Applied Differential Calculus

Self-assessment: Test 2

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Problem 1 A particular solution of $y'' + 2y' + y = e^{-t} \ln t$, for $t > 0$, is given by

$$y_p(t) = \frac{1}{2} t^2 e^{-t} \ln t - \frac{3}{4} t^2 e^{-t} .$$

Using this information, calculate the general solution of the following second-order differential equation

$$y'' + 2y' + y = e^{-t} \ln t + (1 - t)\, e^{-t} .$$

Problem 2 Solve the following second-order differential equation

$$y'' + 3y' + 2y = \sin(e^{x})$$

and verify the obtained result.

Problem 3 Consider the following differential equation

$$(x-1)\,y'' + y' \;=\; 0$$

and assume that the solution is given by the power series $y(x) = \sum_{n=0}^{\infty} a_n\, x^n$.

(a) Find the recurrence relation satisfied by the coefficients a_n .

(b) Apply the initial conditions $y(0) = 0$ and $y'(0) = 1$, then write the first three non-zero terms of the used power series.

Problem 4 Solve the initial value problem : $xy'' - (2x+1)y' + (x+1)y = 0$, with $y(1) = 0$, $y'(1) = e$ knowing that, obviously, $y = e^x$ is a solution of the homogeneous ODE.
