Formal Languages and Automata Theory
Exercises Finite Automata
Unit 3 – Part 2

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* Several exercises are based on the ones proposed in the following books:
  * Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón. *Teoría de autómatas y lenguajes formales.* McGraw-Hill (2007).
  * Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga. *Teoría de lenguajes, gramáticas y autómatas.* Publicaciones R.A.E.C. (1997).
  * Pedro Isasi, Paloma Martínez y Daniel Borrajo. *Lenguajes, Gramáticas y Autómatas. Un enfoque práctico.* Addison-Wesley (1997).
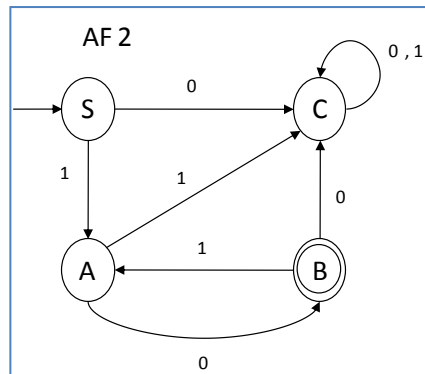
1. Indicate the graph of a DFA which recognizes each one of the following languages. The alphabet is $\{0, 1\}$.

   a) The language $\{0\}$

   b) The language $0^m 1^n 0^p$ ($m \geq 0, n \geq 0, p \geq 1$)

2. Indicate which is the language recognized by the FA2.

3. [**Exam**] We have a door with only one lock. To open it, it is necessary to use three different keys (called *a*, *b*, and *c*), in a predefined order, which is following described:

   - Key *a*, then key *b*, then key *c*, or
   - Key *b*, then key *a*, then key *c*.

   If this order is not followed, then the lock is blocked (for instance, if the key *a* is used and following it is introduced again).

   Once the door is open, the introduction of keys in the lock (in every possible order) does not affect the closing device (i.e. the door remains open).

   Consider that the names of the different keys are symbols of an alphabet, over which a language L whose words are the valid sequences for the opening of the door is defined. For instance, *abcbc* is a word included in the language.
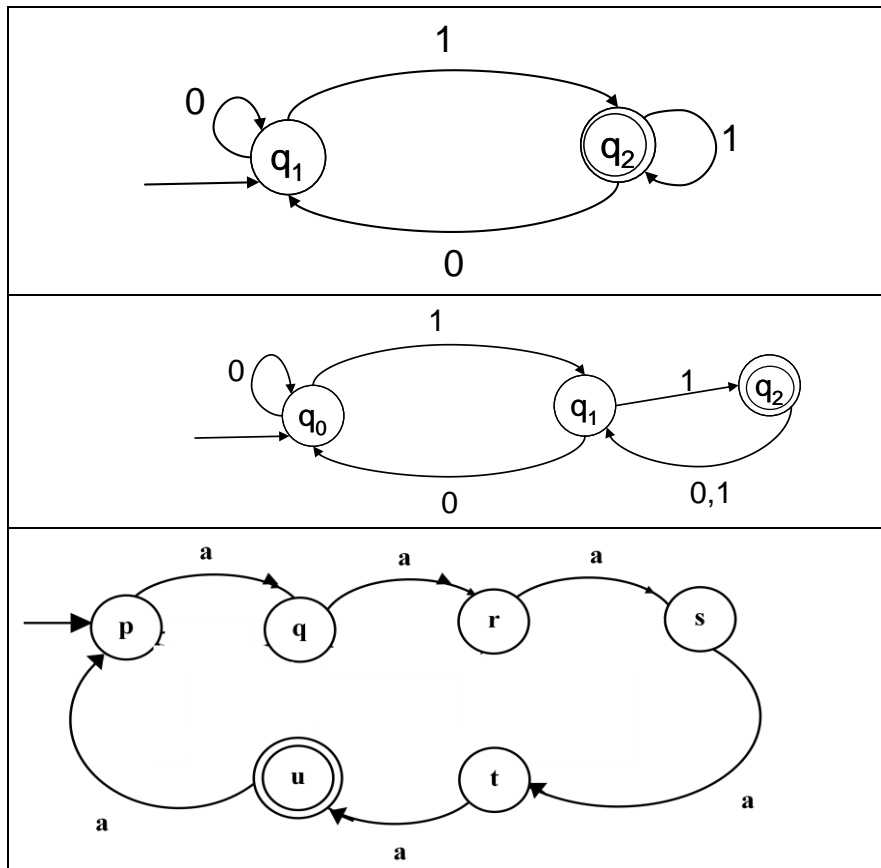
   It is required:

   a) Design a finite automata FA which accepts L.
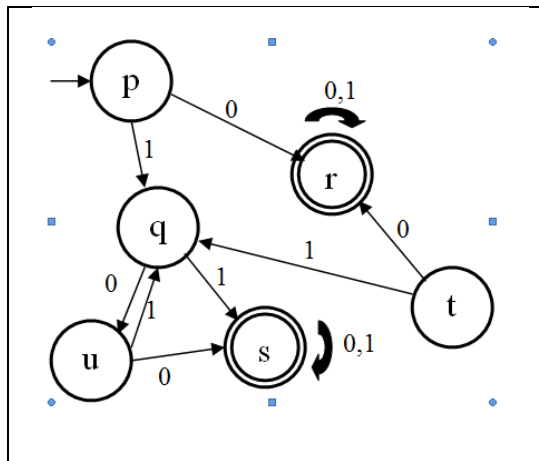   b) (**After Unit 5**) Grammar (clean and well-formed) which generates words in L.

