Solutions to exercises on Fundamentals

J. Daniel García Sánchez (coordinator)
David Expósito Singh
Javier García Blas

Computer Architecture
ARCOS Group
Computer Science and Engineering Department
University Carlos III of Madrid

1. Exam exercises

Exercise 1 *June 2015.*

An application allows to process a very high resolution image where a certain fraction can be parallelized, while other part must be run sequentially.

Assume that there is no upper bound to the number of processes that can be used for the parallelization. We need to get a global speedup of at least 10 for the parallel version.

Express the fraction of code that must be parallelized as a function of the parallelism degree (number of processes running in parallel).

Solution 1

$$S = \frac{1}{(1-F) + \frac{F}{n}}$$

$$S \times \left((1-F) + \frac{F}{n}\right) = 1$$

$$S - S \times F + \frac{S \times F}{n} = 1$$

$$n \times S - n \times S \times F + S \times F = n$$

$$n \times S\breve{}n = n \times S \times F - S \times F$$

$$S \times F \times (n-1) = n \times (S-1)$$

$$F = \frac{n \times (S-1)}{S \times (n-1)}$$

For the case of $S = 10$

$$F = \frac{n \times (10-1)}{10 \times (n-1)} = \frac{9 \times n}{10 \times n - 10}$$

As F must be less or equal than 1:

$$F \leq 1$$

$$\frac{9 \times n}{10 \times n - 10} \leq 1$$

$$9 \times n \leq 10 \times n - 10$$

$$n \geq 10$$

Exercise 2 *January 2014.*

A single core computer runs a finance risk assessment application. The application is computation intensive (computations take 90 % of total execution time). The remaining 10 % is devoted for waiting for I/O operations.

The time that the applications is running computation instructions is divided into 75 % for floating point operations and 25 % for other instructions. Executing a floating point operation requires, on average, 12 CPI. The rest of instructions require, on average, 4 CPI.

Migrating this application to a new machine is being evaluated. The following alternatives are considered. In both alternatives, there is no improvement in the I/O time for disk.

- **Alternative A**: A single-core process with clock frequency 50 % higher than the original machine, where floating point instructions require 10 % more cycles per instruction and the rest of instructions require 25 % less cycles per instruction.
- **Alternative B**: A four-core process with a clock frequency 50 % lower than the original machine, where floating point instructions require 20 % less cycles per instruction and the rest of instructions the same number of cycles per instructions.

State the following questions giving an appropriate reasoning:

1. Which is the global speedup/slowdown for the application in case A?
2. Which is the global speedup/slowdown for the application in case B, assuming that the computation part can be fully parallelized while the I/O part cannot be improved at all?

Solution 2

Time for executing instructions in the original computer will be:

$$T_{orig} = 0{,}75 \times 12 \times IC \times P + 0{,}25 \times 4 \times IC \times P = (9 + 1) \times IC \times P \tag{1}$$

Alternative A Time for executing instructions in computer A will be:

$$T_A = (0{,}75 \times (1,1 \times 12) + 0{,}25 \times (1{,}25 \times 4)) \times IC \times \frac{P}{1{,}5} = \frac{(9{,}9 + 1{,}25) \times IC \times P}{1{,}5} = \frac{11{,}15}{1{,}5} \times IC * P \tag{2}$$

$$T_A = 7{,}433 \times IC \times P \tag{3}$$

Speedup due to instructions will be:

$$S_A^I = \frac{T_{orig}}{T_A} = \frac{10}{7{,}433} = 1{,}345 \tag{4}$$

Applying Amdahl's law, the global speedup will be:

$$S_A = \frac{1}{0{,}1 + \frac{0{,}9}{1{,}345}} = 1{,}3 \tag{5}$$

Alternative B In this case, assuming complete parallelization for the computing part, we may consider the number of instructions to be executed in each core is one fourth from the original.

