Applied Differential Calculus

Self-Assessment: Test 1

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Problem 1 Consider the first-order differential equation

$$(3kx^2 y + e^y) + (x^3 + kxe^y - 2yk^2)\, y' = 0\,,$$

where k is a real parameter.

  (a) Find the value of k that makes the equation exact.

  (b) Solve the equation for that value of k.

Problem 2 Solve the following first-order differential equation

$$y = (x + \sqrt{xy})\, y'$$

for $x > 0$, together with the initial condition $y(1) = 1$.

Problem 3 Consider the first-order differential equation

$$\left(\sin^2 x + 4xye^{xy^2} - x\right) y' + 2y\sin x \cos x + 2y^2 e^{xy^2} - y = 0\,.$$

  (a) Classify the equation, justifying your answer.

  (b) Find the general solution of the equation.

Problem 4 Given the Ordinary Differential Equation (ODE):

$$-5x^4 + 2y + xy' = 0 \quad \text{with} \quad x > 0,$$

i) Classify this ODE.

ii) Solve the ODE with initial condition $y(1) = 2$.