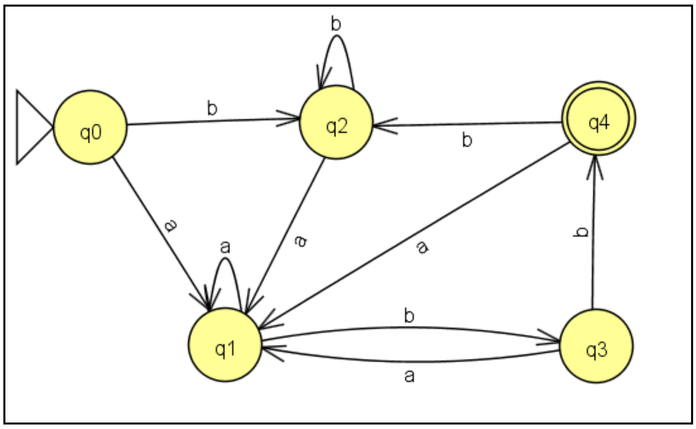
	UNIVERSIDAD CARLOS III DE MADRID **FORMAL LANGUAGES AND AUTOMATA THEORY** **COMPUTER SCIENCE DEGREE. CONTINUOUS ASSESSMENT - PARTIAL 1** **Date: 2012 October 17th** **Last name(s): ___________________________** **First name: ______________________________** **NIA: ___________________________________**

Duration: 45 minutes

1. (**3.5 points**) Given the following Finite Automaton



a. Is this a DFA or a NFA? Explain in detail.

It is a DFA given that:

- For every pair (state, input symbol) there is only one possible transition.
- There are not λ-transitions.

b. Construct the minimal equivalent DFA. Explain in detail.

We have to apply the algorithm related to equivalence classes. Q/E0 is calculated by creating a class with final states and a second class with non-final states:

C0 = {q4}

C1 = {q0, q1, q2, q3}

<u>Q/E1</u>

We take the class C1, given that the class C0 just contains one state.

f(q0, a) = q1 f(q1, a) = q1 f(q2, a) = q1 f(q3, a) = q1

f(q0, b) = q2 f(q1, b) = q3 f(q2, b) = q2 f(q3, b) = q4

Then, we can conclude that {q0, q1, q2} are equivalent states and a new class has to be created for q4:

C0 = {q4}

C1 = {q0, q1, q2}

C2= {q3}

<u>Q/E2</u>

We take again the class C1, given that the classes C0 and C2 just contain one state.

f(q0, a) = q1   f(q1, a) = q1   f(q2, a) = q1

f(q0, b) = q2   f(q1, b) = q3   f(q2, b) = q2

Then, we can conclude that {q0, q2} are equivalent states and a new class has to be created for q1:

C0 = {q4}

C1 = {q0, q2}

C2= {q3}

C3 = {q1}

<u>Q/E3</u>

We take again the class C1, given that the classes C0, C2 and C3 just contain one state.

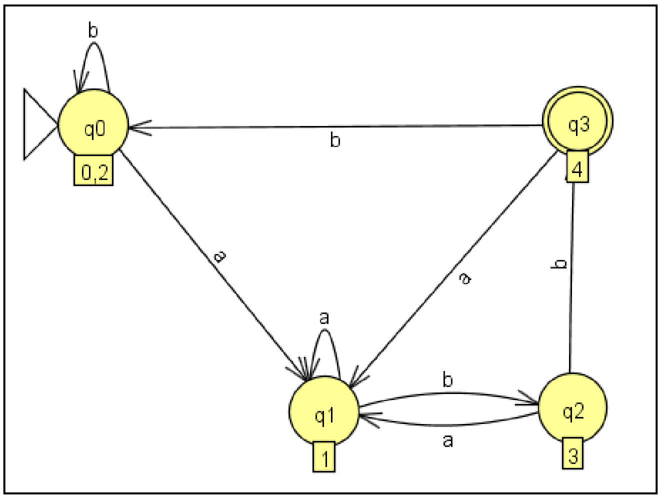
f(q0, a) = q1   f(q2, a) = q1

f(q0, b) = q2   f(q2, b) = q2

Then, we can conclude that {q0, q2} are equivalent states and:

Q /E3 = Q/E2 = Q/ E

The minimal DFA is as follows:



c. Which is the language that is recognized? Explain in detail.

