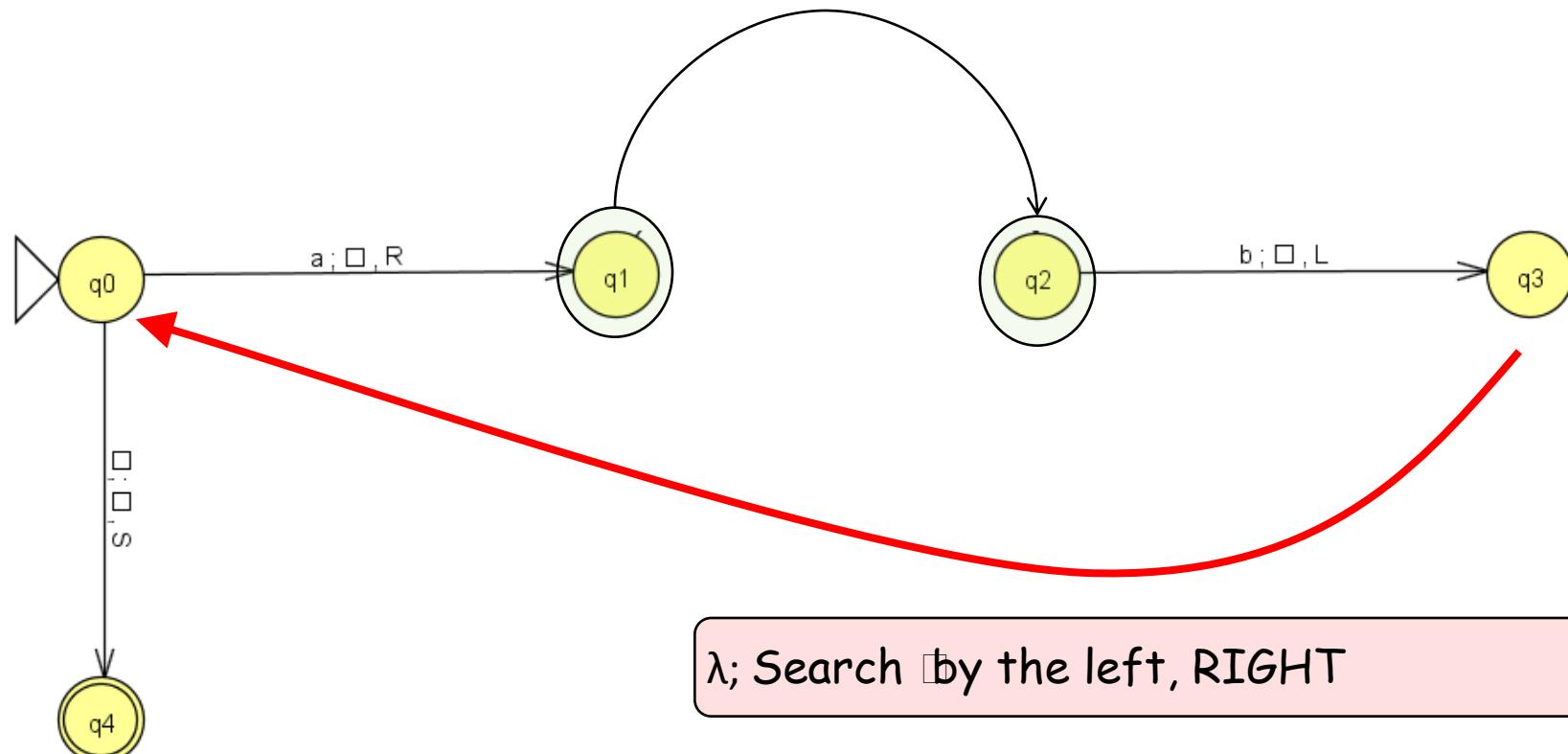


# Variations of Turing Machines

## Turing Submachines (Subroutines)

**Example:**  $L = \{a^n b^n, n \geq 0\}$   
(using submachines)

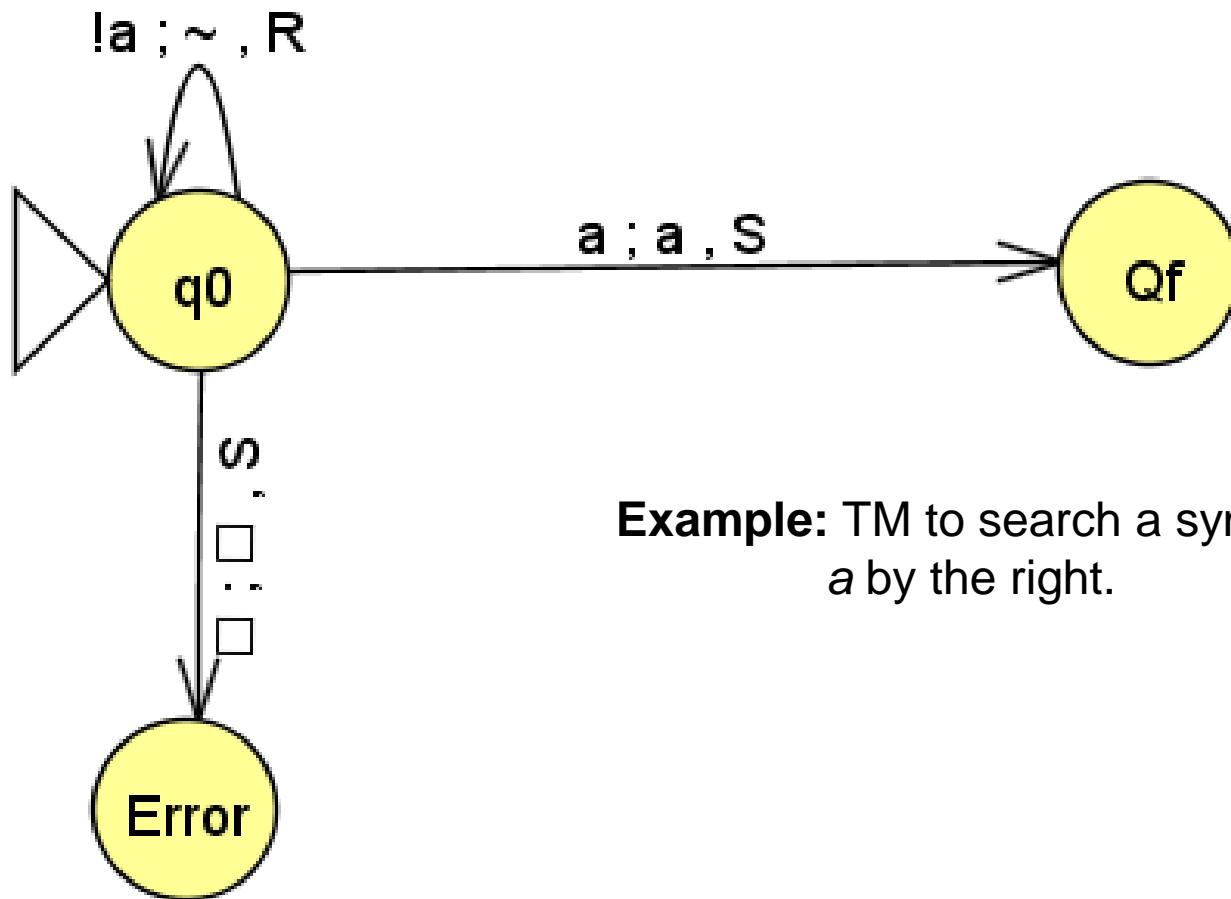
$\lambda$ ; Search  $\square$  by the right, LEFT



$\lambda$ ; Search  $\square$  by the left, RIGHT

# Variations of Turing Machines

## Turing Submachines (Subroutines)



**Example:** TM to search a symbol  $a$  by the right.



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# OUTLINE

- Definition of Turing Machine
- Variations of Turing Machines
- **Universal Turing Machine**
- Additional issues



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# Universal Turing Machine

## Universal Turing Machine

- It is possible to define a TM that simulates the execution of EVERY TURING MACHINE.

This machine is called

## UNIVERSAL TURING MACHINE (U)





## Universal Turing Machine

- **Universal Turing machine: U**
  - Input:
    - Description of the TM.
    - Input string (word):  $w$ .
  - What does  $U(TM, w)$  simulate?
    - It simulates the operation of TM, transition by transition.
    - If TM accepts  $w$ , then U stops in a final state for that word.
    - If TM does not accept  $w$ , U does not stop or it stops in a non-final state for this word.





## Universal Turing Machine

- **Universal Turing Machine: U**
  - It simulates a TM.
  - What does it happen if TM never stops?



# Universal Turing Machine

## Universal Turing Machine: Undecidability

- HALTING PROBLEM
  - Remember: “*The output of the machine—i.e., the solution to a mathematical query—can be read from the system once the TM has stopped.*”
  - However, in the case of Gödel’s undecidable propositions, the machine would never stop, and this became known as the “halting problem.”
    - It is not possible to decide if, given an input word  $w$ , the machine will stop or will be always running.
    - There is not a TM that answers this question.



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# OUTLINE

- Definition of Turing Machine
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- Universal Turing Machine
- **Additional issues**



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# Design issues

- The solution of the problem is the transition table, but this is not enough to complete an exercise.
- It is essential to think and describe the algorithm before to be implemented, as well as the behavior of each one of the states.
- It is precise to evaluate the machine with a significant set of inputs. There are maybe “difficult” cases that have to be considered.
- Many TM operates on input tapes with a specific configuration, that normally we do not verify.





# Design issues

- Any solution is not valid, you must think the algorithm very well and verify that it works for every case.
- If TM must operate as if it had “memory”, additional states can be incorporated (in more complicated machines, the input data can be written in specific positions of the tape).
- If a TM must remember positions within the tape, they usually use additional markers.
- If the task can be divided in several sequential subtasks (or subprogram), then do it.

