Abstract

The main objective of this chapter is the study of anisotropic heterogeneous laminate and sandwich beams. A brief introduction to the theory of laminate are presented, including some consideration about the modelization of the failure of composite materials.

1. Introduction

The first composite materials developed for aeronautical industry were glass-fiber-reinforced plastic (GFRP) used for radomes and helicopter blades but found limited use in the main structural elements of aircrafts, such as wings and fuselages, due to its poor stiffness.

New fibers were developed in the decade of 1960s. Aramid fibers are stiffer than glass but their strength is similar and they are difficult to machine. Boron fibers were the first to provide enough strength and stiffness for primary structures. However, boron fibers were replaced by carbon-fiber-reinforced-plastics (CFRP) with similar properties but much cheaper.

The strength of CFRP is approximately three times that of aluminum alloys and, moreover, the density of CFRP is about half that of aluminum. Due to these excellent specific properties, CFRPs are widely used in aeronautical industry because a slight reduction in the aircraft weight means a great reduction in energy and fuel consumption. For this reason, the use of composite materials has been extended in the last few decades.

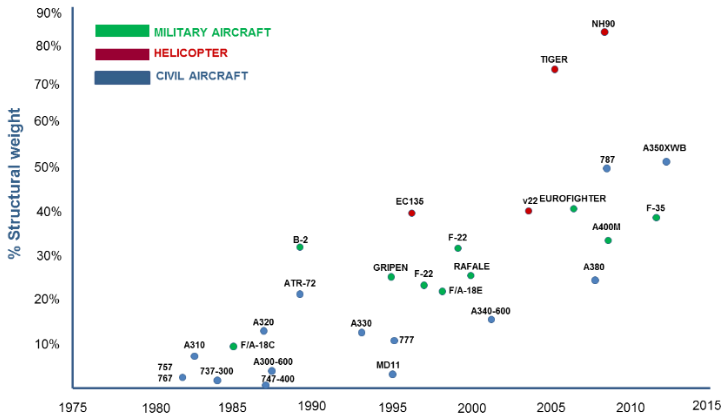

Figure 1. Evolution of the percentage of composite material in the structural weight of aircraft.

For instance, the airbus A350 XWB will be made out of 53% of composites (mainly CFRP), 19% aluminum alloy, 14% titanium alloy, 6% steel, and 8% miscellaneous.

However, the use of composite materials in structural applications requires a full knowledge of their characteristics. Composite materials consist of stiff and strong fibers (such as glass or carbon) in combination with a polymeric matrix. A sheet of fiber-reinforced plastic (FRP) is anisotropic because its properties depend on the fiber direction. The strength and the stiffness

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of the sheet is much higher in fiber direction than in perpendicular direction. Therefore, a composite laminate is composed by several plies with different fiber directions to match those of the major loads.

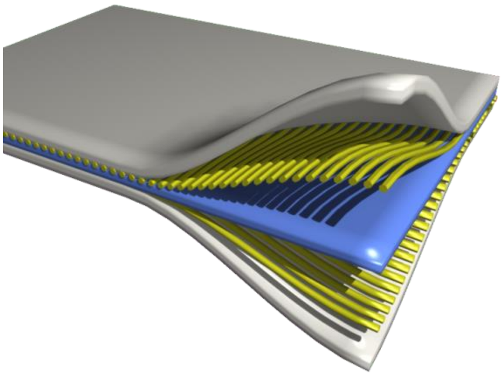

Figure 2. Sketch of a laminate

2. Laminate theory

The theoretical contents of this section can be found in the document “Laminate theory”. This document includes the explanation of the next concepts:

- Anisotropic behaviour
- Lamina stiffness matrix
- Laminate kinematic equations
- Internal forces in a laminate
- Laminate stiffness matrices
- Laminate configurations
- Laminate equivalent constants

