

First Assessment (October 2010)

<i>LAST NAME(s)</i> (Capital letters)			
<i>FIRST NAME</i> (Capital letters)			
NIA		DNI	

INSTRUCTIONS FOR THE EXAM

- Read these instructions carefully before starting the exam.
- Do not forget to write your name, NIA and DNI in every answer sheet.
- Pay attention to what it is asked in each question and/or problem, given that it is not the same: to explain, to list, to describe, to define, etc., always, sometimes, at least.
- **The duration of the exam (Test + problems) is 45 minutes.**

Problem 1 (3.5 points)

1.1 Given the following grammar $G = (\{a,b\}, \{S,A,B\}, S, P)$ with P :

$S ::= Aba$

$A ::= a$

$Ab ::= AAbA \mid ABb \mid AbB$

$B ::= A \mid AB$

Prove that it is an ambiguous grammar.

To prove that the given grammar is an ambiguous grammar, we have to find at least one word accepted by the grammar by means of different parse trees or left-most derivations:

$S \rightarrow Aba \rightarrow AAbAa \rightarrow aAbAa \rightarrow aABbAa \rightarrow aaBbAa \rightarrow aaAbAa \rightarrow aaabAa \rightarrow aaabaa$

$S \rightarrow Aba \rightarrow AbBa \rightarrow ABbBa \rightarrow aBbBa \rightarrow aABbBa \rightarrow aaBbBa \rightarrow aaAbBa \rightarrow aaabBa \rightarrow aaabAa \rightarrow aaabaa$

1.2 Given the following grammar with axiom H :

$H ::= aH \mid FG$

$F ::= bF \mid bcb$

$G ::= Gb \mid c$

a. **Indicate the type of grammar in the Chomsky Hierarchy. Explain in detail.**

It is a type-2 grammar, given that in each production there is a single nonterminal symbol. It is not a type-3 grammar given that the complete set of production rules does not fulfill the properties to be a right-linear or left-linear type-3 grammar.

b. **Detail the language that it generates.**

The language that it generates can be denoted as $a^n b^m (cbc)^s$ $n \geq 0, m \geq 0, s \geq 0$

1.3. Indicate a grammar to generate palindromes over the alphabet {0, 1}. Describe which is the type of the grammar in the Chomsky Hierarchy.

A possible grammar is:

$$G = (\{0,1\}, \{S\}, S, P)$$

with P:

$$S \rightarrow 0 \mid 1 \mid \lambda \mid 0S0 \mid 1S1$$

For the same reasons described above, it is a Type-2 grammar.

Problem 2 (4 points)

Calculate a well-formed grammar equivalent to the following one, explaining in detail whether the different problems are present or not and the respective solution.

$$G = (\{0, 1, 2\}, \{S, A, B, C, D\}, S, P)$$

$$P = \{S ::= AABC, (A ::= \lambda \mid 1A0), (B ::= 1B \mid 1), (C ::= 1C1 \mid 0C0 \mid \lambda), (D ::= C)\}$$

Unnecessary symbols: The terminal symbol “2” does not appear in the right side of any production rule, so it is unnecessary. The nonterminal symbol D can not be reached from the axiom, so it is also unnecessary.

Unnecessary rules: The rule $D ::= C$ can be eliminated due to the nonterminal D has been eliminated in the previous point.

Not-generating rules: There are two not-generating rules $A ::= \lambda$ and $C ::= \lambda$.

$A ::= \lambda$ is removed by adding: $S ::= ABC \mid BC$ and $A ::= 10$

$C ::= \lambda$ is removed by adding: $S ::= AAB \mid AB \mid B$ and $C ::= 11 \mid 00$

Redenomination rules: Now, we have the production rule $S ::= B$. This production rule can be eliminated by adding the production rules $S ::= 1B \mid 1$

The resulting well-formed grammar is:

$$G = (\{0, 1\}, \{S, A, B, C\}, S, P)$$

$$P = \{S ::= AABC \mid ABC \mid BC \mid AAB \mid AB \mid 1B \mid 1, (A ::= 1A0 \mid 10), (B ::= 1B \mid 1), (C ::= 1C1 \mid 0C0 \mid 11 \mid 00)\}$$