CHAPTER 3

AUTO EVALUATION EXERCISE

The deflection of a square plate of side **a**, which supports a lateral load represented by the function q(x, y) is given by:

$$w(x,y) = w_0 \cos\frac{\pi x}{a}\cos\frac{3\pi y}{a}$$

where **x** and **y** are referred to axes whose origin coincides with the centre of the plate and $\mathbf{w_0}$ is the deflection at the centre. If the flexural rigidity of the plate is **D** and Poisson's ratio is ν determine:

a. The loading function q(x,y)
b. The support conditions of the plate
c. The bending moments at the centre of the plate.

Ans.

a) Using the equilibrium equation for the plate:
$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{D}$, the following expression for the loading function is obtained:

$$q(x,y) = \frac{100D\pi^4 w_0 \mathrm{Cos}[\frac{\pi x}{a}]\mathrm{Cos}[\frac{3\pi y}{a}]}{a^4}$$

b) The expression for the Mx bending moment is:

$$M_x(x,y) = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) = \frac{D\pi^2 w_0(1+9\nu)Cos[\frac{\pi x}{a}]Cos[\frac{3\pi y}{a}]}{a^2}$$

Then, the following conditions hold at the plate edges:

$$w\left(\frac{a}{2},0\right) = w\left(-\frac{a}{2},0\right) = w\left(0,\frac{a}{2}\right) = w\left(0,-\frac{a}{2}\right) = 0$$

$$M_x\left(\frac{a}{2},0\right) = M_x\left(-\frac{a}{2},0\right) = M_x\left(0,\frac{a}{2}\right) = M_x\left(0,-\frac{a}{2}\right) = 0$$

Therefore, these conditions correspond to a plate with four simply supported edges

c) The Mx bending moment at the centre of the plate is:

$$M_x(0,0) = \frac{D\pi^2 \mathrm{w}_0(9+\nu)}{a^2}$$

The expression for the My bending moment is:

$$M_y(x,y) = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) = \frac{D\pi^2 w_0(1+9\nu)Cos[\frac{\pi x}{a}]Cos[\frac{3\pi y}{a}]}{a^2}$$

Therefore, the My bending moment at the centre of the plate is:

$$M_y(0,0) = \frac{D\pi^2 \mathrm{w}_0(9+\nu)}{a^2}$$