Formal Languages and Automata Theory
Exercises Languages and Formal Grammars
Unit 4 – Part 1

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* Several exercises are based on the ones proposed in the following books:

- Enrique Alfonseca Cubero, Manuel Alfonseca Cubero, Roberto Moriyón Salomón. *Teoría de autómatas y lenguajes formales.* McGraw-Hill (2007).
- Manuel Alfonseca, Justo Sancho, Miguel Martínez Orga. *Teoría de lenguajes, gramáticas y autómatas.* Publicaciones R.A.E.C. (1997).
- Pedro Isasi, Paloma Martínez y Daniel Borrajo. *Lenguajes, Gramáticas y Autómatas. Un enfoque práctico.* Addison-Wesley (1997).

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1. Create a grammar to generate the following languages:
   a. { a, aa, aaa }
   b. { a, aa, aaa, aaaa, aaaaa, …)
   c. { λ, a, aa, aaa }
   d. { λ, a, aa, aaa, aaaa, aaaaa, …)

   The notation used to denote each one of the languages is:
   a. { a^n | n ∈[1, 3] }
   b. { a^n | n > 0 }
   c. { a^n | n ∈ [0, 3] }
   d. { a^n | n≥ 0 }

2. Given the grammars G=({c,d}, {S,A,T}, S, Pi}where:

P_1: S → λ | A A → AA | c	P_2: S → λ | A A → cAd | cd	P_3: S → λ | A A → AcA | c	P_4: S → cA A → d | cA | Td T → Td | d	P_5: S → λ | A A → Ad | cA | cd

   Determine the associated language.

3. Create a grammar to generate the following languages:
   a. { a^n b^n | n > 0 }
   b. { a^n b^m | n > 0, 0 < m < n }
   c. { a^n b^m | n > 0, 0 ≤ m < n }

4. Determine the type of the following grammars into the Chomsky Hierarchy. Justify your answer.
   a. G=({a,b}, {A,B,S}, S, P), P={S::=aA, A::=bB, A::=aA, A::=a, B::=λ}
   b. G=({a,b,c}, {A,B,C,S}, S, P), P={S::=aAb, S::=Ba, S::=λ, aAbC::=aAbB, aAbC::=aabC, BCc::=AaCc, BCc::=BaAbc, C::=Ca, C::=a}
   c. G=({house, garden, cat}, {S, CASTLE, FOREST, TIGER}, S, P), P={ S::=TIGER garden, S::=FOREST CASTLE, FOREST::=λ, garden CASTLE TIGER house::=garden FOREST TIGER house, cat CASTLE FOREST ::=cat FOREST house TIGER FOREST, FOREST::=TIGER house, FOREST::=garden}
   d. G=({x,y}, {C,A,B,S}, S, P), P={S::=Cx, S::=Cy, S::=By, S::=Ax, S::=x, S::=y, A::=Ax, A::=Cx, A::=x, B::=By, B::=yA, C::=xA}
   e. G=({a,b,c}, {S,B}, S, P), P={S::=abc, S::=aBSc, Ba::=aB, Bb::=bb}

5. Given the grammar G,

G=({a,b,c}, {S,A,B}, S, P), P={S::=λ, S::=aAc, A::=aA, A::=Ac, A::=B, B::=b, B::=Bb}

It is required:

a. Specify the type of G in the Chomsky Hierarchy. Justify your answer.
b. Determine the language L generated by the grammar.
c. Construct two different derivation trees for a word in L(G).
d. Verify if the following sentential forms are valid in G, and write a derivation chain to generate the valid ones.
    1) aaAcc
    2) ac
    3) ababBcc
    4) abbccc

6. Obtain the grammar corresponding to the language L={$a^n b^m c^p a^q b^n$, tal que q=p+m; n,m≥1; p≥0}

7. Obtain a grammar for the language with alphabet {a, b, c, d} that consists of all the strings that can be formed by combining these symbols except those that contain the substring"bc".

8. Obtain a grammar for the language L = {$x^n y^m z^k$ | m, n, k ≥0, k=m+n }

9. Obtain a grammar for the language {$ab^n a$ / n=0, 1, …}

10. Obtain a type-0 grammar for the language L={$a^n b^n c^n$} where n≥1.

11. Obtain the language generated by the grammar G=({0,1},{S,A,B,C},S,P), where P:

    S → BAB
    BA →BC
    CA → AAC
    CB → AAB
    A→0
    B →1

12. Design a grammar to generate natural numbers.