The words that are accepted by this FA can be expressed by means of the following mathematical expression:

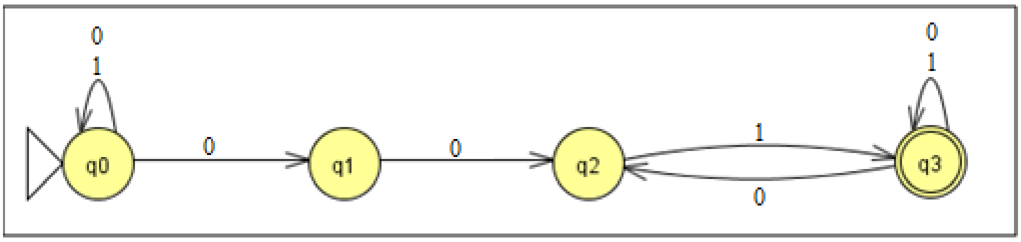
$$(b^n aa^n b(aa^n b)^n b(aa^n b(aa^n b)^n b)^n b)^n b^n aa^n b(aa^n b)^n b(aa^n b(aa^n b)^n b)^n \quad n \geq 0$$

2. (**3.5 points**) **Given the input alphabet {0,1}:**

a. Construct a NFA to recognize words having the form X0ZY where:

- **X and Y are every possible combination of 0's and/or 1's (λ included).**
- **Z is a word which consists of one or more '01' subwords (at least one).**

A valid NFA to recognize the words described by the restrictions of the problem is as follows:



b. Construct an equivalent DFA. Explain in detail.

We apply the algorithm based on λ-closure to calculate an equivalent DFA. The initial state of the equivalent DFA can be calculated as follows:

A = λ-closure{q0} = {q0}

We have the following transitions from q0 in the NFA:

q0 → {q0,q1} with *0*
q0 → q0 with *1*

This implies the following transitions from the state A of the equivalent DFA:

A → λ-closure{q0,q1} = {q0,q1} = B with *0*
A → λ-closure{q0 } = {q0} = A with *1*

Now we have to calculate transitions from the new state B in the DFA. In the initial NFA we have the following transitions from the states {q0, q1}:

q0 → {q0,q1} q1 → {q2} with *0*
q0 → q0 with *1*

This implies the following transitions from the state B of the equivalent DFA:

B → λ-closure{q0,q1} = {q0,q1,q2} = C with *0*
B → λ-closure{q0 } = {q0} = A with *1*

We have to repeat to calculate now transitions from the new state C in the DFA. In the initial NFA we have the following transitions from the states {q0, q1, q2}:

q0 → {q0,q1} q1 → {q2} with *0*
q0 → q0 q2 → q3 with *1*

This implies the following transitions from the state C of the equivalent DFA:

C → λ-closure{q0, q1, q2} = {q0,q1,q2} = C with *0*
C → λ-closure{q0, q3} = {q0,q3} = D with *1*

We repeat the process with the new state D:

D → λ-closure{q0, q1, q2, q3} = {q0, q1, q2,q3} = E with *0*
D → λ-closure{q0, q3} = {q0,q3} = D with *1*

Finally, we calculate the transitions for the state E:

E → λ-closure{q0, q1, q2, q3} = {q0, q1, q2,q3} = E with *0*
E → λ-closure{q0, q3} = {q0,q3} = D with *1*

Final states in the equivalent DFA are those containing state q3 of the NFA (i.e., D and E). The equivalent DFA is as follows: