

# Formal Languages and Automata Theory

## Exercises Finite Automata

### Unit 3 – Part 3

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\* Several exercises are based on the ones proposed in the following books:

- Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón. *Teoría de autómatas y lenguajes formales*. McGraw-Hill (2007).
- Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga. *Teoría de lenguajes, gramáticas y autómatas*. Publicaciones R.A.E.C. (1997).
- Pedro Isasi, Paloma Martínez y Daniel Borrajo. *Lenguajes, Gramáticas y Autómatas. Un enfoque práctico*. Addison-Wesley (1997).

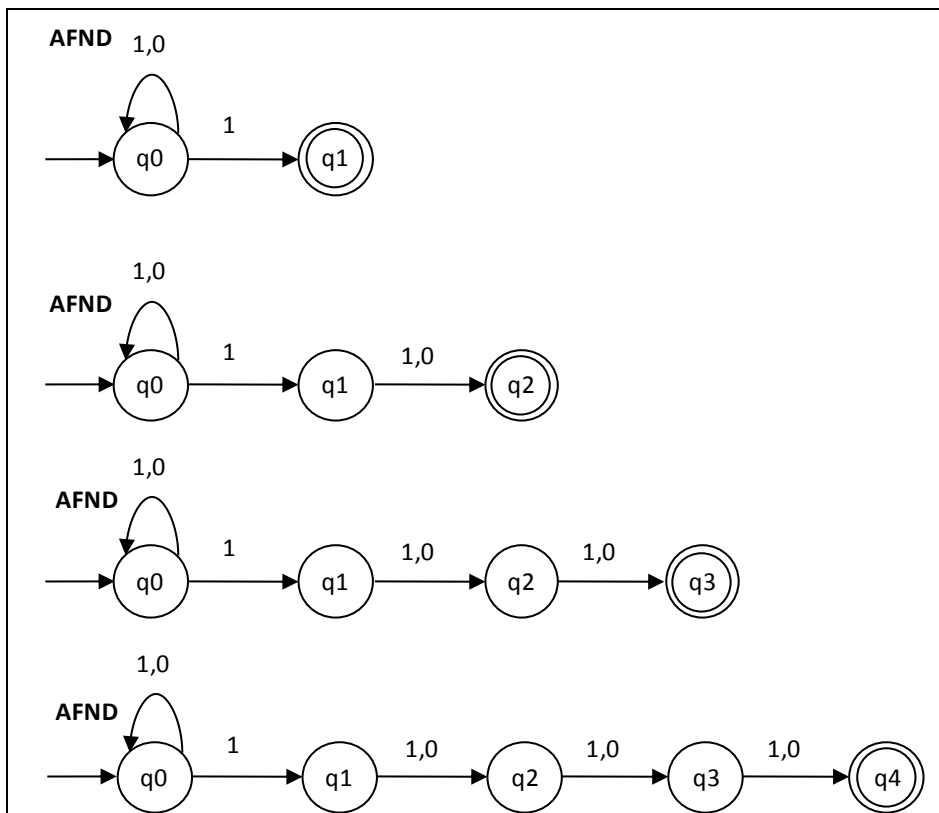


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## Formal Languages and Automata Theory

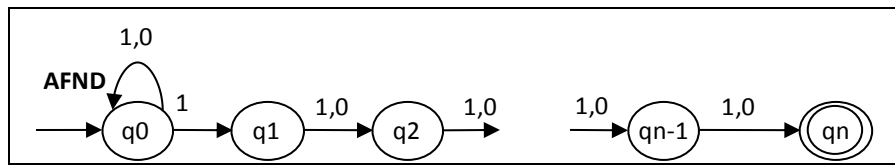
1. Indicate the graph of a NFA, only including the number of indicated states, which recognizes each one of the following languages. The alphabet is always  $\{0,1\}$ .
  - a) The language  $0^m 1^n 0^p$  ( $m \geq 0, n \geq 0, p \geq 1$ ) with only 3 states.
  - b) The language  $\{0\}$  with only 2 states.
  