Each ply is assumed as an orthotropic homogenous material. Ply behavior is linear elastic up to failure. In this kind of materials nine elastic properties have to be considered. Due to the low thickness of the plies a plane stress state can be assumed, and thus only five elastic properties are needed (two young modulus in direction 1 and 2, shear modulus and Poisson ratio in plane 12). Therefore the constitutive equation for a lamina can be written as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \dfrac{E_1}{1-\nu_{12}\cdot\nu_{21}} & \dfrac{\nu_{21}\cdot E_2}{1-\nu_{12}\cdot\nu_{21}} & 0 \\ \dfrac{\nu_{12}\cdot E_1}{1-\nu_{12}\cdot\nu_{21}} & \dfrac{E_2}{1-\nu_{12}\cdot\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Assuming Small displacements and Kirchhoff hypotheses the constitutive equation of a laminate can be written. With the Kirchhhoff hypotheses the displacement of each point of the laminate can be expressed as a function of the displacements and the rotations of the mid-plane.

$$u(x,y,z)=u_o(x,y)-\phi_x\cdot z \qquad \phi_x=\frac{\partial w_o}{\partial x}$$
$$v(x,y,z)=v_o(x,y)-\phi_y\cdot z \qquad \phi_y=\frac{\partial w_o}{\partial y}$$
$$w(x,y,z)=w_o(x,y)$$

Where u, v and w are the displacements along the axis xyz, and ϕx and ϕy are the rotations around x- and y-axis, respectively.

The constitutive equation of the laminate are:

$$\{N\}=[A]\cdot\{\varepsilon^o\}+[B]\cdot\{k\}$$

$$\{M\}=[B]\cdot\{\varepsilon^o\}+[D]\cdot\{k\}$$

Were $\{\varepsilon_o\}$ are the strains of the mid-plane and $\{k\}$ the curvatures of the laminate; [A], [B] and [D] the stiffness matrix; and $\{N\}$ and $\{M\}$ the applied forces per unit length. The applied forces are showed in Fig. 3.

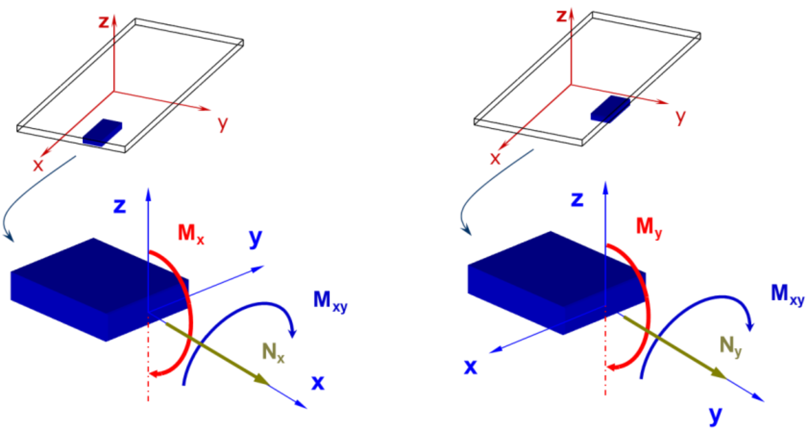

Figure 3. Applied forces per unit length in a laminate

The stiffness matrices can be calculated by the following equations:

$$[A]=\sum_{i}^{N}[\bar{Q}]_i \cdot h_i$$

$$[D]=\frac{1}{3}\cdot\sum_{i}^{N}[\bar{Q}]_i \cdot \left(Z_i^3 - Z_{i-1}^3\right)$$

$$[B]=\frac{1}{2}\cdot\sum_{i}^{N}[\bar{Q}]_i \cdot \left(Z_i^2 - Z_{i-1}^2\right)$$

The matrix [A] relates axial forces (membrane forces) with plain strains. This matrix is Independent on stacking sequence. If $A_{1S} \neq 0$ and $A_{2S} \neq 0$ there is a coupling effect between axial and shear forces. Matrix [D] relates bending moments with curvatures and it is strongly dependent on the stacking sequence. If $D_{1S} \neq 0$ and $D_{2S} \neq 0$ there is a coupling effect between bending and torsion moments. [B] is the coupling matrix and relates bending moments with curvatures. If $B_{1S} \neq 0$ and $B_{2S} \neq 0$ there is a coupling effect between axial forces and torsion moment.

Equivalent elastic properties of a laminate can be defined using laminate theory.

$$E_1^o = \frac{1}{a_{11}^*} \qquad \nu_{21}^o = -\frac{a_{21}^*}{a_{11}^*} \qquad E_1^f = \frac{1}{d_{11}^*} \qquad \nu_{21}^f = -\frac{d_{21}^*}{d_{11}^*}$$

$$E_2^o = \frac{1}{a_{22}^*} \qquad \nu_{12}^o = -\frac{a_{12}^*}{a_{22}^*} \qquad E_2^f = \frac{1}{d_{22}^*} \qquad \nu_{12}^f = -\frac{d_{12}^*}{d_{22}^*}$$

$$G_{12}^o = \frac{1}{a_{ss}^*} \qquad G_{12}^f = \frac{1}{d_{ss}^*}$$

Where:

$$\left[A^*\right]=\frac{[A]}{H} \qquad \left[D^*\right]=\frac{12\cdot[D]}{H^3}$$

$[a^*] = [A^*]^{-1}$ $[d^*] = [D^*]^{-1}$ (Only valid for symmetric laminates)

In this section several failure criteria to estimate the failure load of laminates are presented. This content is include in the document entitled "Failure criteria". These criteria are classified in three families:

- Global failure criteria
- Intralaminar failure criteria
- Delamination criteria

One exercise are included in this section to complete the theoretical concepts. In this exercise the design of a structure are proposed using a laminate and an aluminium alloy. The equations of laminate theory and the Tsai-Hill failure criterion are used.

The following readings are recommended for further study of this section.

Suggested reading (Lamina level)

- Chapter 5. Ply Mechanics
  E. J. Barbero. Introduction to Composite Materials Design. CRC Press. 2010

- Chapter 3. Ply Properties
  Daniel Gay. Composite Materials: Design and Applications, Third Edition. CRC Press. 2014

- Chapter.4 The plane-stress assumption
  M. W. Hyler. Stress analysis of fiber-reinforced composite materials. McGraw Hill, 1998

Suggested reading (Laminate level)

- Chapter 6. Macro-mechanics
  E. J. Barbero. Introduction to Composite Materials Design. CRC Press. 2010

- Chapter 7. Strength
  E. J. Barbero. Introduction to Composite Materials Design. CRC Press. 2010

- Cap.6 Classical Lamination theory: The kirchhoff Hypothesis
  M. W. Hyler. Stress analysis of fiber-reinforced composite materials. McGraw Hill, 1998

- Cap.5 Elastic behaviour of multidirectional laminates
  I.M. Daniel y O. Ishai. Engineering Mechanics of Composite Materials. Oxford University Press

- Cap 5: Classical and First-order theories of laminated composite plates
  J.N. Reddy. Mechanics of laminated composite plates. Theory and analysis. CRC Press. 1997

3. Composite Beams

The theoretical contents are included in the document entitled “composite beams”. The methods to estimate stresses and displacements in laminate beams (solid) and thin-walled beams are presented.

The stresses in a laminate beam can be estimated by the laminate theory. Using the equivalent elastic properties of a laminate, the conventional formulations of Strength of Materials (for isotropic beams) can be used to estimate the displacements and rotation of these beams.

For beam subjected to uniaxial loads: $\langle EA \rangle = \dfrac{w}{a_{11}}$

For beam subjected to bending loads: $\langle EI \rangle = \dfrac{w}{d_{11}}$

Were **w** is the width of the beam, and $\mathbf{a_{11}}$ and $\mathbf{d_{11}}$ are the first terms of the plane and bending flexibility matrices, respectively.

In this section only a brief introduction to the calculation of thin walled beams are presented. To further study of this kind of beams, a study of chapter 8 of the book "Introduction to Composite Materials" is suggested.

Two exercises that cover the theoretical concepts of this chapter are proposed, exercises 4.2 and 4.5. The first one is focused in the theoretical concepts related to laminate beams and the second in thin walled beams.

The following readings are recommended for further study of this section.

