



APPLIED DIFFERENTIAL CALCULUS
LECTURE 4: Boundary value problems.
PROBLEMS

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Problem 1

Find the solution of the following boundary value problem:

$$y'' + 5y = 0; \quad y(0) = 1; \quad y(\pi) = 0.$$

Problem 2

Solve the boundary value problem:

$$y'' + y = 0; \quad y(0) = 1; \quad y(\pi) = \alpha,$$

where α is a given real number.

Problem 3

Find the solution of the following boundary value problem:

$$y'' + 5y = 0; \quad y(0) = 0; \quad y(\pi) = 0.$$

Problem 4

Solve the boundary value problem:

$$y'' + y = 0; \quad y(0) = 0; \quad y(\pi) = 0.$$

Problem 5 Given the following boundary value problem:

$$X'' + \lambda X = 0; \quad X'(0) = 0, \quad X'(\pi/3) = 0,$$

find the values of the constant parameter $\lambda \geq 0$ yielding non-zero solutions.

Problem 6 Solve the following boundary value problem:

$$X'' + \lambda X = 0; \quad X'(0) = 0, \quad X(1) = 0,$$

and find the values of the constant parameter $\lambda > 0$ yielding non-zero solutions.

Problem 7 Solve the following boundary value problem:

$$X'' + \lambda X = 0; \quad X(0) = 0, \quad X'(1) = 0,$$

and find the values of the constant parameter $\lambda > 0$ yielding non-zero solutions.

Problem 8 Solve the following boundary value problem:

$$X'' + \lambda X = 0; \quad X(0) = 0, \quad X(1) = 0,$$

and find the values of the constant parameter $\lambda > 0$ yielding non-zero solutions.

Problem 9 Find the eigenvalues and eigenfunctions of the following problem and expand $f(x) = e^x$ in terms of the eigenfunctions:

$$\begin{aligned} u'' + \lambda u &= 0, & x \in (0, 1), \\ u(0) &= 0, & u'(1) = 0. \end{aligned}$$
