
# Applied Differential Calculus

**Self-assessment: Test 3**

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**Problem 1** Solve the following system of first-order differential equations

$$\begin{cases} x_1' = -4x_1 + 2x_2\,, & x_1(0) = 2 \\ x_2' = -\dfrac{5}{2}x_1 + 2x_2\,, & x_2(0) = -3\,. \end{cases}$$

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**Problem 2** (i) Prove that the differential equation $y'' - 6y' + 13y = 0$ is equivalent to the following system of first-order differential equations

$$X' = \begin{bmatrix} 0 & 1 \\ -13 & 6 \end{bmatrix} X\,, \quad \text{where} \quad X = X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}.$$

(ii) Solve the system in (i) knowing that

$$X(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

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**Problem 3** Given the ODE: $y'' + 2y' - 3y = 0$, write it as a system of two first order ODEs, classify the equilibrium point $(0, 0)$ and draw its phase portrait, indicating explicitly any real eigendirections that may be relevant.

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**Problem 4** Solve the following system of first order linear ODEs with initial condition $x_1(0) = 1$, $x_2(0) = 1$:

$$\begin{cases} x_1' = 2x_1 - 3x_2 \\ x_2' = 6x_1 - 4x_2 \end{cases}$$

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