Suggested reading

- Chapter 8. Beams
  E. J. Barbero. Introduction to Composite Materials. Design. CRC Press. 2010

- Chapter. 4: Beams, columns and Rods of Composite Materials
  J.R. Vinson y R.L. Sierakowski. The Behaviour of Structures Composed of Composite Materials. Kluwer Academic Publishers. 2002.

- Chapter 6: One- Dimensional Analysis of Laminated Plates
  J.N. Reddy. Mechanics of laminated composite plates. Theory and analysis. CRC Press. 1997

4. Sandwich Structures

In some application sandwich structures can be used. This kind of structures can be defined as shows in Fig.4, by the combination of two thin plates of high stiffness with a thick plate of a material of low density. These structures have high strength and stiffness under bending moments.

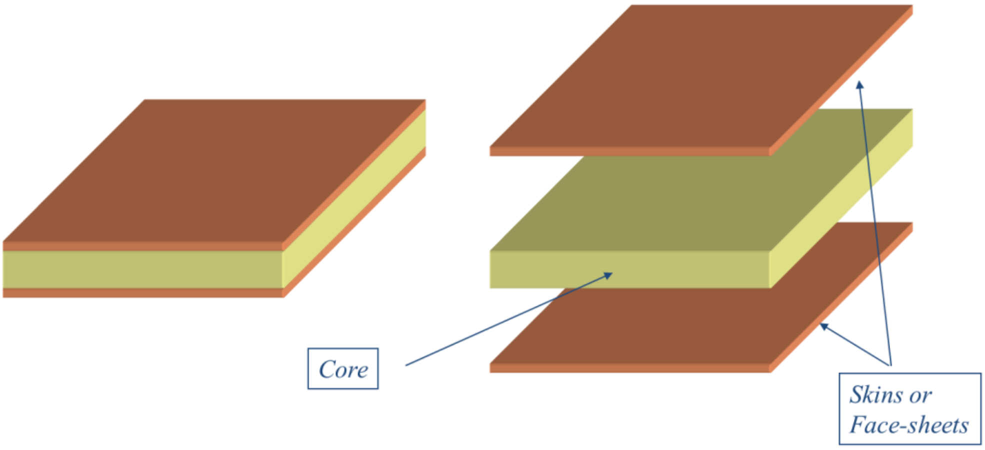

Figure 4. Definition of a sandwich structure

A sandwich structure can be modelled as a laminate made from three plies of different materials. Usually two additional simplification are made:

- The axial force is only supported by the skins (only normal stress appear in the skins).
- The shear stress is constant in the core.

One exercise are included in this section to complete the theoretical concepts

Finally, chapter 4 includes an auto-evaluation exercise. The students must use this exercise to check if they have a deep understanding of the main concepts of this chapter. This exercise includes different questions that cover all the theory explained in chapter 4.

The following readings are recommended for further study of this section.

Suggested reading

- Chapter 3. DERIVATION OF THE GOVERNING EQUATIONS FOR SANDWICH PLATES
  Jack R. Vinson. The Behavior of Sandwich Structures of Isotropic and Composite Materials. CRC Press. 1999.

- Chapter 4. Sandwich structures.
  Daniel Gay. Composite Materials: Design and Applications, Third Edition. CRC Press. 2014

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5. References

- Daniel Gay. Composite Materials: Design and Applications, Third Edition. CRC Press. 2014
- E. J. Barbero. Introduction to Composite Materials. Design. CRC Press. 2010
- J.R. Vinson y R.L. Sierakowski. The Behaviour of Structures Composed of Composite Materials. Kluwer Academic Publishers. 2002.
- Jack R. Vinson. The Behavior of Sandwich Structures of Isotropic and Composite Materials. CRC Press. 1999.
- J.N. Reddy. Mechanics of laminated composite plates. Theory and analysis. CRC Press. 1997
- M. W. Hyler. Stress analysis of fiber-reinforced composite materials. McGraw Hill. 1998.
- I.M. Daniel and O. Ishai. Engineering Mechanics of Composite Materials. Oxford University Press.