$$T_B = (0{,}75 \times 0{,}8 \times 12 + 0{,}25 \times 4) \times \frac{IC}{4} \times \frac{P}{0{,}5} = (7{,}2 + 1) \times \frac{2}{4} \times IC \times P = \tag{6}$$

$$T_B = 4{,}1 \times IC \times P \tag{7}$$

The speedup due to instructions will be:

$$S_B^I = \frac{T_{orig}}{T_B} = \frac{10}{4{,}1} = 2{,}439 \tag{8}$$

Applying Amdahl's law, the global speedup will be:

$$S_B = \frac{1}{0{,}1 + \frac{0{,}9}{2{,}439}} = 2{,}132 \tag{9}$$

Exercise 3 *October 2013.*

In your organization, there is an application with the following characteristics:

- The application spends 80 % of time executing instructions and 20 % of the remaining time waiting for disk operations.
- The time that the application spends executing instructions is distributed in 20 % for floating point instruction (which require 8 CPI) and 80 % for the rest of instructions (which require 6 CPI).

You are evaluating the migration to a new machine in which instructions require 25 % more of CPI but whose clock frequency is double.

What is the global speedup for the application?

Solution 3

$$T_{inst}(orig) = 0{,}2 \cdot 8 \cdot IC \cdot P + 0{,}8 \cdot 6 \cdot IC \cdot P = (1{,}6 + 4{,}8) \cdot IC \cdot P = 6{,}4 \cdot IC \cdot P$$

$$T_{inst}(mej) = 0{,}2 \cdot 10 \cdot IC \cdot \frac{P}{2} + 0{,}8 \cdot 7{,}5 IC \cdot \frac{P}{2} = (1 + 3) \cdot IC \cdot P = 4 \cdot IC \cdot P$$

$$S_{inst} = \frac{6{,}4}{4} = 1{,}6$$

$$S = \frac{1}{0{,}2 + \frac{0{,}8}{1{,}6}} = \frac{1}{0{,}2 + 0{,}5} = \frac{1}{0{,}7} = 1{,}42$$

Exercise 4 *October 2013.*

Given a processor consuming a dynamic power **P**, there are two alternatives for reducing the consumed dynamic power.

1. Decrease voltage to the half keeping the same value for clock frequency.
2. Decrease frequency to the half keeping the same value for the voltage.

Reason which of them gets a higher dynamic power reduction and quantify the value for this reduction.

Solution 4

If we decrease voltage to the half we have:

$$\frac{P_{nuevo}}{P_{ant}} = \frac{(V \cdot 0{,}5)^2 \cdot f}{V^2 \cdot f} = \frac{0{,}25 \cdot V^2 \cdot f}{V^2 \cdot f} = 0{,}25$$

If we decrease frequency to the half we have:

$$\frac{P_{nuevo}}{P_{ant}} = \frac{V^2 \cdot f \cdot 0{,}5}{V^2 \cdot f} = 0{,}5$$

Consequently, we get a higher reduction for the dynamic power in the case of reducing voltage to the half.

Exercise 5 *January 2013.*

Given a computer working continuously and without failures until a time $t = 20$ months.

1. Define what is the reliability of a system. What is the reliability of that computer for $t = 0$, $t = 24$ y $t = 30$? (given t measured in months).
2. If during two years of usage, the computer has two failures whose repair times are 3,85 and 4,15 days respectively, what would be its availability?

Solution 5

Reliability (R) is the probability that the system lifetime (X) is greater than a given time t. That is $P[X > t]$. Reliability is a function of time and fulfills the following:

$$R(t = 0) = 1$$

$$R(t = \infty) = 0$$

$$R(0 < t < \infty) \in [0, 1]$$

In the example:

$$R(t = 0) = 1, R(t = 24) = 0, R(t = 30) = 0$$

Availability (A) is the fraction of time that the system is working correctly, or free of errors. Formally, the average availability will be:

$$A = \frac{MTTF}{MTTF + MTTR}$$

Where $MTTF$ is the average time between failures and $MTTR$ is the average time to repair. Consequently, the availability of the system in the example:

$$MTTR = 3{,}85 + 4{,}15 = 8$$

$$MTTF = 365 * 2 - 8 = 722$$

$$A = \frac{722}{730} = 0,9890 \rightarrow 98{,}9\,\%$$