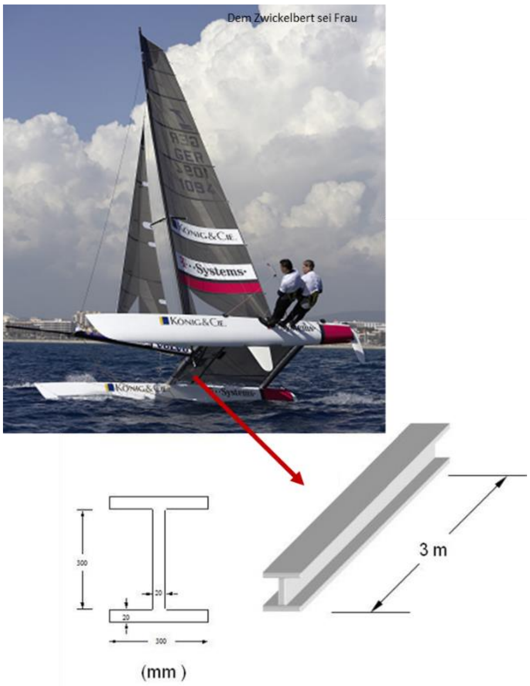
CHAPTER 4

AUTO EVALUATION EXERCISE

A catamaran is being designed using composite materials. The union between the two hulls is done via a 3 meters long beam with a double T section. The beam is manufactured by a symmetrical laminate composed by the same number of plies at 0 °, 45 ° and -45 °.



Modelling the structure as a single-span beam supported at its ends and subjected to a uniformly distributed load:

1. Determine the maximum stresses and the beam sections in which they appear.
2. Assuming that the bending moment would be supported solely by the flanges, propose a model to express this moment as a pair of axial forces acting on each flange.
3. Calculate the strain distribution in the upper flange.
4. Estimate the stresses that appear on the plies at 0 °of the laminate.
5. Considering exclusively the plies at 0 °, determine the load that produces the failure.

DATA:

Material: Kevlar/epoxy

Density: 1350 kg/m^3

X = 1410 MPa

X'=280 MPa

Y = 28 MPa

S = 45 MPa

E_1 = 85 MPa

E_2 = 5,6 GPa

ν_{21} = 0,34

G_{12} = 2,1 GPa

Y' = 141 MPa

Stiffness matrix of the lamina $[Q] = \begin{bmatrix} 85{,}65 & 1{,}92 & 0 \\ 1{,}92 & 5{,}64 & 0 \\ 0 & 0 & 2{,}10 \end{bmatrix}$ GPa

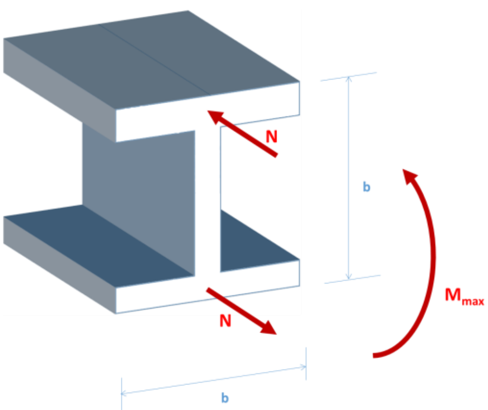
Ans:

1. Using the Equilibrium equations:

Maximum shear force (In the supports): $\frac{qL}{2}$ (in N)

Maximum bending moment (in the middle cross-section): $\frac{qL^2}{8}$ (in N/m)

2. Assuming that the bending moment would be supported solely by the flanges, the bending moment can be represented by a pair of forces.



$$N = \frac{M}{b}$$

According to laminate theory

$N_x = -\frac{qL^2}{8b^2}$ (top flange)

$N_x = \frac{qL^2}{8b^2}$ (bottom flange)

3. Laminate $[0/+45/-45]_{nS}$

Plane stiffness matrix:

$$[A] = 2 \cdot ([\bar{Q}]_0 + [\bar{Q}]_{45} + [\bar{Q}]_{-45}) \cdot n \cdot h$$

$$[A] = \frac{e}{3}\begin{bmatrix} 137.416 & 45.280 & 0 \\ 45.280 & 57.406 & 0 \\ 0 & 0 & 45.828 \end{bmatrix} \text{(GN/m)}$$

$$e = 6nh$$

Strain in the flanges:

$$\{\mathcal{E}\} = \begin{Bmatrix} 1.475 \cdot 10^{-9} \\ -1.163 \cdot 10^{-9} \\ 0 \end{Bmatrix} N_x$$

4. Stress in plies at 0º

$$\{\sigma\} = [\bar{Q}]_0 \cdot \{\varepsilon\}$$

$$\{\sigma\} = \begin{Bmatrix} 124.1 \\ -3.79 \\ 0 \end{Bmatrix} N_x \text{ (Pa)}$$

5. Appliyng the maximum stress criterion:

q_{max} = 180 kN/m