UNIVERSIDAD CARLOS III DE MADRID

FORMAL LANGUAGES AND AUTOMATA THEORY

GRADO EN INGENIERÍA INFORMÁTICA.

Final Exam – January 2011-12 17th January 2012

Problem 1 (1 point)

Given the alphabet Σ={a,b,c,d} and the regular expression: $\alpha = a + ab + a\,(b+c)^*$

Calculate the following derivatives:

- $D_a(\alpha)$
- $D_b(\alpha)$
- $D_c(\alpha)$
- $\delta(D_a(\alpha))$
- $\delta(D_b(\alpha))$
- $\delta(D_c(\alpha))$
- Given $\beta = D_a(\alpha)$, obtain $D_b(\beta)$

Problem 2 (2 points)

We want to design a device for a diving chronometer that prevents its involuntary use. The device includes three buttons: *a, b* and *c*.

> The button *a* moves the pointer 10 minutes forward; the button *b* moves the pointer 20 minutes forward, and the *c*, 30 minutes.

To start the chronometer it is necessary to complete 60 minutes by means of pressing 3 buttons (the same or not) from the initial state (0 minutes).

It is required:

a. Construct a Finite Automata that indicates that the pointer has been moved 60 minutes forward by pressing 3 buttons. Explain in detail.

b. Obtain a grammar corresponding to the language accepted by the previous automaton, expressed in Chomsky Normal Form. Explain in detail.

Note: It is not required to define drain states into the designed FA.

Problem 3 (2 points)

Construct a Push-Down Automaton by empty stack to recognize the language L:

L = { ZZ^{-1} / Z = (a+b+c)* } where Z^{-1} represents the opposite order of the element in the expression Z.

(Example, the words *abccba*, *baab,* and *cc* are included in the language).

1. Describe formally the PDA_E, also detailing if it is a deterministic or a non-deterministic automaton and the reasons why.
2. Explain which is the shortest word accepted and show the acceptance of a word in L with length equal to 4.
3. Transform the PDA_E into an equivalent PDA_F, also showing the acceptance of the same word of the previous section by the latter automaton.

Problem 4 (2 points)

Construct a Turing Machine to enumerate the complete set of binary numbers in the tape. A blank symbol (□) must be used as a separator between each one of the numbers. The Turing Machine begins with a 0 in the tape and completes the list of numbers from right to left, i.e., the contents in the tape will be the following successively:

□0□ → □1□0□ → □10□1□0□ → □11□10□1□0□ → …

where □ represents an empty cell in the tape.

The designed TM must be explained in detail.

Note: The operation of the TM could be considered as follows:

1. Copy the current number at the left.
2. Increase this number by one unit.
3. Repeat the process.