2. Given the language  $(01)^n$  with  $n \geq 0$ , indicate which of the following finite automata generates this language. In addition, obtain the minimal equivalent DFA for the selected automaton.
  - a.  $FA = [\{0,1\}, \{A,B,C,F\}, f, A, \{F\}]$   
 $f(A,0)=B, f(A,\lambda)=\lambda, f(C,0)=B, f(B,1)=C, f(B,1)=\lambda$
  - b.  $FA = [\{0,1\}, \{A,B,C,F\}, f, A, \{F\}]$   
 $f(A,0)=B, f(A,\lambda)=F, f(C,0)=B, f(B,1)=C, f(B,1)=F$
  - c.  $FA = [\{0,1\}, \{A,B,C,F\}, f, A, \{F\}]$   
 $f(A,B)=0, f(A,F)=\lambda, f(C,B)=0, f(B,C)=1, f(B,F)=1$
  - d.  $FA = [\{0,1\}, \{A,B,C,F\}, f, A, \{F\}]$   
 $f(B,0)=A, f(F,\lambda)=A, f(B,0)=C, f(C,1)=B, f(F,1)=B$
  
3. Indicate the graph of the DFA which corresponds to the NFAs indicated in the following figures.



How many states will the equivalent DFA to the following NFA have?



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4. Obtain the minimal equivalent DFA for the following Non-Deterministic Finite Automata. Describe the intermediate transformations: NFA  $\rightarrow$  DFA  $\rightarrow$  Minimal DFA.

a)  
 $NFA_A = (\{a, b, c\}, \{Q_0, Q_1, Q_2, Q_3\}, f, Q_0, Q_3)$

$f(Q_0, a) = Q_1$  ;  $f(Q_0, b) = Q_2$  ;  $f(Q_0, c) = Q_3$   
 $f(Q_1, a) = Q_2$  ;  $f(Q_1, b) = Q_3$  ;  $f(Q_1, c) = Q_1$   
 $f(Q_2, a) = Q_3$  ;  $f(Q_2, b) = Q_1$  ;  $f(Q_2, c) = Q_3$

b)  
 $NFA_B = (\{a, b, c\}, \{Q_0, Q_1, Q_2, Q_3, Q_7\}, f, Q_0, Q_3)$

$f(Q_0, a) = Q_0, Q_2, Q_3$  ;  $f(Q_0, c) = Q_1$   
 $f(Q_1, c) = Q_3$  ;  
 $f(Q_2, b) = Q_2, Q_3$   
 $f(Q_7, b) = Q_2, Q_3$

c)  
 $NFA_C = (\{a, b, c\}, \{Q_0, Q_1, Q_2, Q_3, Q_4, Q_5\}, f, Q_0, Q_5)$

$f(Q_0, a) = Q_1$  ;  $f(Q_1, a) = Q_1, Q_5$   
 $f(Q_1, b) = Q_2$  ;  $f(Q_2, b) = Q_2, Q_3, Q_5$   
 $f(Q_3, b) = Q_2, Q_3$  ;  $f(Q_3, c) = Q_4$   
 $f(Q_4, b) = Q_2, Q_3$  ;  $f(Q_4, c) = Q_3$

d)  
 $NFA_D = (\{c, f, d\}, \{Q_0, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}, f, Q_0, Q_6)$

$f(Q_0, c) = Q_1, Q_4$  ;  $f(Q_0, f) = Q_2, Q_6$  ;  $f(Q_1, c) = Q_1$   
 $f(Q_1, f) = Q_3$  ;  $f(Q_1, d) = Q_4$  ;  $f(Q_2, c) = Q_0$   
 $f(Q_3, c) = Q_3$  ;  $f(Q_3, f) = Q_3$  ;  $f(Q_4, c) = Q_4$   
 $f(Q_4, f) = Q_5$  ;  $f(Q_4, d) = Q_5$  ;  $f(Q_5, c) = Q_5$   
 $f(Q_5, f) = Q_5$

5. Draw the graph of a Determinist Finite Automaton. The alphabet is always  $\{0, 1\}$ . The problem can be solved by directly designing the DFA, or by starting from the NFAs obtained in problem 4.

- a) The language  $0^m 1^n 0^p$  ( $m \geq 0, n \geq 0, p \geq 1$ ) (the NFA included only three states)
- b) The language  $\{0\}$  (the NFA included only two states)

6. Given the NFA (with lambda transitions) described by the following table, obtain the minimal equivalent DFA.

	a	b	c	$\lambda$
$\rightarrow p$	p	q		q
q	q	p,r		r
r			s	p
* s	s			



7. Indicate the graph of the DFA corresponding to the following NFA:

