



APPLIED DIFFERENTIAL CALCULUS
LECTURE 7: Fourier series and separation of variables: Laplace equation.
PROBLEMS

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Problem 1

Find the solution of the following boundary value problem:

$$\begin{aligned}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= 0, \quad x \in (0, 1), \quad y \in (0, 1) \\ u(0, y) &= 0, \quad u(1, y) = 0, \quad y \in [0, 1], \\ \frac{\partial u}{\partial y}(x, 0) &= 1, \quad u(x, 1) = 0, \quad x \in [0, 1].\end{aligned}$$

Problem 2 Find the solution of the following boundary value problem:

$$\begin{aligned}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= 0, \quad x \in (0, 1), \quad y \in (0, 1) \\ u(0, y) &= 0, \quad u(1, y) = \sin \frac{\pi y}{2}, \quad y \in [0, 1], \\ \frac{\partial u}{\partial y}(x, 0) &= 0, \quad \frac{\partial u}{\partial y}(x, 1) = 0, \quad x \in [0, 1].\end{aligned}$$

Problem 3 Find the solution of the following boundary value problem:

$$\begin{aligned}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= 0, \quad x \in (0, 1), \quad y \in (0, 1) \\ u(0, y) &= 0, \quad u(1, y) = 0, \quad y \in [0, 1], \\ \frac{\partial u}{\partial y}(x, 0) &= e^{2x}, \quad u(x, 1) = 0, \quad x \in [0, 1].\end{aligned}$$