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4. Calculate the Q/E of the following automata:
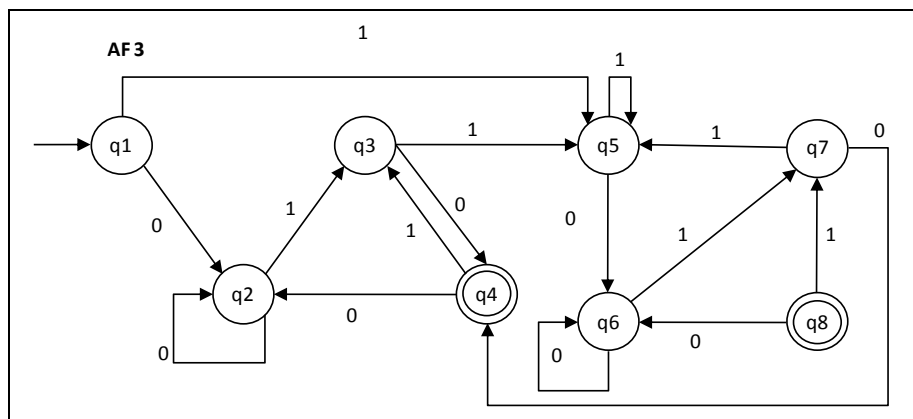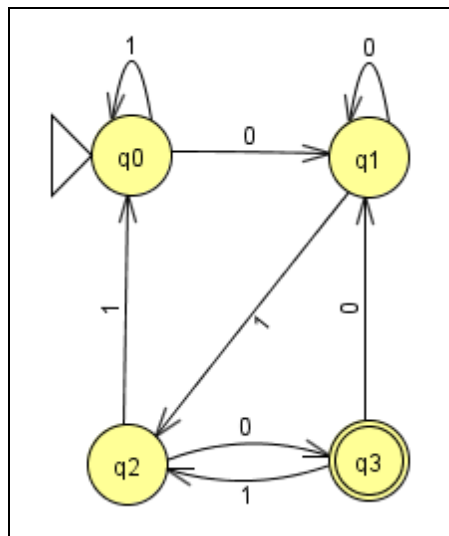

5. Given the following DFA, obtain the minimal equivalent DFA.

6. Verify if the following two DFA are equivalent (by obtaining the minimal DFA for each one of them).

7. Obtain the minimal DFA for the following DFA.

DFA_1=({a,b,c},{Q0,Q1,Q2,Q3,Q4},f,Q0,Q3)

f(Q0, a) = Q1 ; f(Q0, b) = Q2 ; f(Q0, c) = Q3
f(Q1, a) = Q2 ; f(Q1, b) = Q3 ; f(Q1, c) = Q1
f(Q2, a) = Q3 ; f(Q2, b) = Q1 ; f(Q2, c) = Q3
f(Q3, a) = Q4 ; f(Q3, b) = Q4 ; f(Q3, c) = Q4
f(Q4, a) = Q4 ; f(Q4, b) = Q4 ; f(Q4, c) = Q4

DFA_2; =({a,b,c}, {Q0,Q1,Q3,Q4,Q5,Q6,Q8},f,Q0,{Q3,Q4,Q6,Q8})

f(Q0, a) = Q4 ; f(Q0, b) = Q5 ; f(Q0, c) = Q1
f(Q1, a) = Q5 ; f(Q1, b) = Q5 ; f(Q1, c) = Q3
f(Q3, a) = Q5 ; f(Q3, b) = Q5 ; f(Q3, c) = Q5
f(Q4, a) = Q4 ; f(Q4, b) = Q8 ; f(Q4, c) = Q1
f(Q5, a) = Q5 ; f(Q5, b) = Q5 ; f(Q5, c) = Q5
f(Q6, a) = Q5 ; f(Q6, b) = Q8 ; f(Q6, c) = Q5
f(Q8, a) = Q5 ; f(Q8, b) = Q6 ; f(Q8, c) = Q5

DFA_3=({a,b,c},{Q0,Q1,Q2,Q3,Q4,Q6,Q7,Q8,Q9},f,Q0,{Q7,Q8})

f(Q0, a) = Q1 ; f(Q0, b) = Q6 ; f(Q0, c) = Q6
f(Q1, a) = Q7 ; f(Q1, b) = Q2 ; f(Q1, c) = Q6
f(Q7, a) = Q7 ; f(Q7, b) = Q2 ; f(Q7, c) = Q6
f(Q2, a) = Q6 ; f(Q2, b) = Q8 ; f(Q2, c) = Q6
f(Q8, a) = Q6 ; f(Q8, b) = Q8 ; f(Q8, c) = Q4
f(Q4, a) = Q6 ; f(Q4, b) = Q9 ; f(Q4, c) = Q3
f(Q9, a) = Q6 ; f(Q9, b) = Q8 ; f(Q9, c) = Q4
f(Q3, a) = Q6 ; f(Q3, b) = Q9 ; f(Q3, c) = Q4
f(Q6, a) = Q6 ; f(Q6, b) = Q6 ; f(Q6, c) = Q6

DFA_4=({c,f,d},{Q0,Q5,Q8,Q9,Q10,Q11,Q12},f,Q0,Q10)

f(Q0, c) = Q9 ; f(Q0, f) = Q10 ; f(Q0, d) = Q8
f(Q9, c) = Q9 ; f(Q9, f) = Q11 ; f(Q9, d) = Q12
f(Q10, c) = Q0 ; f(Q10, f) = Q8 ; f(Q10, d) = Q8
f(Q11, c) = Q11 ; f(Q11, f) = Q11 ; f(Q11, d) = Q8
f(Q12, c) = Q12 ; f(Q12, f) = Q5 ; f(Q12, d) = Q5
f(Q5, c) = Q5 ; f(Q5, f) = Q5 ; f(Q5, d) = Q8
f(Q8, c) = Q8 ; f(Q8, f) = Q8 ; f(Q8, d) = Q8