Duration of the exam: 45 minutes TEST + 2 hours PROBLEMS
Maximum mark: 5 POINTS

PROBLEM 1: Maximum mark 1.25 points.

Obtain formally, applying the method of the derivates of regular expressions, the right-linear Type-3 grammar (G3RL) that generates the language described by the regular expression R_0.

$$R_0 = (a + b)^* \, b \, (\lambda + a)$$

Explain, using examples of words with length 1, 2 and 3, how these words are represented by the RE and generated by the equivalent G3RL.

PROBLEM 2: Maximum mark 1.25 points.

We want to develop a system for detecting errors in the transmission of words over the alphabet $S = \{a, b, c\}$ and included in the language $L_1 = S^+$. To do this, the L_2 language is generated by adding a sequence of bits to the words in L_1 (so many bits as transmitted symbols). The added bits indicate whether the current symbol is equal to previous one or not in the words of L_1. 0 is used to indicate that the current symbol is equal to the previous one, and 1 to indicate that there is a change. The transmission of the first symbol is always corresponded with a 0. The sequence of bits will be interpreted reading it from right to left, in opposite order to the sequence of corresponding letters of the message.

Example: caabcc011010

Word in L_2											
transmitted symbol						Control bits					
c	a	a	b	c	c	0	1	1	0	1	0
1	2	3	4	5	6	7	8	9	10	11	12

(Order in the transmission is the last row, labeled 1–12)

The symbol c (number 1) is not interpreted as a change; therefore, the bit 12 is a 0
The symbol a (number 2) supposes a change with respect to the previous one; then, the bit 11 is a 1
The symbol a (number 3) is not a change with respect to the previous one; then, the bit 10 is a 0
The symbol b (number 4) supposes a change with respect to the previous one; then, the bit 9 is a 1
The symbol c (number 5) supposes a change with respect to the previous one; then, the bit 8 is a 1
The symbol c (number 6) does not change with respect to the previous one; then, the bit 7 is a 0.

Design, from the L2 grammar, an automaton to recognize this language.

PROBLEM 3: Maximum mark 1.25 points.

Design a Turing Machine to order, from lowest to highest, two natural numbers in unary code separated by the symbol #. The tape initially contains *number1#number2*, and it will provide *LowestNumber#HighestNumber*. It is mandatory to place the header in the first digit of the smallest number once the result is provided.

For example, the input 1111#11 generates the output 11#1111; and the input 11#1111 generates the output 11#1111. It is required:

a) Detailed description of the algorithm implemented by the Turing Machine.
b) Formal definition with the seven elements of the Turing Machine. Include the transition diagram (not the list, nor the table of the transition function).
c) Explain the meaning of:
   - each symbol of the alphabet of the tape not defined in this wording,
   - each one of the states and transitions, or group of states and transitions.